The Space Dynamics

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PNPI Winter Scool, 2006

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The fundamental physical object of the *Global Time Theory* is a three-dimensional curved space dynamically developing in global time.
The equations of its dynamics are derived from the Lagrangian, and the Hamiltonian of gravitation turns out to be nonzero.
The General Relativity solutions are shown to be a subset of the GTT solutions with zero energy density.

General Relativity and Global Time Theory

General relativity (GR) is built on the Riemannian geometry of space-time. Solutions of its basic equations, the Einstein equations, determine full four-dimensional space-time, past, present and future simultaneously.

The Global Time Theory (GTT) considers a three-dimensional space as a fundamental physical object that develops dynamically in *global time* which is common for all points of the space.

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The Dynamical Geometry

The space is represented by a set of all its points.

- The frame of references in which coordinates of the space points $(\bar{x}^i)$ do not change with time, is *an absolute inertial frame*.
- Then, in some other frame of references with a time-dependent coordinates transformation $x^i = f^i(\bar{x}, t)$, there exists *a field of absolute velocities*

$$V^i = \frac{\partial x^i}{\partial t}, \tag{1}$$

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The invariant Derivatives on the Time

The Dynamical Geometry

- For scalar:

$$D_t\, f(x,t) = \frac{\partial f}{\partial t} + V^i\, \partial_i\, f,$$

- For Tensor:

$$D_t\, Q^i_{jk} = \frac{\partial}{\partial t} Q^i_{jk} - V^i_{;s} Q^s_{jk} + V^s_{;j} Q^i_{sk} + V^s_{;k} Q^i_{js} + V^s Q^i_{jk;s}. \quad (2)$$

- Especially important is invariant derivative on the time of the metric tensor:

$$D_t\, \gamma_{ij} = \frac{\partial \gamma_{ij}}{\partial t} + V_{i;j} + V_{j;i}. \quad (3)$$

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The Space Action

- The space deformation velocity tensor

$$\mu_{ij} = \frac{1}{2\,c} D_t\, \gamma_{ij} = \frac{1}{2\,c}(\dot{\gamma}_{ij} + V_{i;j} + V_{j;i}). \qquad (4)$$

- determines the space dynamics action:

$$S = \frac{c^4}{16\,\pi\,k} \int (\mu^i_j\, \mu^j_i - (\mu^j_j)^2 + R)\sqrt{\gamma}\, d_3\, x\, dt + S_m, \qquad (5)$$

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The Dynamical Equations

Variation of the action with respect to the metric tensor γ_{ij} leads to six dynamic equations. Also variation with respect to the absolute velocity field generates three *to the equations of connectivity*. Thus the set of equations of the space dynamics consists of *nine* partial differential equations of the second order. By introducing momenta

$$\pi^i_j = \sqrt{\gamma}(\mu^i_j - \delta^i_j \, \mu^s_s),$$

and varying the action by six components of spacial metrics,

The Dynamical Equations

we obtain the six equations of dynamics:

$$\dot{\pi}^i_j = b^i_j + \sqrt{\gamma}\, G^i_j + \sqrt{\gamma}\,(T^i_j - V^i\; T^0_j), \tag{6}$$

where

$$b^i_j = -\delta^i_j \frac{\sqrt{\gamma}}{2} (\mu^k_l \mu^l_k - \mu^k_k \mu^l_l) - \partial_s (V^s\, \pi^i_j) + V^i{}_{,s}\; \pi^s_j - V^s{}_{,j}\; \pi^i_s, \tag{7}$$

G^i_j is the Einstein's tensor of three-dimensional space. The variation (only the kinetic part of action) by three components of the field of absolute velocities gives three equations of links:

$$\nabla_i\, \pi^i_j = \sqrt{\gamma}\, \frac{8 \pi k}{c^4}\, T^0_j. \tag{8}$$

Hamiltonian

From the action the energy density and the Hamiltonian $H_\gamma = \int \rho \sqrt{\gamma}\, d_3x$ are determined as usually in field theory:

$$\rho = \frac{c^4}{16\,\pi\,k}\,(\mu^i_j\,\mu^j_i - (\mu^j_j)^2 - R). \tag{9}$$

$H = T + U$, where T is kinetic energy and U potential one:

$$T = \frac{c^4}{16\,\pi\,k}\int(\mu^i_j\,\mu^j_i - (\mu^j_j)^2)\sqrt{\gamma}\,d_3x; \quad U = \frac{c^4}{16\,\pi\,k}\int R\sqrt{\gamma}\,d_3x.$$

An important feature of this Hamiltonian is its indeterminate sign.

