QCD EVOLUTION and Density Matrix POSITIVITY

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Outline

- QCD factorization: positivity constraints for non-perturbative inputs
- Constraints vs QCD: evolution equations as master equations
- Positivity constraints for spin-dependent distributions
- Convexity: exploring large and small x behaviour
- Positivity for BFKL equations and its non-linear generalizations
- Irreversibility and scale arrow(s)
- Conclusions

QCD factorization

- Separation of short (pQCD) and large (npQCD) distances
- npQCD ingredients – parton distributions
- Well-defined as a hadronic matrix elements of quark fields (DGLAP and ERBL) factorization or
- Impact factors (BFKL factorization)

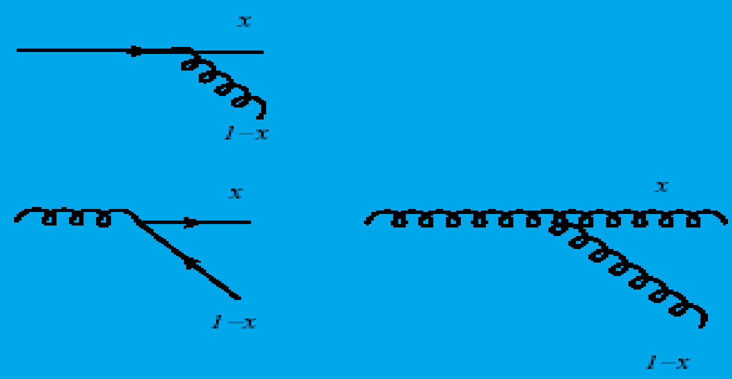
Parton distributions and density matrix positivity

- Inclusive processes – parton distributions are the density matrices of quarks and gluons in hadrons
- Density matrix positivity $\lambda_i \geq 0$
- Counterpart of unitarity $\sum_i \lambda_i = 1$
- More important for more complicated density matrices
- Simplest example – positivity of spin-averaged parton distributions

Positivity and QCD evolution

- Compatibility of NP and PQCD ingredients
- Hint for more elaborated positivity constraints
- Key point – evolution equations as master equation

DGLAP equation as master equation



Integration over transverse momentum - collinear $log\mu^2 = t$ ("time"!) . Its coefficient - recovered by differentiation.

$$\frac{dq(x)}{dt} = \frac{\alpha_s}{2\pi}[\int_x^1 dy\frac{q(y)}{y}P(\frac{x}{y}) - q(x)\int_0^1 P(z)dz]. \quad (1)$$

Probabilistic interpretation stressed already by Gribov and Lipatov (gain-loss equation) and later by Altarelli and Parisi

DGLAP equation as master equation -II

- Master form – simple transformation of variables to get the loss term from 0 to x:

$$\frac{dq(x)}{dt} = \frac{\alpha_s}{2\pi}[\int_x^1 dy \frac{q(y)}{y} P(\frac{x}{y}) - \int_0^x dy \frac{q(x)}{x} P(\frac{y}{x})]$$

- This is a master equation

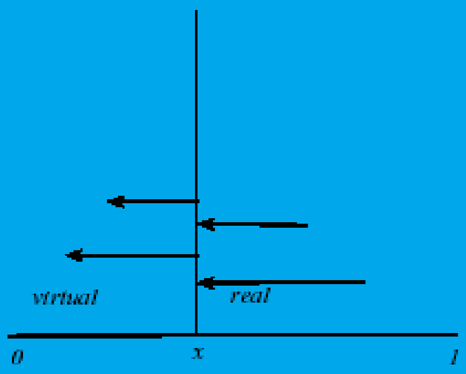
$$\frac{dq(x)}{dt} = \int_0^1 dy (w(y \to x) q(y) - w(x \to y) q(x))$$

with

$$w(y \to x) = \frac{\alpha_s}{2\pi} P(\frac{x}{y}) \frac{\theta(y > x)}{y}$$

DGLAP as master equation - III

Directed evolution:



The cancellation of the IR divergencies between the contributions of the real and virtual gluons emission is coming from the equality of in- and out- flows for $y \sim x$, following from the continuity condition.

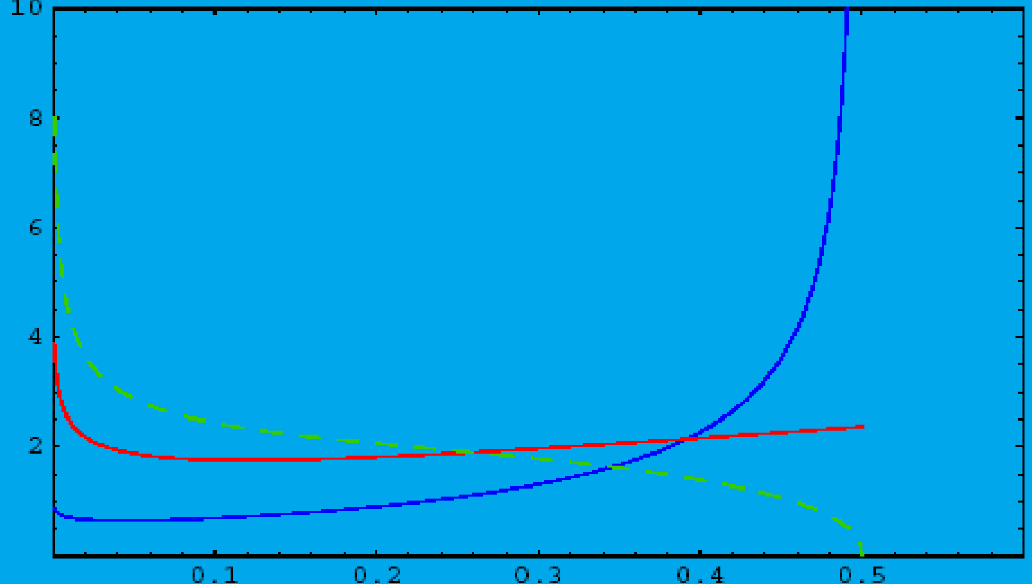
DGLAP as master equation - IV

Also, the conservation of the vector current

$$\int_0^1 dx \frac{dq(x)}{dt} = \int_0^1 dxdy(w(y \to x)q(y) - w(x \to y)q(x)) = 0, \quad (6)$$

comes from the integration of an antisymmetric function in a symmetric region.

Flow in the direction of low x - decrease at large x and growth of small x. Numerical example:



DGLAP – spin - dependent case

Positivity in spin-dependent case seen from the coupled equations for helicities

$$\frac{dq_+(x)}{dt} = \frac{\alpha_s}{2\pi}(P_{++}(\frac{x}{y}) \otimes q_+(y) + P_{+-}(\frac{x}{y}) \otimes q_-(y)),$$
$$\frac{dq_-(x)}{dt} = \frac{\alpha_s}{2\pi}(P_{+-}(\frac{x}{y}) \otimes q_+(y) + P_{++}(\frac{x}{y}) \otimes q_-(y)). \quad (12)$$

as, P_{++}, P_{+-} are positive, decrease term only in diagonal in helicity.

Practically, equations for $\Delta q = q_+ - q_-$, which are decoupled from q, are considered. Examples (LSS): $s-$quarks, Gluons (grow due to axial anomaly, $\alpha_S \int dx \Delta G = const$), total quark spin $\Delta\Sigma$.

Quark-gluon mixing

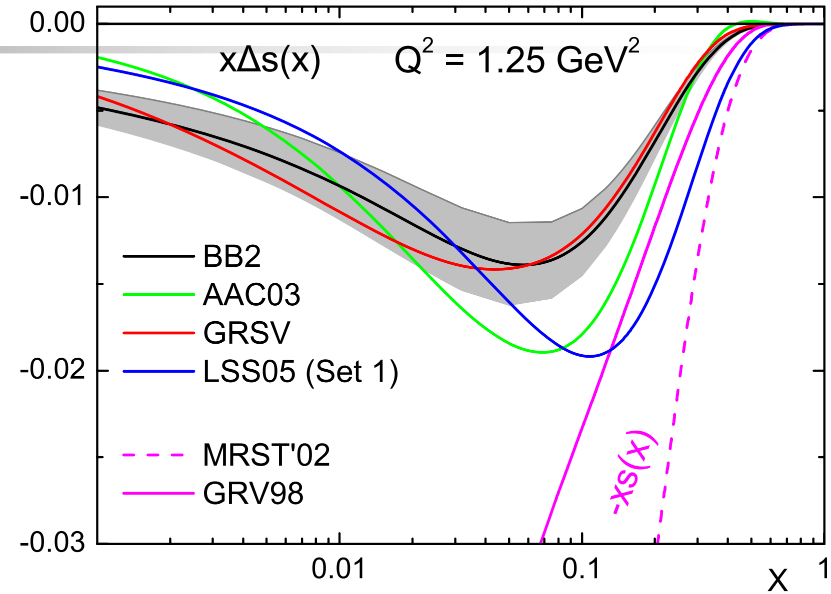
Quark-gluon mixing:

