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Nucleon Form Factors in QCD

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Outline

Experiment:

- G_E^p/G_M^p and the LT/PT controversy
- Form factors in time-like region using ISR technique

Theory 1:

- Lattice calculations
- Dispersion relations

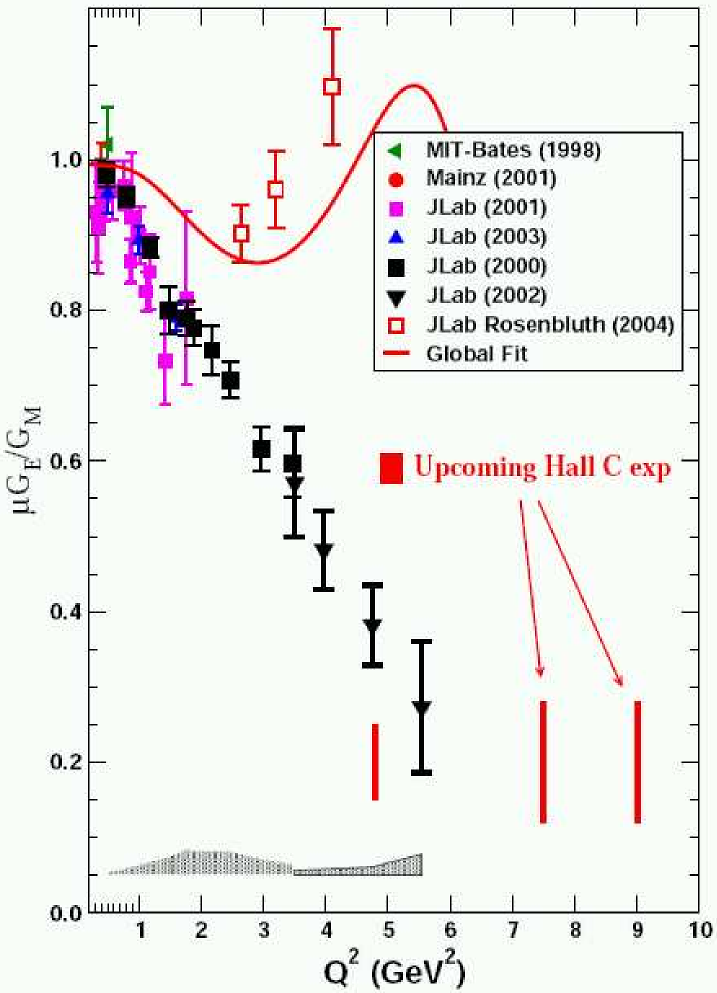
Theory 2:

- Status of perturbative factorization
- Time-like form factors: Sudakov resummation
- Light-Cone Sum Rules

Present and . . .

Future G_E/G_M at JLab in Hall C

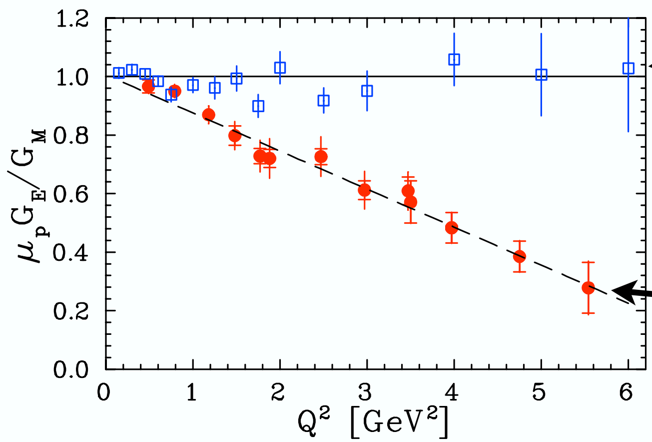
- FPP has been built at Dubna and will be installed in the HMS
- A large calorimeter has been assembled with help of IHEP and Yerevan.
- Scheduled to run in 2007.
- With the 12 GeV upgrade at JLab can reach $Q^2 = 14$