The Space virial Theorem

The sum of equations (6), in absence of external sources (for proper gravitation), gives

$$\dot{\pi} + \partial_s (V^s \pi) = -3\,T - \frac{1}{2} R \sqrt{\gamma} = -3\,T + U, \tag{10}$$

where T and U are the densities of the kinetic and the potential energy, respectively.

The space virial theorem can be applied to the almost stationery fields in space, on the boundaries of which $V^n = 0$.

Averaging (10) over time, we obtain:

$$U = 3\,T; \quad E = T + U = 4\,T. \tag{11}$$

Under the aforementioned conditions, the kinetic and full energies are positive.

The proper time of the moving observer

In GR, as Einstein constantly emphasized, the special relativity theory is valid in local: in small, the space and time are described by the Minkowski metric (tangential space-time).
The global construction of GTT does not superimpose any restrictions on local properties. The special relativity theory, as local structure of space-time, can be also naturally incorporated in GTT. For any moving observer its proper time is defined via a global time and velocity with respect to the space $v^i = \dot{x}^i - V^i$:

$$d\tau = dt \sqrt{1 - \frac{1}{c^2} \gamma_{ij} v^i v^j}. \qquad (12)$$

The Spherical Universe dynamics

For three-dimensional sphere with radius r, the scalar curvature

$$R = \frac{3}{r^2}, \quad \sqrt{\gamma} = r^3.$$

The Hamiltonian is negative:

$$H = -3\,\frac{c^4}{16\pi k}\;r\,(\dot{r}^2 + 1) = -E. \tag{13}$$

The Hamiltonian conservation leads to a differential equation of the first order which is the Friedman's equation, that have a cicloida solution, but without some matter density.

The field of spherical mass

The spherically symmetric metric can be transformed to

$$dl^2 = dr^2 + R^2(r)(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2),$$

and the field of velocities is radial: $V^r = V(r)$. Solely the radial link-equation is nontrivial:

$$\nabla_i \pi^i_r = \frac{2}{r} R'' = 0,$$

from which $R = r$, and the space turns out to the flat.
From dynamical equation

$$r\, V^2 = const \equiv 2\, k\, M \geq 0,$$

M must be positive.

The Painleve metrics

The field of radial velocities

$$V^r = V = \sqrt{\frac{2\,k\,M}{r}}$$

leads to the four-dimensional metric

$$ds^2 = \left(c^2 - \frac{2kM}{r}\right)\,dt^2 + 2\sqrt{\frac{2kM}{r}}\,dt\,dr - dr^2 - r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2).$$

In 1921 this metric was obtained by Painlevé [4] by time transformation in the Shwarzshield metric.

The Vortex field

The task of space vortexes has no analog in GR and is the proper task of GTT.

The metric is stationary, axially-symmetric, and can be transformed into

$$dl^2 = e^{w(r,\vartheta)}\,(dr^2 + r^2\,d\vartheta^2) + r^2\,\sin^2\vartheta\,d\varphi^2. \tag{14}$$

with one metric function $w(r,\vartheta)$.

The absolute velocities field is also dependent on r and ϑ, and has one component – the vortex field $V^\varphi = \Omega(r,\vartheta)$.

The Vortex field

The unique nontrivial link for V^{φ} in the absence of current yields the equation for Ω:

$$\Omega_{,rr} + \frac{4}{r}\,\Omega_{,r} + \frac{1}{r^2}\,(\Omega_{,\vartheta\vartheta} + 3\,\mathrm{ctg}\vartheta\,\Omega_{,\vartheta}) = 0. \qquad (15)$$

Note, that this second order linear differential equation is independent on the metric function $w(r, \vartheta)$.

Metrics equations

The metric function $w(r,\vartheta)$ is determined from equations:

$$w_{,r} = \frac{r}{2\,c^2}\left(\Omega_{,\vartheta}^{\,2} - r^2\,\Omega_{,r}^{\,2} - 2\,\mathrm{ctg}\vartheta\, r\,\Omega_{,r}\,\Omega_{,\vartheta}\right)\,\sin^4\vartheta;$$

$$w_{,\vartheta} = \frac{r^2}{2\,c^2}\left(\mathrm{ctg}\vartheta\left(r^2\,\Omega_{,r}^{\,2} - \Omega_{,\vartheta}^{\,2}\right) - 2r\,\Omega_{,r}\,\Omega_{,\vartheta}\right)\,\sin^4\vartheta. \qquad (16)$$

Upon completing these relationships along with (15) for Ω all equations of dynamics and links are satisfied.