$$
\begin{aligned}
\frac{dq_+(x)}{dt} &= \frac{\alpha_s}{2\pi}(P^{qq}_{++}(\frac{x}{y}) \otimes q_+(y) + P^{qq}_{+-}(\frac{x}{y}) \otimes q_-(y)) \\
&\quad + P^{qG}_{++}(\frac{x}{y}) \otimes G_+(y) + P^{qG}_{+-}(\frac{x}{y}) \otimes G_-(y), \\
\frac{dq_-(x)}{dt} &= \frac{\alpha_s}{2\pi}(P^{qq}_{+-}(\frac{x}{y}) \otimes q_+(y) + P^{qq}_{++}(\frac{x}{y}) \otimes q_-(y)) \\
&\quad + P^{qG}_{+-}(\frac{x}{y}) \otimes G_+(y) + P^{qG}_{++}(\frac{x}{y}) \otimes G_-(y), \\
\frac{dG_+(x)}{dt} &= \frac{\alpha_s}{2\pi}(P^{Gq}_{++}(\frac{x}{y}) \otimes q_+(y) + P^{Gq}_{+-}(\frac{x}{y}) \otimes q_-(y) \\
&\quad + P^{GG}_{++}(\frac{x}{y}) \otimes G_+(y) + P^{GG}_{+-}(\frac{x}{y}) \otimes G_-(y)), \\
\frac{dG_-(x)}{dt} &= \frac{\alpha_s}{2\pi}(P^{Gq}_{+-}(\frac{x}{y}) \otimes q_+(y) + P^{Gq}_{++}(\frac{x}{y}) \otimes q_-(y) \\
&\quad + P^{GG}_{+-}(\frac{x}{y}) \otimes G_+(y) + P^{GG}_{++}(\frac{x}{y}) \otimes G_-(y)). \qquad (13)
\end{aligned}
$$

Contain the decoupled equations for spin-averaged and spin-dependent terms: positivity is preserved.

Impact of positivity constraints on $x\Delta s(x, Q^2)$

GRSV: Glück et al., hep-ph/0011215
BB: Blümlein, Böttcher, hep-ph/0203155
AAC: Goto et. al., hep-ph/0312112
LSS'05: Leader at al., hep-ph/0503140
JHEP 0506:033,2005



$$| x\Delta f(x, Q_0^2) | \leq xf(x, Q_0^2)_{GRV}$$

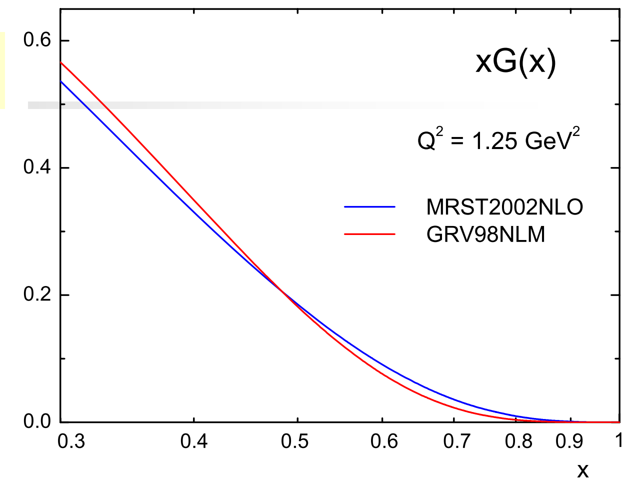
$$| x\Delta f(x, Q_0^2) |_{LSS} \leq xf(x, Q_0^2)_{MRST02}$$

GRSV, BB and AAC have used the **GRV** **unpolarized** PD for constraining their PPD, while LSS have used those of **MRST'02**.

As a result, $x|\Delta s(x)|$ (LSS) for $x > 0.1$ is **larger** than the magnitude of the polarized strange sea densities obtained by the other groups.

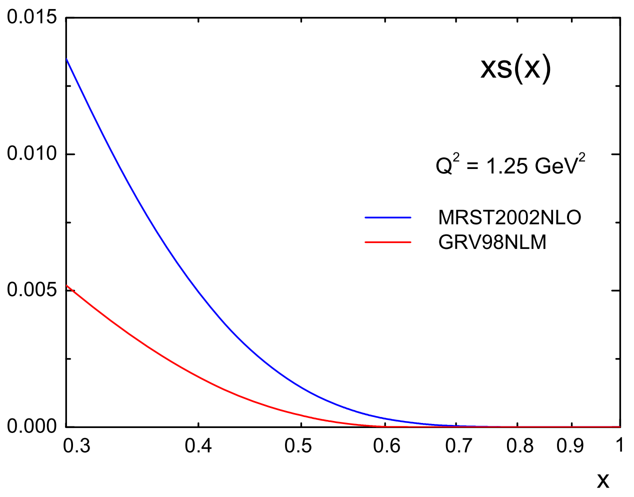
Role of unpolarized PD in determining PPD at large x

- At large x the unpolarized GRV and MRST'02 **gluons** are practically **the same**, while $xs(x)_{GRV}$ is much smaller than that of MRST'02.



- For the adequate determination of $x\Delta s$ and $x\Delta G$ at large x, the role of the corresponding **unpolarized** PD is very important.

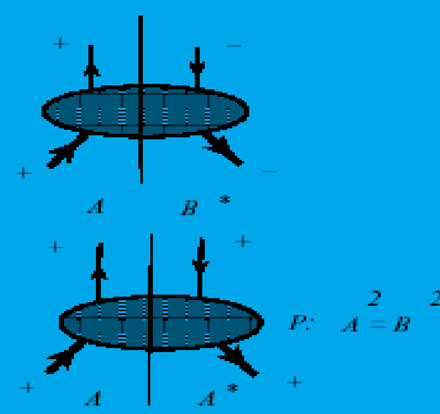
- Usually the sets of unpolarized PD are extracted from the data **in the DIS region** using cuts in Q^2 and W^2 chosen in order to minimize the higher twist effects.



- The latter have to be determined with good accuracy at large x in the **preasymptotic** (Q^2, W^2) region too.

Chiral-odd parton distributions

Non-trivial example of positivity constraint: Soffer inequality for transversity: bound for interference term of the type $A^2 + B^2 > 2AB$: $|h_1| < q+$



Consider

$$
\begin{aligned}
Q_+(x) &= q_+(x) + h_1(x), \\
Q_-(x) &= q_+(x) - h_1(x). \qquad (14)
\end{aligned}
$$

Soffer inequality in QCD

Due to Soffer inequality, both these distributions are positive at some point Q_0^2, and the evolution equations for the NS case take the form

$$\frac{dQ_+(x)}{dt} = \frac{\alpha_s}{2\pi}(P^Q_{++}(\frac{x}{y}) \otimes Q_+(y) + P^Q_{+-}(\frac{x}{y}) \otimes Q_-(y)),$$
$$\frac{dQ_-(x)}{dt} = \frac{\alpha_s}{2\pi}(P^Q_{+-}(\frac{x}{y}) \otimes Q_+(y) + P^Q_{++}(\frac{x}{y}) \otimes Q_-(y)), \tag{15}$$

where the 'super'-kernels at LO are just

$$P^Q_{++}(z) \equiv \frac{[P^{(0)}_{qq}(z) + P^{(0)}_h(z)]}{2} = \frac{C_F}{2}[\frac{(1+z)^2}{(1-z)_+} + 3\delta(1-z)],$$
$$P^Q_{+-}(z) \equiv \frac{[P^{(0)}_{qq}(z) - P^{(0)}_h(z)]}{2} = \frac{C_F}{2}(1-z), \tag{16}$$

where

$$P_+(z) = P(z) - \delta(1-z)\int_0^1 P(y)dy, \tag{17}$$

Decrease- again in diagonal term only.

Short-distance expansion of probability kernel

Transition probability peaked for close points (IR divergence!) - Kramers-Moyal expansion. Convenient in new variables $t = \ln x$ $x/y \to t_1 - t_2$ -translational invariance. Master equation holds for function $f(x) = xq(x)$, as its integral over t conserves

$$\int_0^1 dx q(x) = \int_{-\infty}^0 dt f(t) \tag{7}$$

Another simplificati0on - extend integrals to the who;e t axis $(0 < x < \infty)$ to exclude boundary contributions. As soon as initial distribution stays at $0 < x < 1$, directed transitions guarantee that for the evolved functions. Moments of transition probability in KM expansions coincide with the derivatives of anomalous dimension at $n = 1$

$$\int_0^1 dx \ln^n x P(x) = \frac{d}{dn} \int_0^1 P(x) x^{n-1} dx|_{n=1} \tag{8}$$

Short-distance (Kramers-Moyal) expansion

Differential form of DGLAP:

$$\frac{dq(x)}{dt} = -\frac{1}{x}\exp\left(\frac{\partial}{\partial \ln(1/x)}\frac{d}{dn}\right)xq(x)\gamma(n)|_{n=1} \qquad (9)$$

Fokker-Planck (diffusion) approximation - only two terms are kept

$$\frac{dq(x)}{dt} = \frac{1}{x}\left(v\frac{\partial(xq(x))}{\partial \ln(1/x)} + D\frac{\partial^2(xq(x))}{\partial \ln^2(1/x)}\right)$$

$$v = 5/4 - \pi^2/3 = -2.03987; D = -9/8 + 2\zeta(3) = 1.27911 \qquad (10)$$

Drift towards the region of small x and diffusion. Positivity of the diffusion coefficient (also for evolution of function with arbitrary weight x^m)- convexity of the anomalous dimension curve.