M.Jones, 12.10.05

Nucleon05 – p.15/26

Proton G_E/G_M Ratio



Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

LT

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

$$\tau = Q^2/4M^2$$
$$\varepsilon = \left[1 + 2(1+\tau)\tan^2\theta/2\right]^{-1}$$

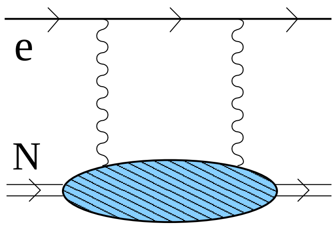
G_E/G_M from slope in ε plot

PT

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\frac{P_T}{P_L}$$

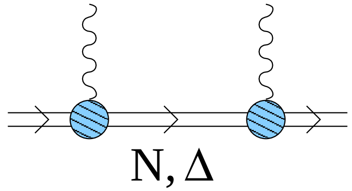
$P_{T,L}$ polarization of recoil proton

W.Melnitchouk, 12.10.05



+ crossed

?



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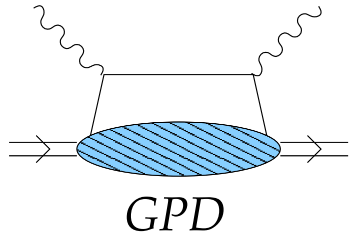


Several calculations exist:

- **Nucleon elastic intermediate state resolves most of LT/PT G_E^p/G_M^p controversy**
- **Δ opposite sign cf. nucleon but smaller**

Blunden, Melnichouk, Tjon

- **GPD-based quark-level calculations indicate substantial short-distance contributions**

Afanasev, Brodsky, Carlson, Chen, Vanderhaeghen

Various proposals to measure the 2γ contribution:

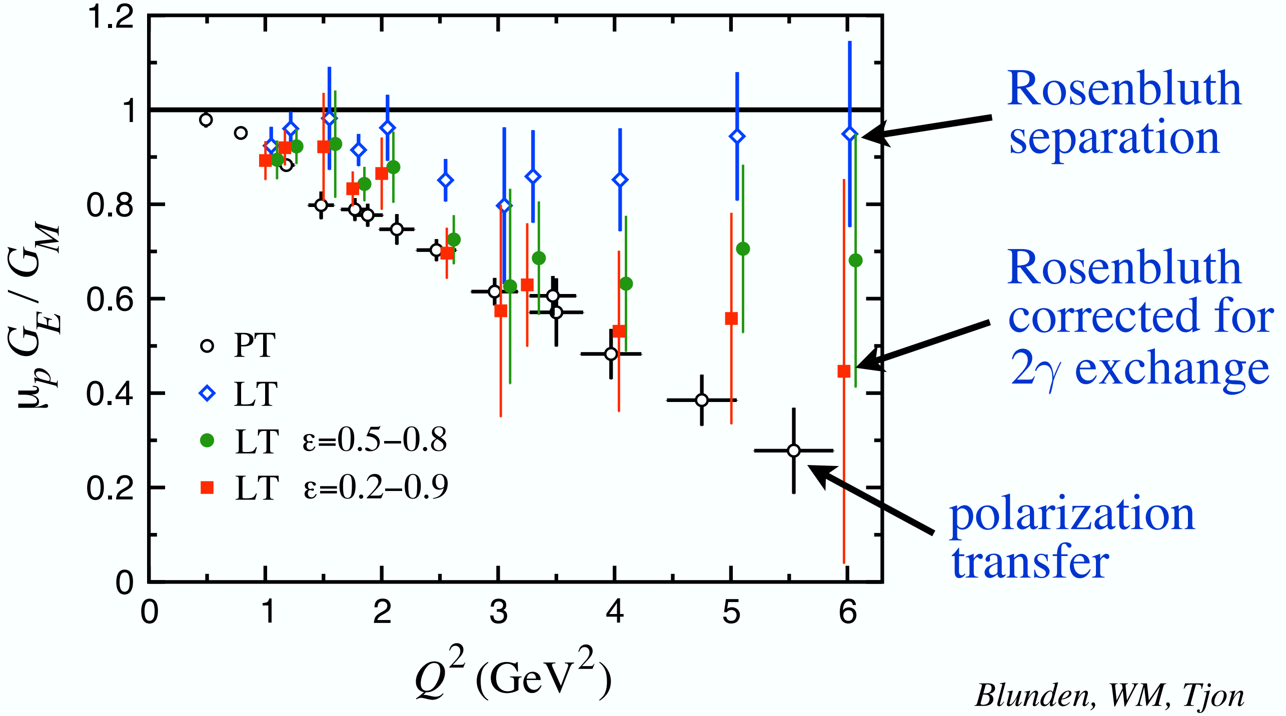
- **1γ exchange changes sign under $e^+ \leftrightarrow e^-$, 2γ exchange invariant**