Multipole solutions

The leader equation (15) is homogeneous along radius r, thus its common solutions can be found in the form of a power series

$$\Omega(r,\vartheta) = \sum_{l=0}^{\infty} \left(A_l\, r^l + \frac{B_l}{r^{l+3}} \right) P_l(\cos\vartheta). \tag{17}$$

The angular part is subject to the differential equation (where $x = \cos\vartheta$):

$$(x^2-1)P_l'' + 4\,x\,P_l' - l\,(l+3)P_l = 0. \tag{18}$$

For each *separate* multipole solution an energy density

$$\varepsilon\sqrt{\gamma} = a^2\, r^{2(l+1)}\, \sin^3\vartheta(P_{l,\vartheta}^{\,2} + l^2\, P_l^2). \tag{19}$$

The Energy of Monopole

A globe with radius R is rotates with angular velocity of Ω *coherently*. Outside – a monopole solution with the energy density

$$\omega(r) = \Omega \frac{R^3}{r^3}; \quad \varepsilon = \frac{9\,\Omega^2\, R^6\, \sin^2\theta}{r^6}. \tag{20}$$

The full energy of space $M\, c^2 \equiv E =$

$$\frac{c^4}{16\pi k} 9\,\Omega^2\, R^6\, 2\pi \int_0^\pi \sin^3\vartheta\, d\vartheta \int_R^\infty \frac{r^2\, dr}{r^6} = \frac{R^3\,\Omega^2}{2\,k}\, c^2. \tag{21}$$

For a globe with diameter 20 cm., that completes one rotation per second, we obtain $M = 300\,000\,000$ kg.

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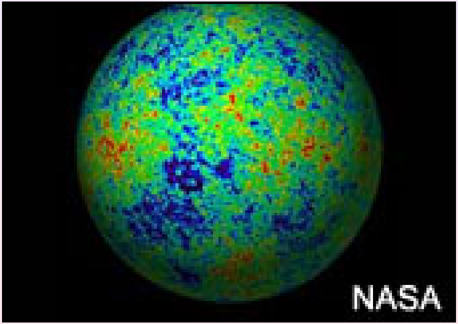
The "evil axis"

The cosmological phenomenon of the "evil axis" demonstrates the viability of the concept of space and time in Global Time Theory.



From first glance, the designation of fluctuations (very small amplitude) of microwave radiation across the sky dome generates a representation of a some oscillating process.

The modes of metrics fluctuations

It follows that the large, observed fluctuation modes create a representation of finiteness of the Universe (closed Friedman model).
The tensor modes of metrics fluctuations, satisfies restrictions

$$\gamma^{ij}\, h_{ij} = 0; \quad \nabla_i\, (\gamma^{ik}\, h_{kj}) = 0; \quad \delta R_{ij}(\mathbf{h}^n) = \frac{\lambda_n}{2\, r^2}\, h^n_{ij}$$

can be computed by **Lie-generation method** and are concentrates in groups, characterized of integer number $n \geq 2$, and each mode has the quantum number $\lambda_n = n^2 + 2\, n + 4$.
At each n there are two polarizations, each having $(n-1)(n+3)$ modes, so the common modes number at concrete n is $N = 2\,(n-1)(n+3)$.

General Relativity. ADM-representation

Arnovitt, Deser and Misner [3] have expressed ten components of the four-dimensional metric tensor as

$$g_{00} = f^2 - \gamma_{ij} V^i V^j; \quad g_{0i} = \gamma_{ij} V^j; \quad g_{ij} = -\gamma_{ij}. \tag{22}$$

Components of an inverse metric tensor are respectively

$$g^{00} = \frac{1}{f^2}; \quad g^{0i} = \frac{V^i}{f^2}; \quad g^{ij} = \frac{V^i V^j}{f^2} - \gamma^{ij}. \tag{23}$$

Ten Einstein equations are obtained then as variation equations for all ten components of the metric tensor.
In GTT, the component $g^{00} = 1$ always and everywhere, then $f = 1$ and can not be varied, so there are nine equations.

Reducing GR to Global Time

If there is the four-dimensional metric $g_{\alpha\beta}$ in arbitrary four coordinates x^α, $\alpha = 0..3$, the variable τ can be reduced to global time by reduction the metrics component g^{00} to one:

$$\bar{g}^{00} = g^{\alpha\beta} \frac{\partial\tau}{\partial x^\alpha} \frac{\partial\tau}{\partial x^\beta} = 1. \tag{24}$$

This differential equation turns out to be Hamilton-Jacoby differential equation for free-falling bodies (laboratories), the common time for which is τ, which is the global time.
Thus *the equivalence principle* is realized, but in contrast to GR, the time of the inertial system exists not only for a local laboratory, but for a great many laboratories in all of space.