Gluons

Gluons. Pure gluodynamics: energy-momentum conservation - master equation for function $xG(x)$. Similar procedure:

$$\frac{dG(x)}{dt} = \frac{1}{x^2}\left(v\frac{\partial(x^2G(x))}{\partial \ln(1/x)} + D\frac{\partial^2(x^2G(x))}{\partial \ln^2(1/x)}\right)$$

$$v = 65/144 - \pi^2/6 = -1.19355;$$

$$D = -395/1728 + \zeta(3) = 0.973469 \qquad (11)$$

The simplest manifestation of irreversibility - positivity

Kinetic form - provides the positivity of distribution if it is positive at lower reference point: But! Not in the back direction - irreversibility.

Simple reason - like for positivity of particles number. Formal - loss term proportional to the function itself.

For moments - convexity of anomalous dimension curve - Nachtmann - now simple physical reason seen - positivity of diffusion coefficient

Preservation of convexity in x-space by DGLAP evolution

- Differential DGLAP operator $\exp(\frac{\partial}{\partial \ln(1/x)}\frac{d}{dn})$
- Commutes with $\frac{\partial}{\partial \ln(1/x)}$
- Derivatives of q(x) evolve in the same way as q(x)!
- Preservation of monotonicity and convexity of pdf’s – may explain the success of the parametrization $x^{-a}(1-x)^{b}$

Convexity and small/large x behaviour

- Small-x: even if intercept is determined by very small x, the convex parametrization leads to its validity at moderate x
- Recent analysis of Ermolaev (developing the approach of Kirschner and Lipatov) – numerically compatible with SLAC data (Soffer, OT)
- PDF in Ermolaev's approach – DIFFERENT from standard – singular in 1/x terms subtracted
- Large x – the similar situation – possibility to apply the analysis of very large x asymptotic for analysis of Bloom-Gilman duality

Positivity for BFKL equation

- "Generalized" master equation – gain and loss probabilities differ

$$\frac{df(x,t)}{dt} = \int dy (w_+(y \to x) f(y) - w_-(x \to y) f(x))$$

- Contains effect of "fission" in addition to diffusion and drift. Is it possible to seprate these effects?
- Consider $f_\sigma(x,t) = f(x,t)\sigma(x)$ so that

$$w_+(x,y) \to w_\sigma(x,y) = w_+(x,y)\sigma(x)/\sigma(y)$$

Separating fission and diffusion in BFKL equation

- Master equation for relative

$$\bar{f}_\sigma(x,t) = f_\sigma(x,t)/ < f_\sigma(t) >, < f_\sigma(t) >= \int dx f_\sigma(x,t)$$

- If $\frac{d < f_\sigma(t) >}{dt} = \lambda_\sigma < f_\sigma(t) >$, master equation is STANDARD

$$\frac{d\bar{f}_\sigma(x,t)}{dt} = \int dy (w_\sigma(y \to x) f_\sigma(y) - w_\sigma(x \to y) \bar{f}_\sigma(x))$$

Separating diffusion and fission – eigenvalue problem

- Fission coefficient –

$$\lambda_\sigma = \int dx(w_\sigma(y \to x) - w_-(y \to x))$$

- Defines the weight function

$$\int dx w_+(y \to x)\sigma(x) = (\lambda_\sigma + \int dx w_-(y \to x))\sigma(y)$$

- BFKL – continious spectrum with varying diffusion, fission and drift and invariant $\lambda_0 = \lambda_\sigma - \frac{v_\sigma^2}{4D_\sigma}$ (=4 ln 2) - minimal diffusion and fission, zero drift

Positivity for BFKL

- Local loss term – positivity of unintegrated gluon distribution
- Caldwell plot – signalling the decrease of ugd
- Non - linear local loss term – also preserves the positivity
- Any saturating non-linearity -> travelling Kolmogorov-Pokrovsky-Piskunov waves (Peschansky et al.)
- Ambiguity of separation between diffusion and fission -> varying coupling constant case
- Positivity for spin-dependent case -> non-trivial relations between BFKL, BKP, $\ln^2 x$ resummation...

Scale arrows

- Master equation – preserves positivity in ONE direction (opposite direction – negative probabilities) – “scale arrows”
- DGLAP and BFKL – different directions of scale arrows. BFKL- pointing to IR, DGLAP – to UV

Comparing longitudinal and transverse arrows

Reasons for irreversibility : possibility to consider evolution equations as a kind of Wilson RG.

DGLAP and ERBL -

$$f(x) = \int_0^{Q^2} dk_T^2 f(x, k_T^2) \tag{18}$$

Wilson RG in the MOMENTUM space. Transverse ”scale arrow” is directed to UV.

BFKL - ”time” is running towards small momenta - Longitudinal ”scale arrow” is directed to IR, one may expect Wilson procedure in the coordinate space. And, indeed, it exists (Jalilian-Marian, Kovner, Leonidov, Weigert)!

Comparing arrows-II

Why longitudinal and transverse scale arrow are directed in a different way?

Possible simple explanation: angular ordering - angular arrow projection to the perpendicular axis - different directions.

Relation to irreversibility in CFT (Zamolodchikov)?

Bogoliubov RG for single moment (DGLAP) - UV fixed point is necessary, but not sufficient ingredient of irreversibility.

Set of ALL moments - convexity of anomalous dimension.

Conclusions

- Positivity of density matrix – number of constraints for npQCD inputs(also GPD's, relation of different twists, fragmentation and fracture functions etc)
- Compatibility with QCD evolution – master form of evolution equation
- Scale arrows – possible relation with Wilson RG and conformal invariance

Spin dependent DIS

- Two invariant tensors

$$W_A^{\mu\nu} = \frac{-i\epsilon^{\mu\nu\alpha\beta}}{pq} q_\beta (g_1(x,Q^2) s_\alpha + g_2(x,Q^2)(s_\alpha - p_\alpha \frac{sq}{pq})) = \frac{-i\epsilon^{\mu\nu\alpha\beta}}{pq} q_\beta ((g_1(x,Q^2) + g_2(x,Q^2)) s_\alpha - g_2(x,Q^2) p_\alpha \frac{sq}{pq})$$

- Only the one proportional to $g_T = g_1 + g_2$ contributes for transverse (appears in Born approximation of PT)
- Both contribute for longitudinal
- Apperance of g_1 only for longitudinal case –result of the definition for coefficients to match the helicity formalism

Generalized GDH sum rule

- Define the integral – scales asymptotically as $\frac{1}{Q^2}$

$$I_1(Q^2) = \frac{2M^2}{Q^2}\Gamma_1(Q^2) \equiv \frac{2M^2}{Q^2}\int_0^1 g_1(x, Q^2)dx\ .$$

- At real photon limit (elastic contribution subtracted) – $\frac{1}{Q^2}+\frac{1}{Q^4}+\ldots$ - Gerasimov-Drell-Hearn SR

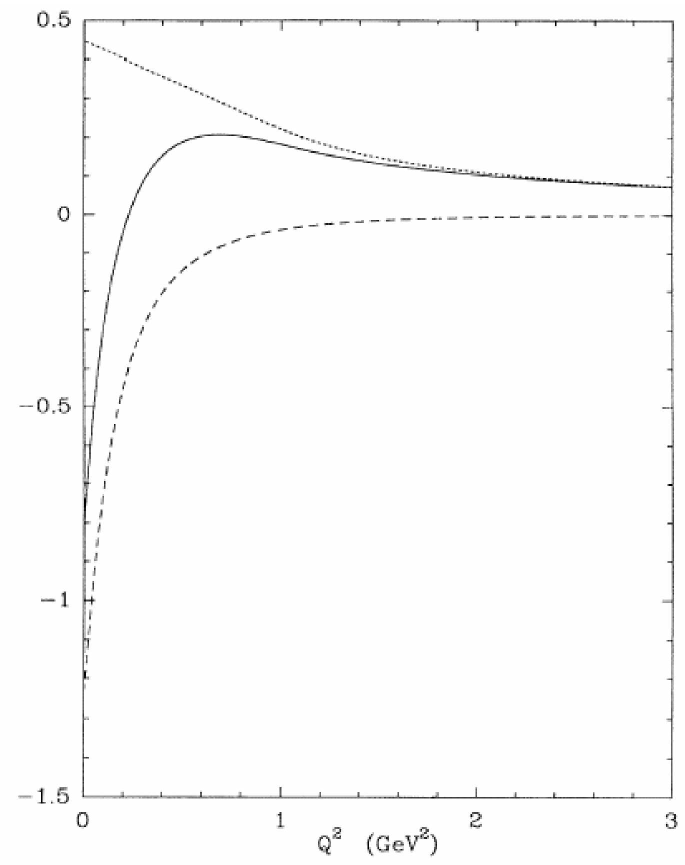
$$I_1(0) = -\frac{\mu_A^2}{4}$$

- Proton- dramatic sign change at low Q!