$\Rightarrow$ **Simultaneous e^+p/e^-p measurements planned in JLAB Hall B (up to $Q^2 = 1$ GeV^2)**

- **Precision ϵ-dependence of σ_{e^-p}; ϵ-dependence of P_T/P_L**

W.Melnitchouk, 12.10.05

G_E^p / G_M^p ratio

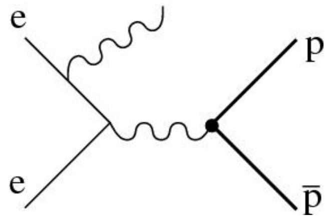


Blunden, WM, Tjon
Phys. Rev. C72 (2005) 034612

⇒ resolves much of the form factor discrepancy

ISR method

V.Druzhinin, 13.10.05



Mass spectrum of pp system in the $e^+e^- \to pp\gamma$ reaction is related to cross section of $e^+e^- \to pp$ reaction at E=m.

$$\frac{d\sigma(e^+e^-\to p\bar{p}\gamma)}{dm\, d\cos\theta}=\frac{2\mathrm{m}}{s}W(s,x,\theta)\sigma(e^+e^-\to p\bar{p})(m),\quad x=\frac{2E_\gamma}{\sqrt{s}}=1-\frac{m^2}{s},$$

$$W(s,x,\theta)=\frac{\alpha}{\pi x}\left(\frac{2-2\mathrm{x}+x^2}{\sin^2\theta}-\frac{x^2}{2}\right),\quad \theta>>\frac{m_e}{\sqrt{s}}.$$

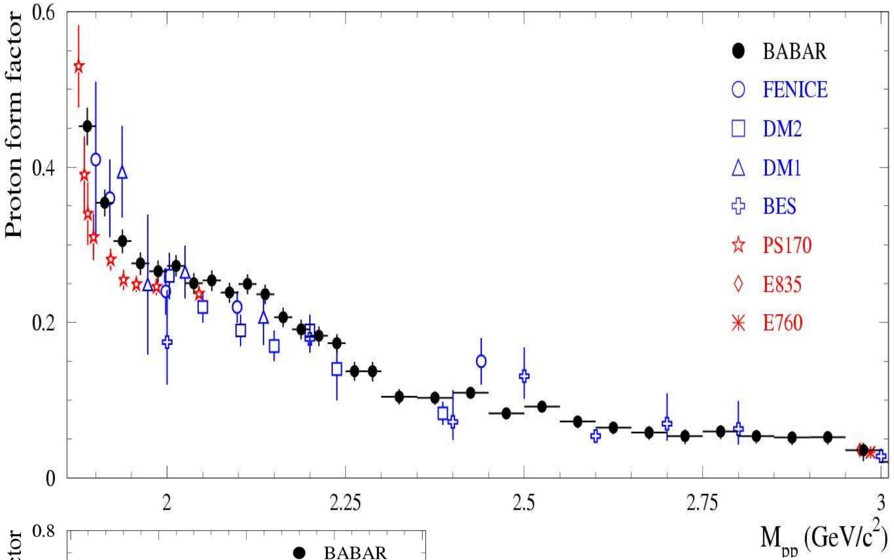
$e^+e^- \to pp$ cross section depends on two form factors, electric G_E and magnetic G_M.