Kerr's metric

For example, Kerr's metric in global time has a radial and an angular component of the absolute velocity field:

$$V^{\varphi} = -\frac{2\,a\,M\,r}{w}; \quad V^{r} = \frac{\sqrt{2\,M\,r(r^2 + a^2)}}{\rho^2}, \qquad (25)$$

where

$$\rho^2 = r^2 + a^2\cos^2\vartheta; \quad w = (r^2 + a^2)\rho^2 + 2\,M\,r\,a^2\sin^2\vartheta,$$

The space metrics

$$\gamma_{11} = \frac{(\rho^2)^2}{w}; \quad \gamma_{22} = \rho^2; \quad \gamma_{33} = \frac{w}{\rho^2}\sin^2\vartheta \qquad (26)$$

has singularity only at $\rho^2 = 0$.

The Quantum Gravity

In a view of GTT, the quantum gravity theory undergoes the most significant modification. The catastrophic relationship GR $H = 0$, putting on hold all quantum dynamics, is removed. In GTT, a quantum gravity, as well as quantum theory of other fields, for example quantum electrodynamics, can be built on the basis of a Schrodinger equation

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H}\,\Psi, \tag{27}$$

defining the dynamics *of a state vector* of space (and other fields) Ψ in global time.

Quantum Friedman's cosmology

For Friedman's model of space, the three-dimensional sphere of variable radius r, Hamiltonian is

$$H = -\frac{p_r^2 + r^2}{2\,r} + \frac{q^2}{2\,r}, \tag{28}$$

where q^2 characterizes a conserved amount of a ultrarelativistic substance.

A stationary cosmological wave equation (in Planck units):

$$u'' - \frac{u'}{r} + (-r^2 + q^2)u = 2\,r\,E\,u. \tag{29}$$

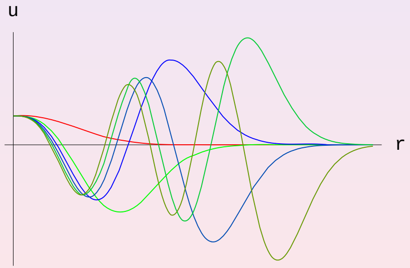
Eigenvalues

Eigenvalues of an energy for first eight such functions at $q = 0$ (the matter is absent, only space dynamics is contributing) and $q = 1$ are listed in the table:

n	q=0	q=1
1	-0.977722	4.42817
2	-3.05247446	-2.3182877
3	-4.16434141	-3.6338011
4	-5.03491431	-4.5990728
5	-5.77537028	-5.3970940
6	-6.43100378	-6.0924244
7	-7.02566164	-6.7165684
8	-7.57373725	-7.2876611

Functions

First six (unnormalized) functions for pure space ($q = 0$) are presented on the graph:

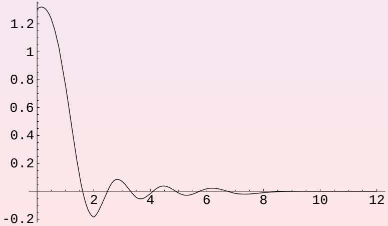


A cosmological uncertainty relation

The quantum effects do not prevent the Big Bang, but

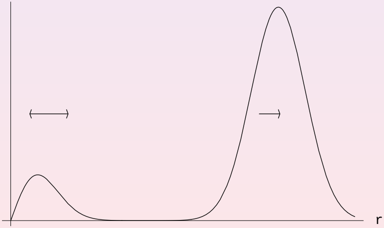
> *The product of maximum and minimum radiuses of the Universe is not less than* $\frac{8\pi k \hbar}{c^3}$.

The wavelet in the space with $n = 8$, having a minimum eigenvalue of radius $r = 0.51$ is presented in the figure.



The moving wavelet

However this wavelet is not stationary and begins to (widen) spread. This expansion is not monotonic: the wavelet (square of the module) breaks into two component, one of which is moving away from zero, but second oscillates near to zero:



Conclusion

- Being built on more advanced mathematical framework, GTT introduces into physics a new (in general old, known at a level of philosophy for a long time) *physical* object: **the space**.
- From the theoretical physics view point the space is the field with nine components $(\gamma_{ij},\ V^i)$ knocked under nine variational equations and have the nontrivial energy density.
- The GR solutions have the zeroes energy density, so almost all GR-solutions belong to GTT.
- The movement the bodies and light in GTT is the same, as in GR, so all effects of GR are implemented to GTT.

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