Decomposition of $g_1 = g_T - g_2$ (J. Soffer, OT '92)

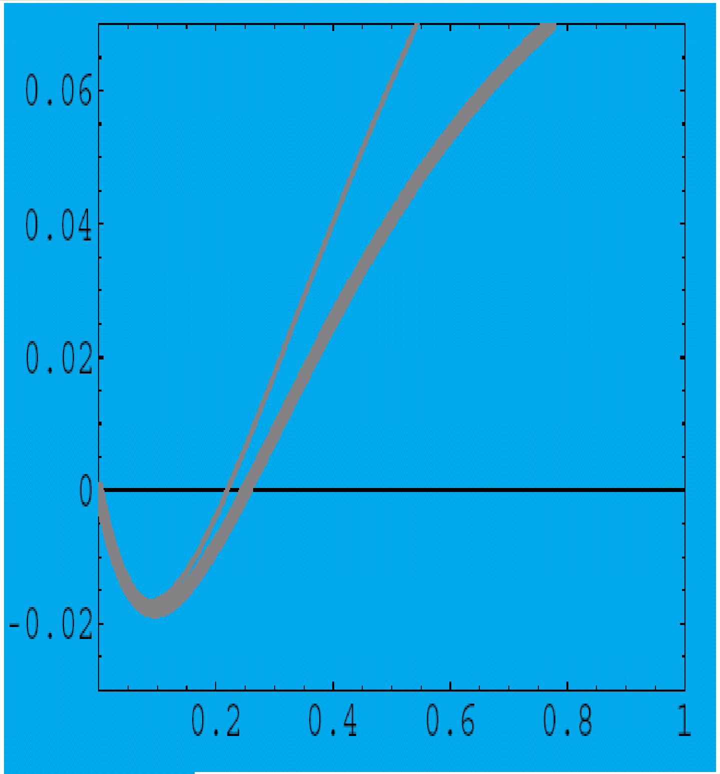
- Supported by the fact that
  $$I_T(0) = +\frac{\mu_A}{4}$$
- Linear in μ_A, quadratic term from g_2
- Natural candidate for NP, like SV(talks!)Z QCD SR analysis – hope to get low energy theorem via WI (C.f. pion F.F. – Radyushkin) - smooth model
- For g_2 -strong Q – dependence due to Burkhardt-Cottingham SR

$$I_2(Q^2) = \frac{1}{4}\,\mu G_M(Q^2)[\mu G_M(Q^2) - G_E(Q^2)]$$



Models for g_T :proton

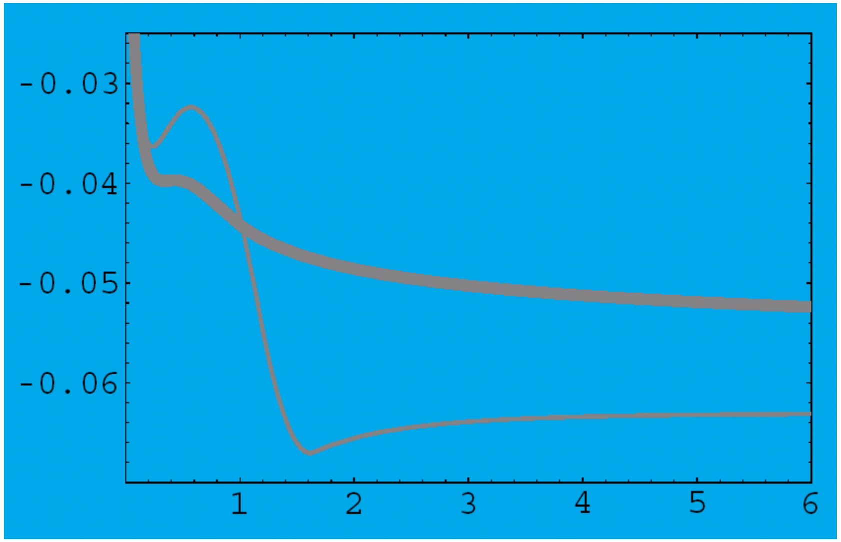
- Simplest - linear extrapolation – PREDICTION (10 years prior to the data) of low (0.2 GeV) crossing point
- Accurate JLAB data – require model account for PQCD/HT correction – matching of chiral and HT expansion
- HT – values predicted from QCD SR (Balitsky, Braun, Kolesnichenko)
- Rather close to the data, like the resonance approach of Burkert and Ioffe (the latter similarity to be discussed below)

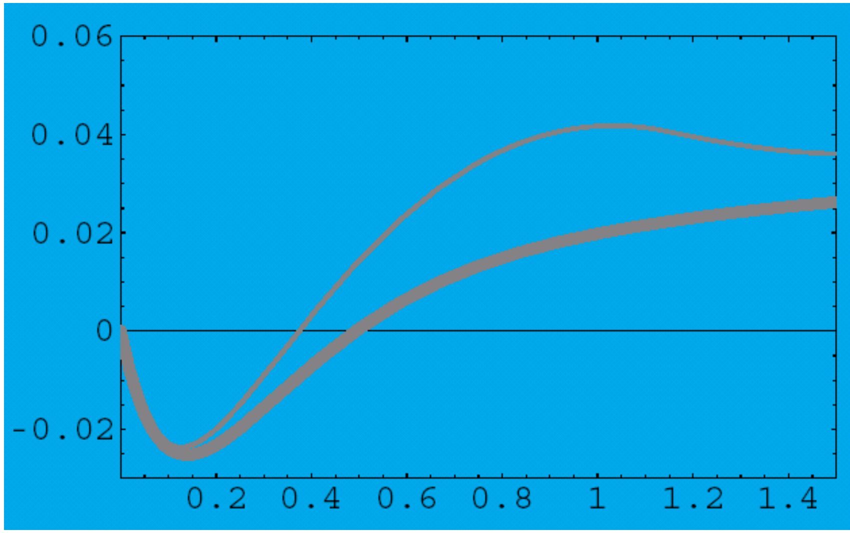


$\Gamma_1(Q^2)$ For Proton

Models for g_T :neutron and deuteron

- Access to the neutron – via the (p-n) difference – linear in μ_A ->
- Deuteron – refining the model eliminates the structures





$\Gamma_1(Q^2)$ for neutron and deuteron

Duality for GDH – resonance approach

- Textbook (Ioffe, Lipatov. Khoze) explanation of proton GGDH structure – contribution of $\Delta(1232)$ dominant magnetic transition form factor
- Is it compatible with g_2 explanation?!
- Yes!– magnetic transition contributes entirely to g_2 and as a result to $g_1 = g_T - g_2$

$\Delta(1232)$ and Bloom-Gilman duality

- Observation (talks of Y. Prok, P. Solvignon, A. Fantoni): $\Delta(1232)$ violates BG duality for g_1
- Natural explanation : $\Delta(1232)$ contributes only via g_2
- For g_2 BG duality is difficult to reach: due to BCSR elastic contribution should compensate all the integral from 0 to 1 (global duality enforced by rotational invariance) while the resonqnces should just slide (talk of C. Carlson) if BG holds
- g_T -natural candidate for BG duality

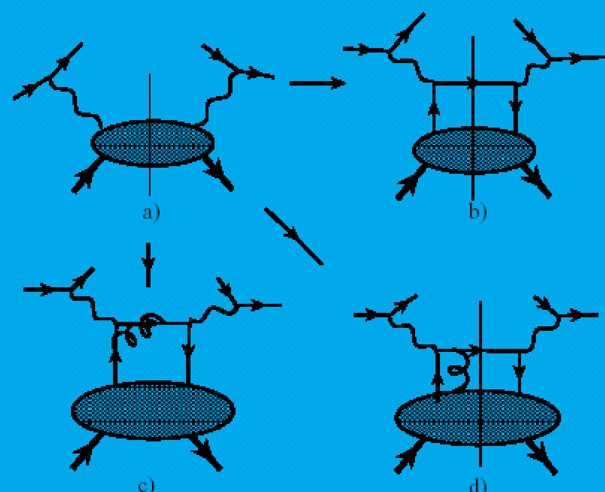
Possible implications for unpolarized

- The best cqndidqtes – structure functions protected against such strong global dependence : F2 - momentum conservation
- Positivity bound: $$A_2 < \sqrt{R(1 + A_1)/2}$$
- As soon as BG holds for A2 – positive deviations for FL and negative for F1 implied

Bloom-Gilman duality in QCD and Borel Sum Rules

- Methods of QCD SR

1. Calculate (handbag+higher twists) contribution to DIS



2. Write the (Borel) dispersion relation (with respect to $s = Q^2/(1-x)$, which is a natural scale of higher twists)

- Only 1/(1-x) - enhanced (dependent on s, rather than Q) higher twist corrections should be considered (Gardi, Kortchemsky,Ross,Tafat)

Bloom-Gilman duality in QCD and Borel Sum Rules -II

3. Take the ansatz for spectral functions which includes RESONANCE contribution below the threshold defined by DUALITY interval and leading perturbative one above that threshold.

$$\rho(s) = \theta(s - s_0)\rho^{pert}(s) + \theta(s_0 - s)\rho^{Res}(s) \qquad (1)$$

4. Put Borel parameter $M \to \infty$ (higher twists corrections disappear) and assume the finite limit of duality interval → BG duality.

Determination of the duality interval from QCD - requires the power corrections calculation.

Different view at High Twist

- Expected to be cancelled to allow for duality with leading term
- Instead - large but determine only the interval for duality with leading term
- Special role of 1/(1-x) enhanced HT

(first indications? - talks of W. Melnitchouk, D; Stamenov, A. Fantoni)

CONCLUSIONS

- Transverse polarization is described by the single invariant amplitude –advantage for duality studies.
- g_T - natural candidate for Bloom-Gilman duality and allows for good description of GGDH SR
- Methods from QCD SR are helpful, in particular BG duality may be quantitatively understood in the framework of Borel sum rules
- Large x HT corrections are important.