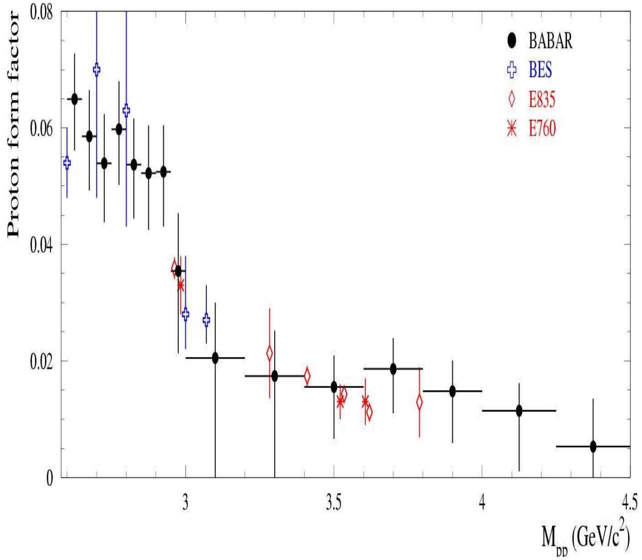
$$\sigma(e^+e^-\to p\bar{p})=\frac{4\pi\alpha^2\beta C}{3\mathrm{m}^2}\left(|G_M|^2+\frac{2\mathrm{m}_p^2}{m^2}|G_E|^2\right)$$

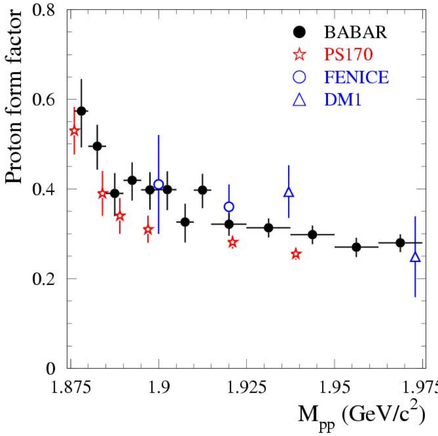
The ratio of form factors $|G_E/G_M|$ can be obtained from the analysis of the proton angular distribution. The terms corresponding G_M and G_E have angular dependence close to $1+\cos^2\theta$ and $\sin^2\theta$, respectively.

Effective proton form factor

V.Druzhinin, 13.10.05







- *Steep behaviour at threshold confirmed*
- *Similar behaviour at threshold is seen in other processes with different quantum numbers*

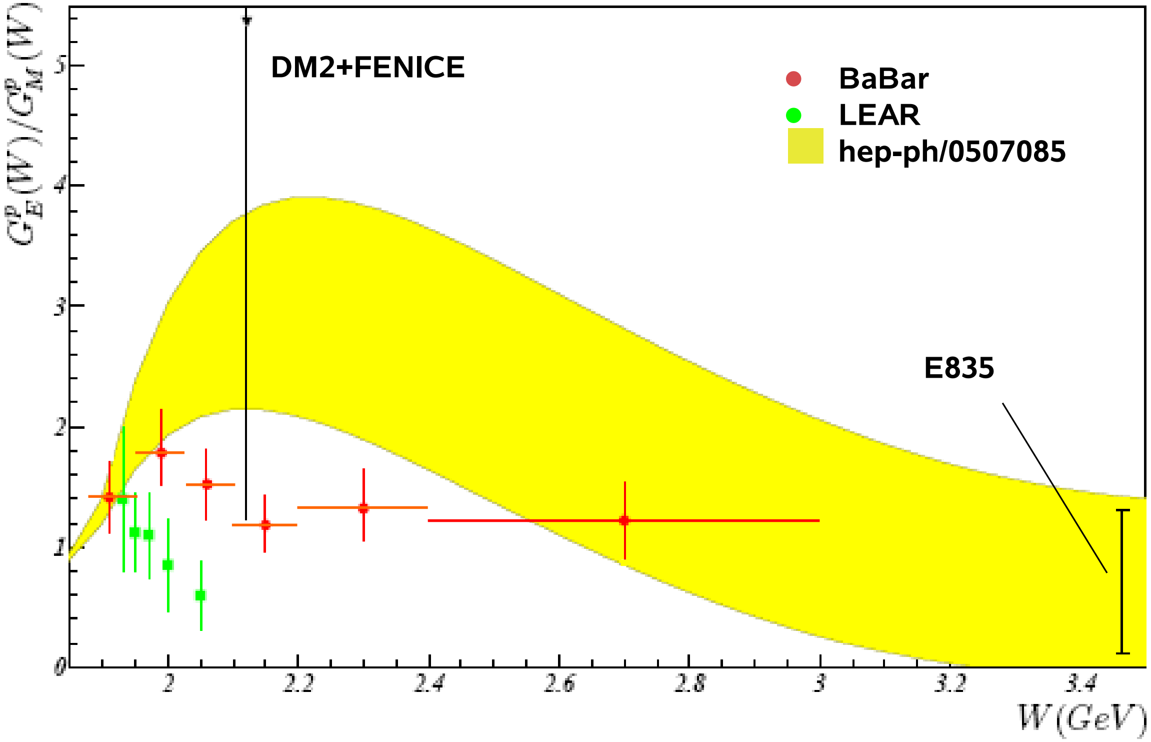
$$BES \quad J/\psi \to \gamma \; p\bar{p}$$