Single Spin Asymmetries

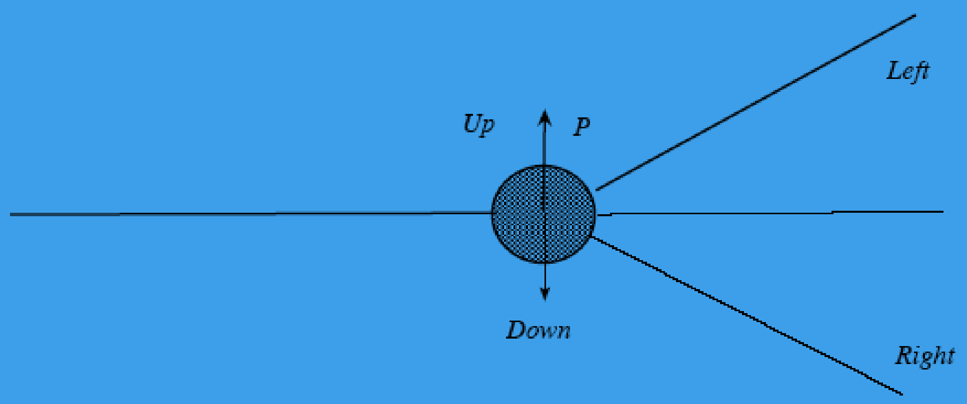
Simpler experimentally – more difficult theoretically. Main properties:

- Parity: transverse polarization
- Imaginary phase – can be seen from the imaginary i in the (quark) density matrix

Various mechanisms – various sources of phases

Non-relativistic Example

Simplest example - (non-relativistic) elastic pion-nucleon scattering $\pi\vec{N} \to \pi N$



$M = a + ib(\vec{\sigma}\vec{n})$ $\vec{n}$ is the normal to the scattering plane.

Density matrix: $\rho = \frac{1}{2}(1 + \vec{\sigma}\vec{P})$,

Differential cross-section: $d\sigma \sim 1 + A(\vec{P}\vec{n})$, $A = \frac{2Im(ab^*)}{|a|^2+|b|^2}$

Phases in QCD-I

- Perturbative (a la QED: Barut, Fronsdal

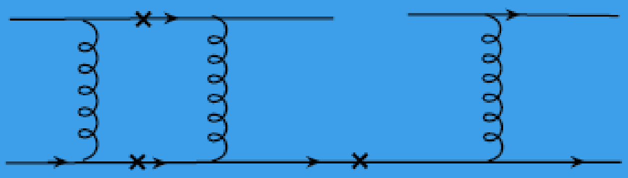
(1960), found at JLAB recently):

Kane, Pumplin, Repko (78) Efremov (78), Efremov, O.T. (80), ...

Perturbative PHASES IN QCD

QCD factorization: where to borrow imaginary parts?

Simplest way: from short distances - loops in partonic subprocess. Quarks elastic scattering (like $q - e$ scattering in DIS):

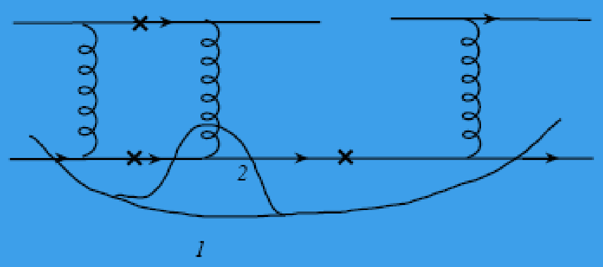


$$A \sim \frac{\alpha_S m p_T}{p_T^2 + m^2}$$

Large SSA "...contradict QCD or its applicability"

Twist 3 correlators

Escape: QCD factorization - possibility to shift the borderline between large and short distances



At short distances - Loop $\rightarrow$ Born diagram
At Large distances - quark distribution $\rightarrow$ quark-gluon correlator.
Physically - process proceeds in the external gluon field of the hadron.
Leads to the shift of α_S to non-perturbative domain AND
"Renormalization" of quark mass in the external field up to an order of hadron's one

$$\frac{\alpha_S m p_T}{p_T^2 + m^2} \rightarrow \frac{M b(x_1, x_2) p_T}{p_T^2 + M^2}$$

Further shift of phases completely to large distances - T-odd fragmentation functions. Leading twist transversity distribution - no hadron mass suppression.

Phases in QCD-II

- Distribution (Sivers, Boer)– no positive kinematic variable producing cut/phase
- Emerge only due to interaction between hard and soft parts of the process: “Effective” or “non-universal” SH interactions by physical gluons – Twist-3 :Efremov, O.T. (fermionic poles, 85); Qiu, Sterman (gluonic poles,91).
- Brodsky-Hwang-Schmidt model:the same SH interactions as twist 3 but non-suppressed by Q: Sivers – leading (twist 2)?

What is “Leading” twist?

- Practical Definition - Not suppressed as M/Q
- However – More general definition: Twist 3 may be suppresses

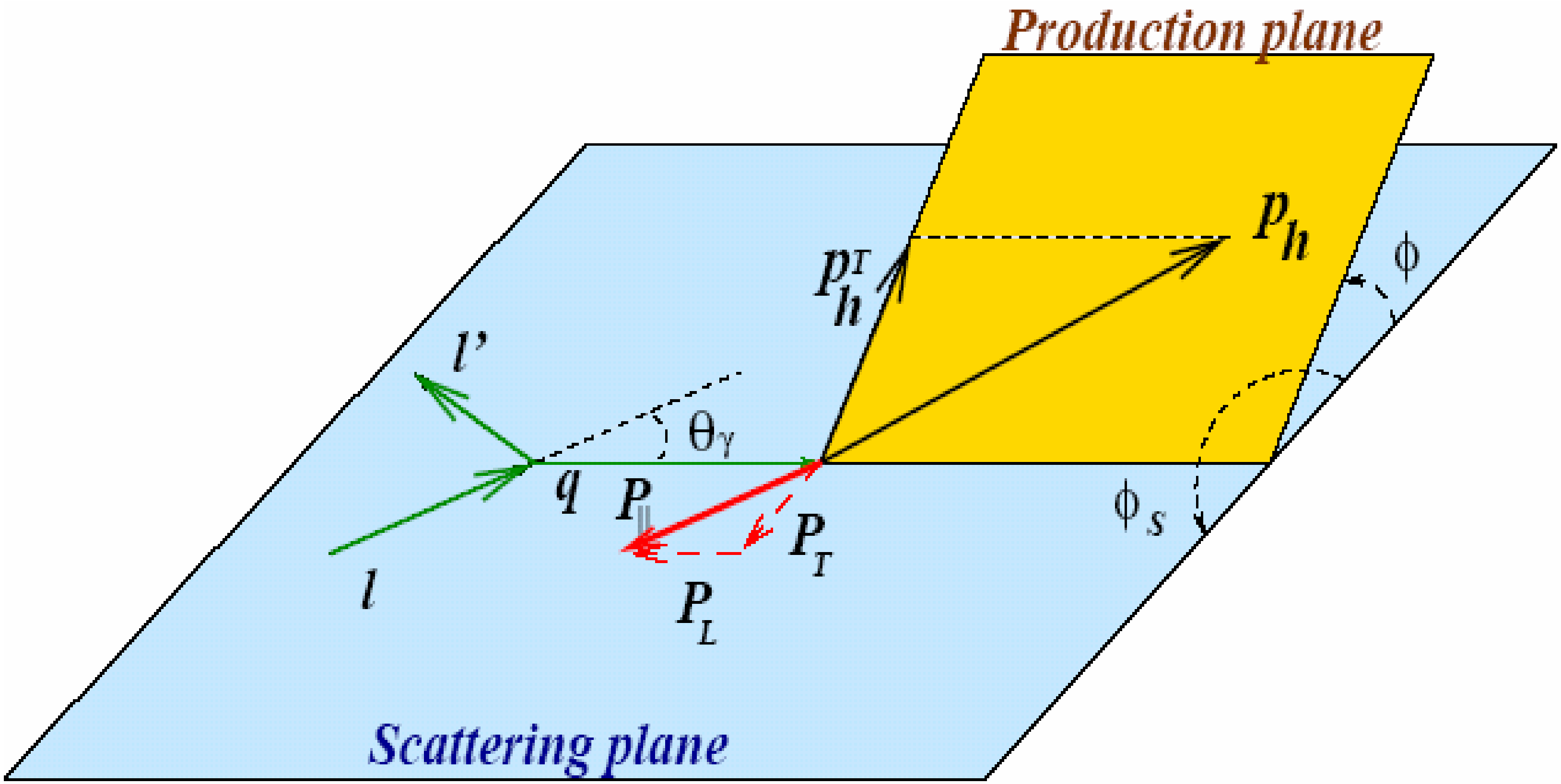
as M/P_T

. Twist 3 may contribute at leading order in 1/Q !

Phases in QCD -III

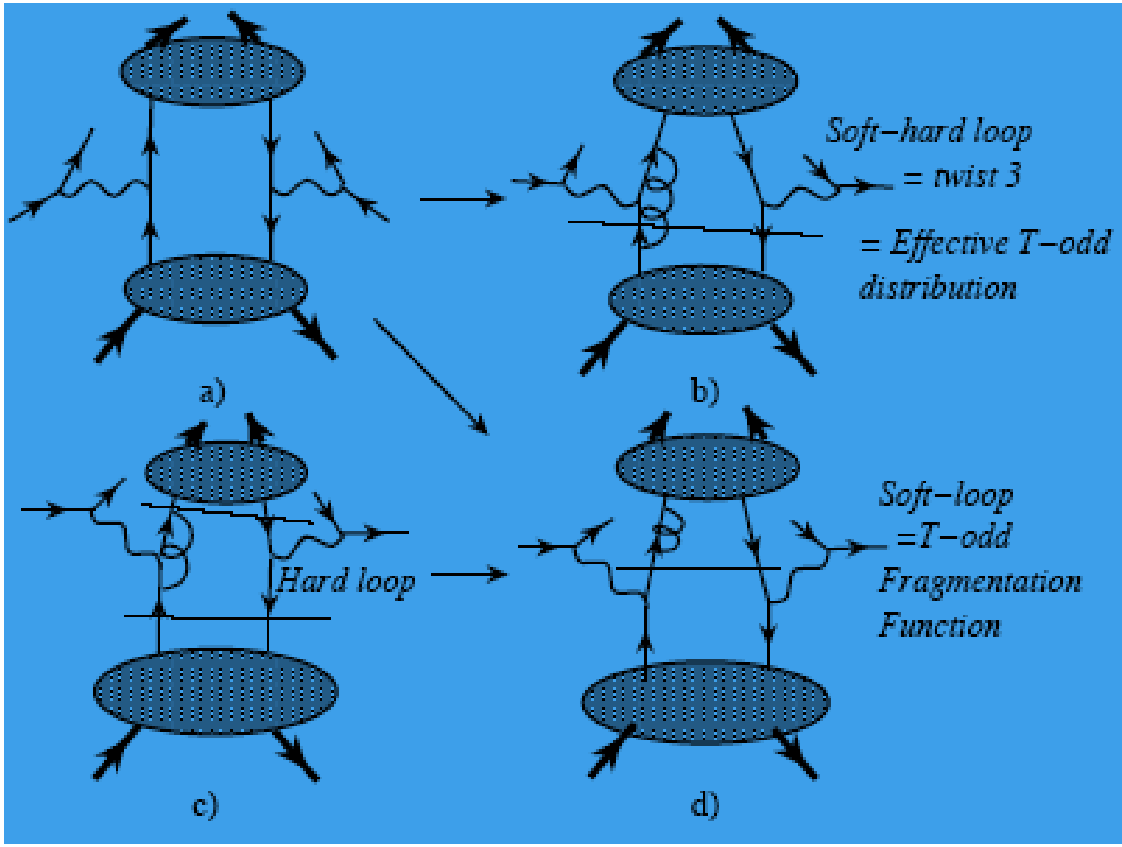
- Non-perturbative - positive variable –
- Jet mass-Fragmentation function: Collins(92);Efremov,Mankiewicz, Tornqvist (92),
- Correlated fragmentation: Fracture function: Collins (95), O.T. (98).

Test ground for SSA : Semi-Inclusive DIS - kinematics



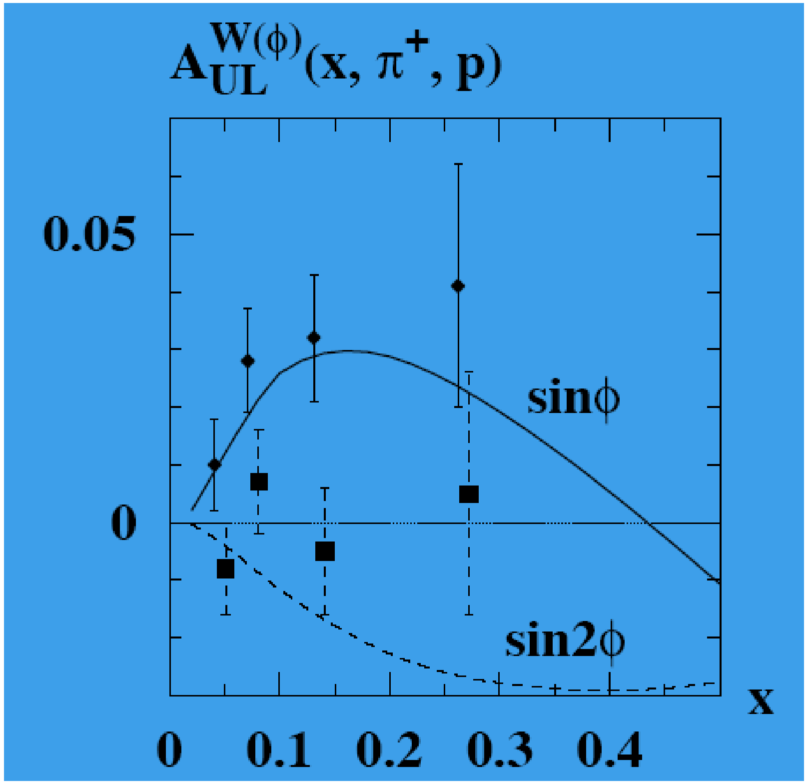
Sources of Phases in SIDIS

- a) Born - no SSA
- b) -Sivers (can be only effective)
- c) Perturbative
- d) Collins