$$BABAR \quad B^+ \to K^+ p\bar{p}, \; B^0 \to D^0 p\bar{p}$$

|GE/GM| ratio

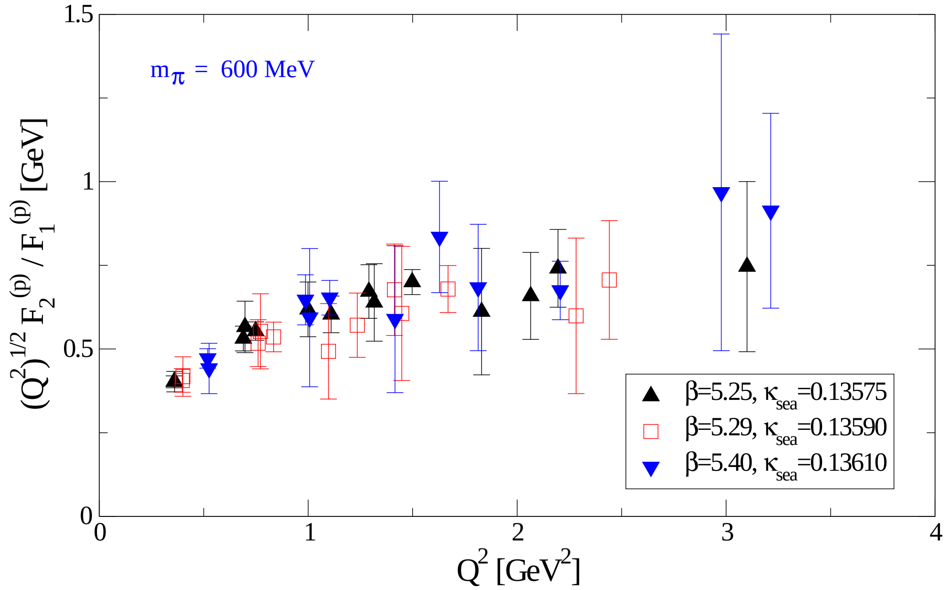
V.Druzhinin, 13.10.05

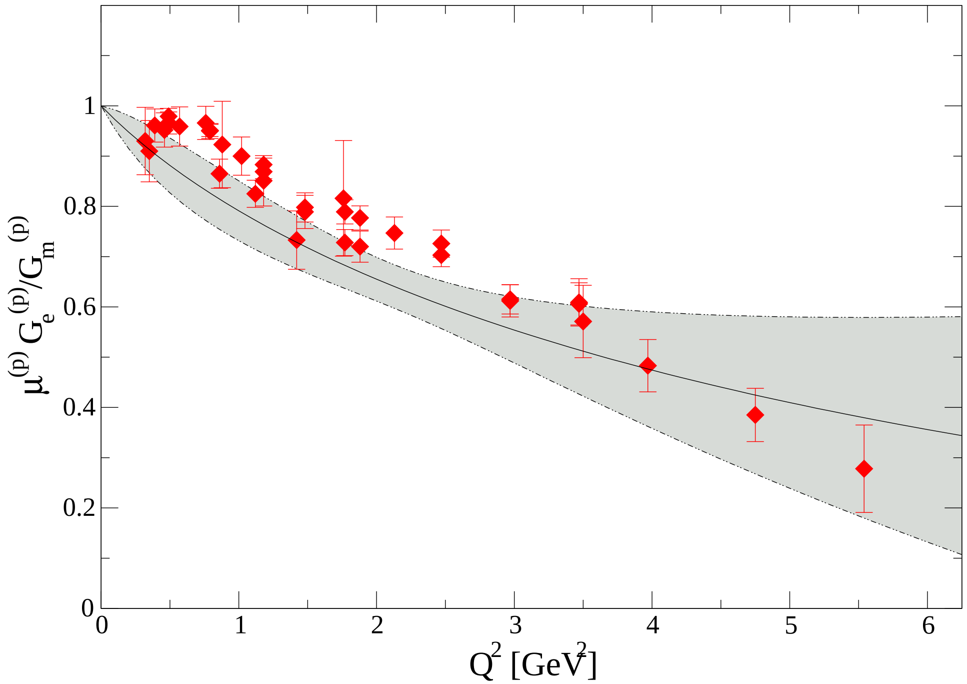
BaBar $|G_E/G_M|$ measurements vs previous ones and dispersion relation prediction (yellow) based on JLab space-like G_E/G_M and analyticity



- **Nonperturbatively $O(a)$ improved Wilson (clover) fermions**
- $N_f = 2$ **dynamical configurations**
- $N_f = 2$ **lattice spacing** $a = 0.07 - 0.11$ **fm;** **Length of the spacial box** $L = 1.4 - 2$ **fm**
- $N_f = 2$ **dynamical configurations**

disconnected contributions neglected!





- **chiral extrapolation compared with JLab data**

M.Goeckeler, 14.10.05

Also N to Δ transition form factors, C. Alexandrou *et al.* PRL94(2005)021601