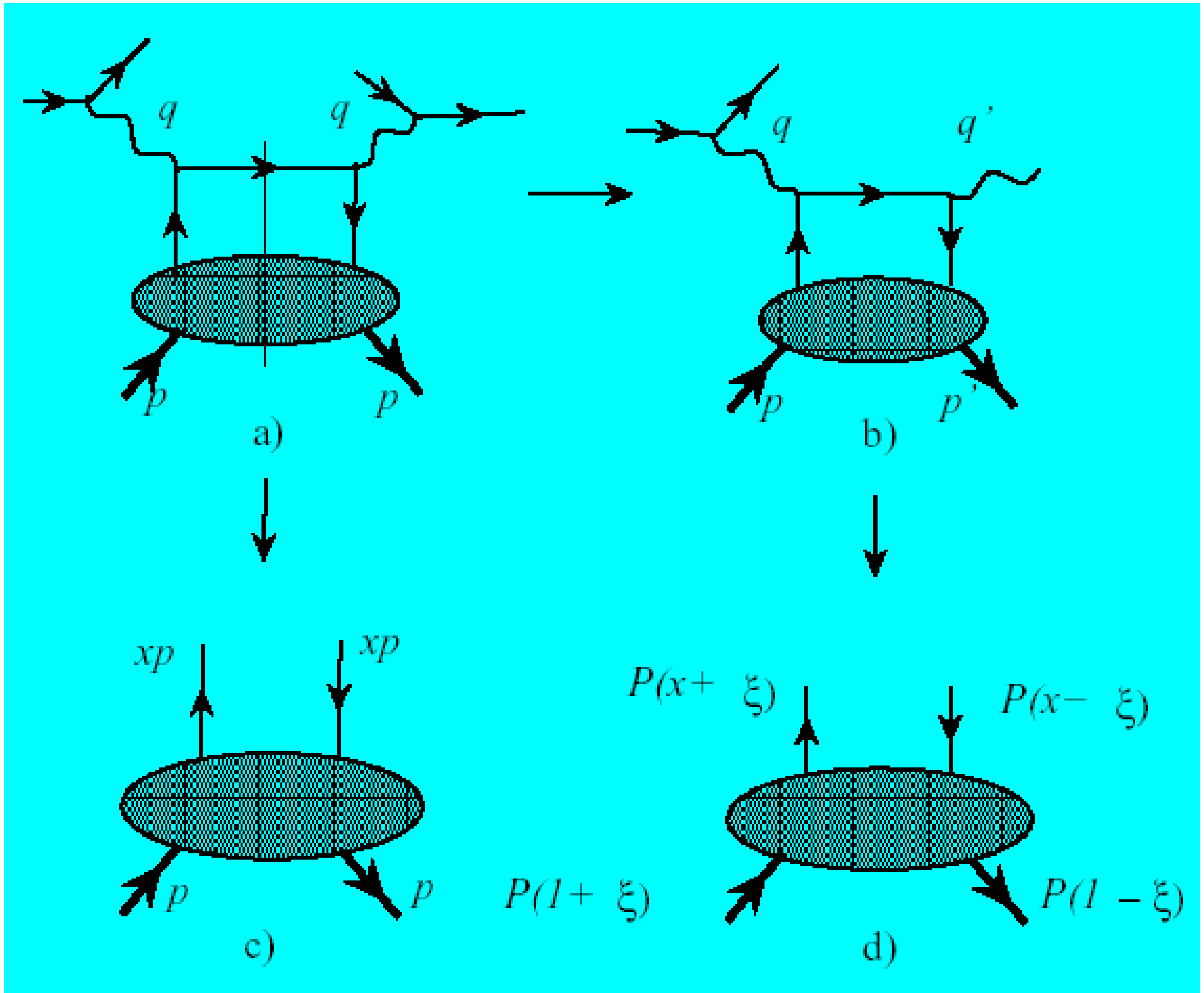


Typical observable SSA in SIDIS

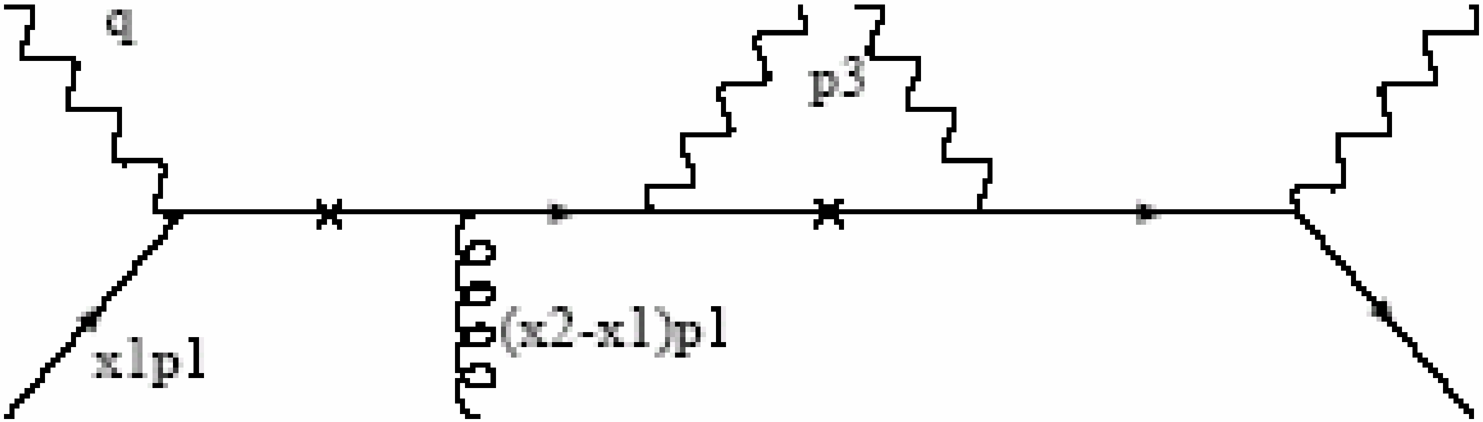
- Theory - Efremov, Goeke, Schweitzer
- Phase - from Collins function - extracted earlier from jets spin correlations qt LEP
- Spin of proton - transversity - from chiral soliton model



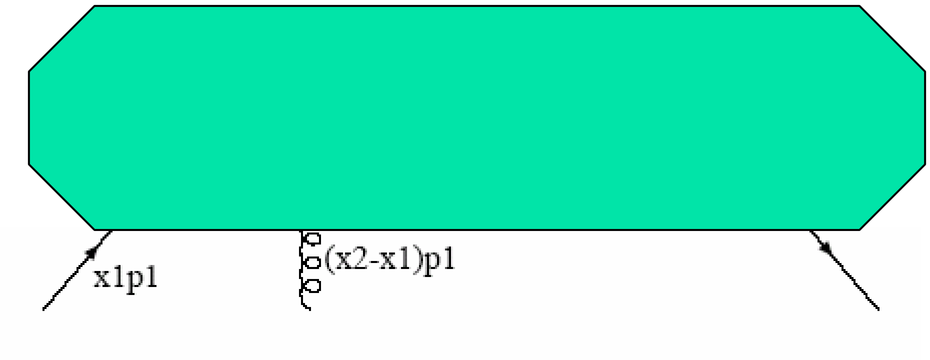
Final Pion -> Photon: SIDIS -> SIDVCS (easier than exclusive) - analog of DVCS



Twist 3 partonic subprocesses for photons SIDIS



Quark-gluon correlators



- Non-perturbative NUCLEON structure – physically mean the quark scattering in external gluon field of the HADRON.
- Depend on TWO parton momentum fractions
- For small transverse momenta – quark momentum fractions are close to each other- gluonic pole: probed if :
- $Q >> P_T >> M$

Low P_T probe small x_2 - x_1=

$$\delta = \frac{p_T^2}{Q^2} \frac{x_{Bj}}{(1-z)^2}$$

Real and virtual photons - most clean tests

- Both initial and final – real :Efremov, O.T. (85)
- Initial - real, final-virtual (or quark/gluon) –Korotkiian, O.T. (94)
- Initial –virtual, final-real: O.T., Srednyak (05, in preparation).

Spin-dependent cross-section

$$d\sigma^{\rightarrow} - d\sigma^{\leftarrow} =$$
$$M\, p_T b_A(0, x_2)(M_{A0} sin(\phi_s) + N_{A0} sin(\phi_s^h)) s_T +$$
$$M\, p_T b_A(x_1, x_2)(M_{A1} sin(\phi_s) + N_{A1} sin(\phi_s^h)) s_T +$$
$$M\, p_T b_V(0, x_2)(M_{V0} sin(\phi_s) + N_{V0} sin(\phi_s^h)) s_T +$$
$$M\, p_T b_V(x_1, x_2)(M_{V1} sin(\phi_s) + N_{V1} sin(\phi_s^h)) s_T$$

Properties of spin-dependent cross-section

- Complicated expressions
- Sivers (but not Collins) angle naturally appears
- Not suppressed as 1/Q provided gluonic pole exist
- Proportional to correlators with arguments fixed by external kinematics-

twist-3 “partonometer”

Low transverse momenta:

$$d\sigma_{total} = f(x_{Bj})8Q^2 \frac{x_{Bj}^2(1+(1-y)^2)(1+(1-z)^2)}{y^2 z\delta} \tag{12}$$

$$d\sigma_{ax1x2} = b_A(x_{Bj}, x_2)8Mp_T \frac{x_{Bj}(1+(1-y)^2)(2-z)}{y^2(1-z)\delta} s_T sin(\phi_s^h) \tag{13}$$

$$d\sigma_{vx1x2} = b_V(x_{Bj}, x_2)8Mp_T \frac{x_{Bj}(1+(1-y)^2)(1+(1-z)^2)}{y^2 z(1-z)\delta} s_T sin(\phi_s^h) \tag{14}$$

$$d\sigma_{a0x2} = -b_A(0, x_2)8Mp_T \frac{x_{Bj}^2(2(1-y)(1-2z)+y^2(1-z))}{y^2 z^2\delta} s_T sin(\phi_s^h)$$

(14) - non-suppressed for large Q if Gluonic pole exists=effective Sivers function; spin-dependent looks like unpolarized (soft gluon)

Experimental options

Natural extension of DVCS studies:

selection of elastic final state –

UNNECESSARY

BUT : Necessity of BH contribution also

- interference may produce SSA

Theoretical Implications

- Twist - 3 SSA survive in Bjorken region provided gluonic poles exist
- The form of SSA - similar to the one provided by Sivers function
- Twist-3 (but non-suppressed as 1/Q) effective Sivers function is found

CONCLUSIONS

- Semi-inclusive DVCS - new interesting hard process
- SSA in SIDVCS - direct probe of twist-3 correlators
- Low transverse momenta - effective twist 3 Sivers function
- Experimentally - naturally to do alongside DVCS

Pion from real photons –simple expression for asymmetry A=

$$\frac{b_A(0,x) - b_V(0,x)}{f(x)} \times$$

$$\times \frac{(1-x_F)(C_F x_F - (x_F+1)C_A/2)}{C_F(1+x_F^2)} \frac{2Mp_T}{m_T^2}$$

Properties of pion SSA by real photons

- Does not sensitive to gluonic poles
- Probe the specific (chiral) combinations of quark-gluon correlators
- Require (moderately) large P_T - may be advantageous with respect to DIS due to the specific acceptance.

Pion beam + polarized target

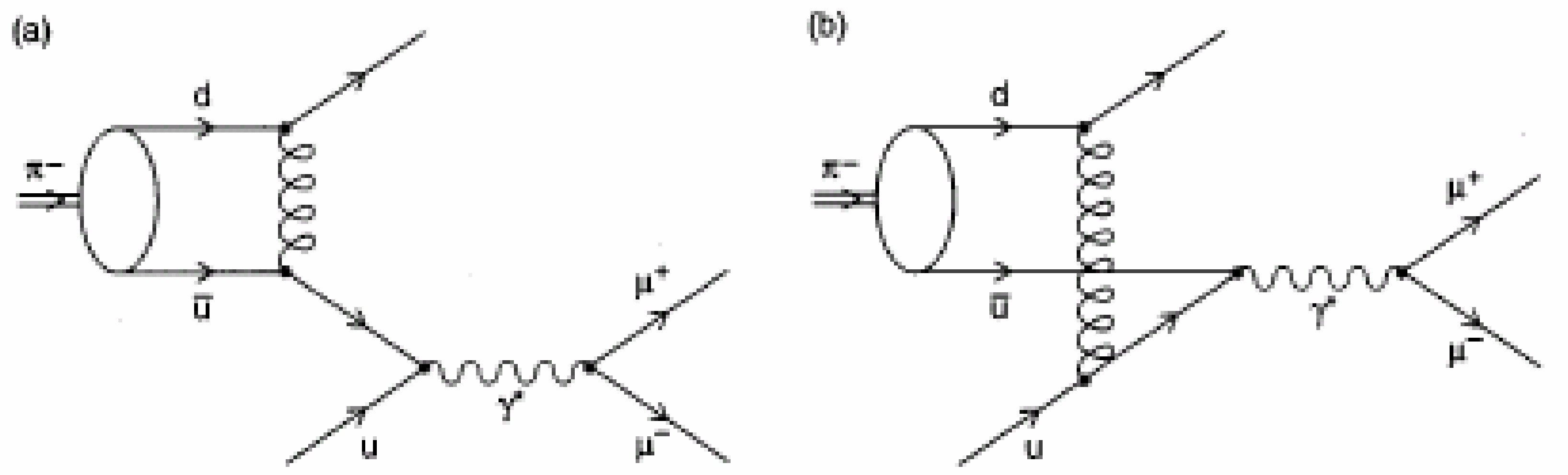
- Allows to study various ingredients of pion structure – rather different from nucleon
- Most fundamental one – pion-light cone

distribution – manifested in SSA in DY:

Brandenburg, Muller, O.T. (95)

Where to measure?! COMPASS(Torino)?!!

Pion Light-cone Distribution in pion-(q)proton scattering

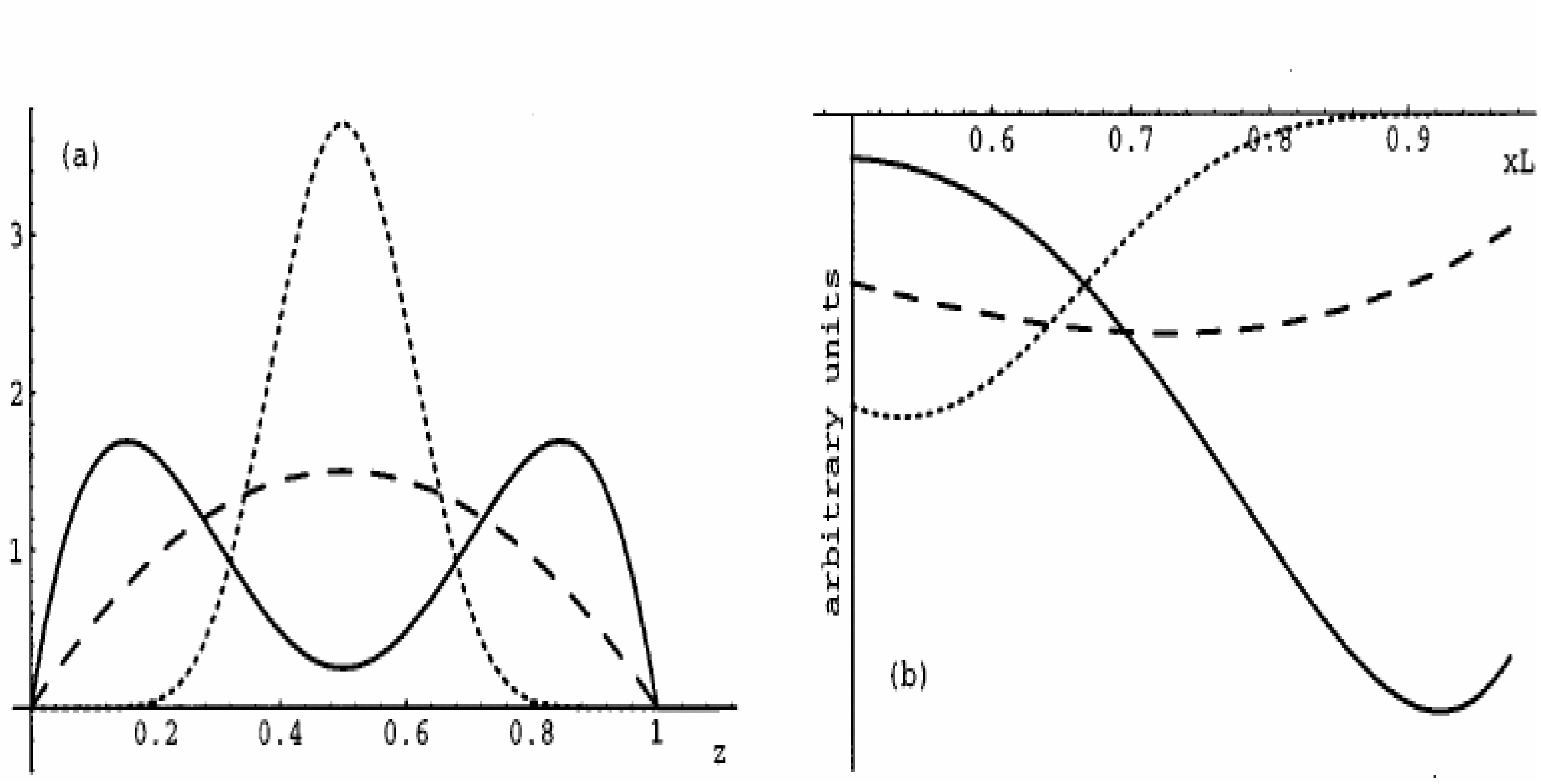


Simplest case-longitudinal polarization- “partonometer”

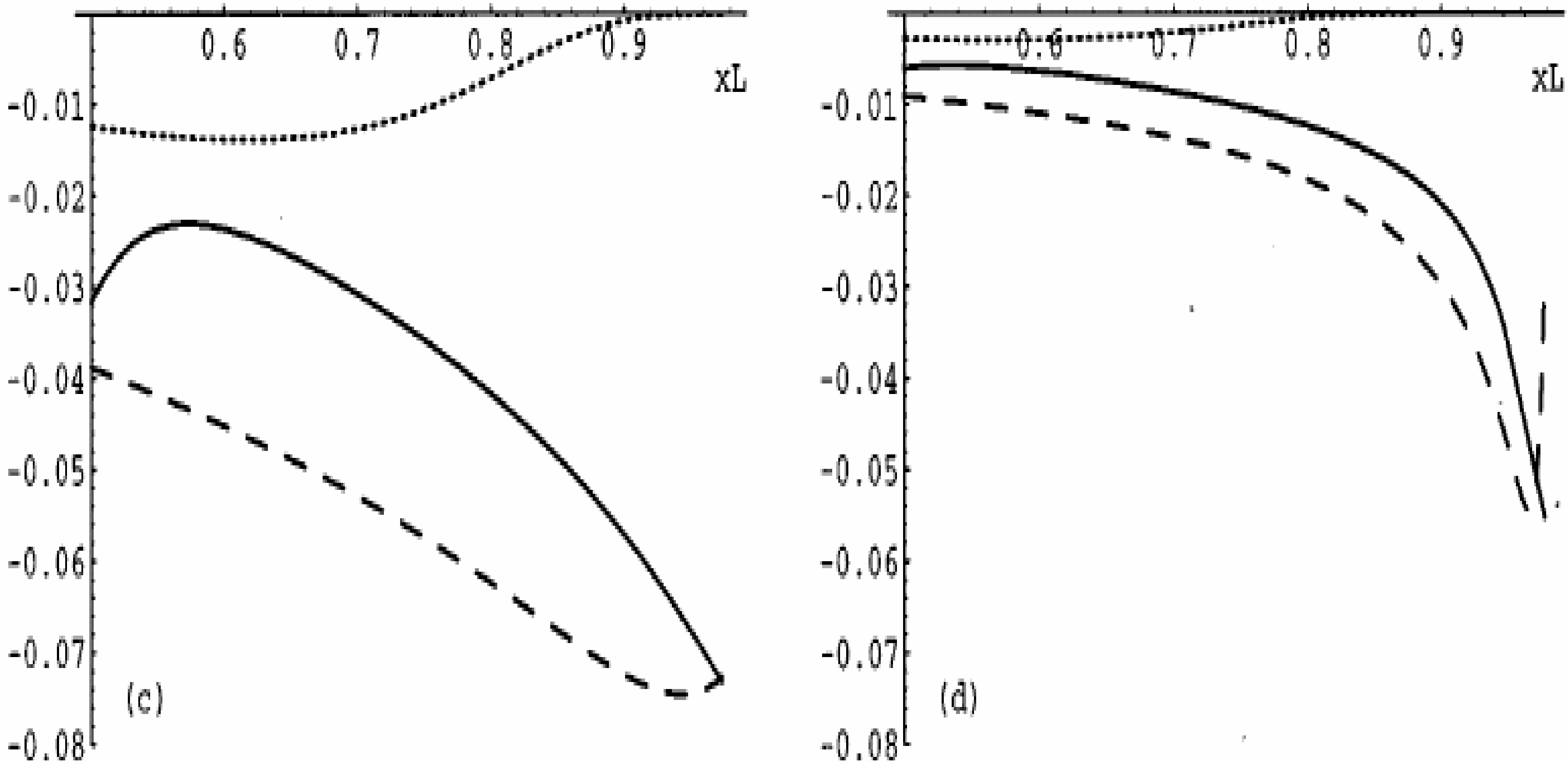
- Two extra terms in angular distribution, proportional to longitudinal polarization

$$\overline{\mu}\sin 2\theta\ \sin\phi + \frac{\nu}{2}\sin^2\theta\ \sin 2\phi$$

Models for light-cone distributions and angular-weighted x-sections



Size of coefficients in angular distributions



Transverse polarization

- Much more complicated – many contributions
- Probe of transversity (X Boer T-odd effective distribution), Sivers function, twist-3 correlations, pion chiral-odd distributions)

CONCLUSIONS-I

- (Moderately) high Pions SSA by real photons – access to quark gluon correlators
- Real photons SSA: direct probe

  of gluonic poles, may be included to DVCS studies

CONCLUSIONS-II

- Pion beam scattering on polarized target – access to pion structure
- Longitudinal polarization – sensitive to pion distrbution
- Transverse polarization – more reach and difficult