SPECTRAL FUNCTIONS – GENERALITIES

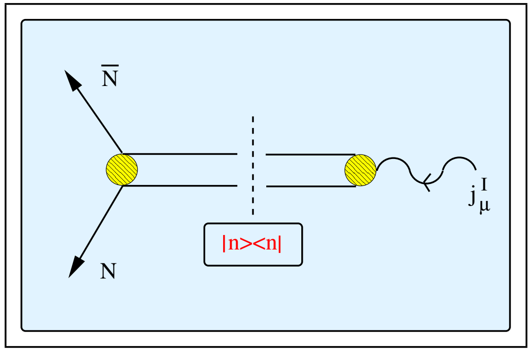
- Spectral decomposition:

$$\mathrm{Im}\,\langle \bar{N}(p')N(p)|J_\mu^I|0\rangle \sim \sum_n \langle \bar{N}(p')N(p)|n\rangle\langle n|J_\mu^I|0\rangle \Rightarrow \mathrm{Im}\,F$$

⋆ on-shell intermediate states

⋆ generates imaginary part

⋆ accessible physical states



- *Isoscalar* intermediate states: $3\pi, 5\pi, \ldots, K\bar{K}, K\bar{K}\pi, \pi\rho, \ldots +$ poles $\longrightarrow t_0 = 9M_\pi^2$
- *Isovector* intermediate states: $2\pi, 4\pi, \ldots +$ poles $\longrightarrow t_0 = 4M_\pi^2$
- Note that some poles are *generated* from the appropriate continua

SPACE-LIKE FORM FACTORS

- present best fit
- 4 effective IS poles
- 3 effective IV poles
- χ^2/dof = 0.84

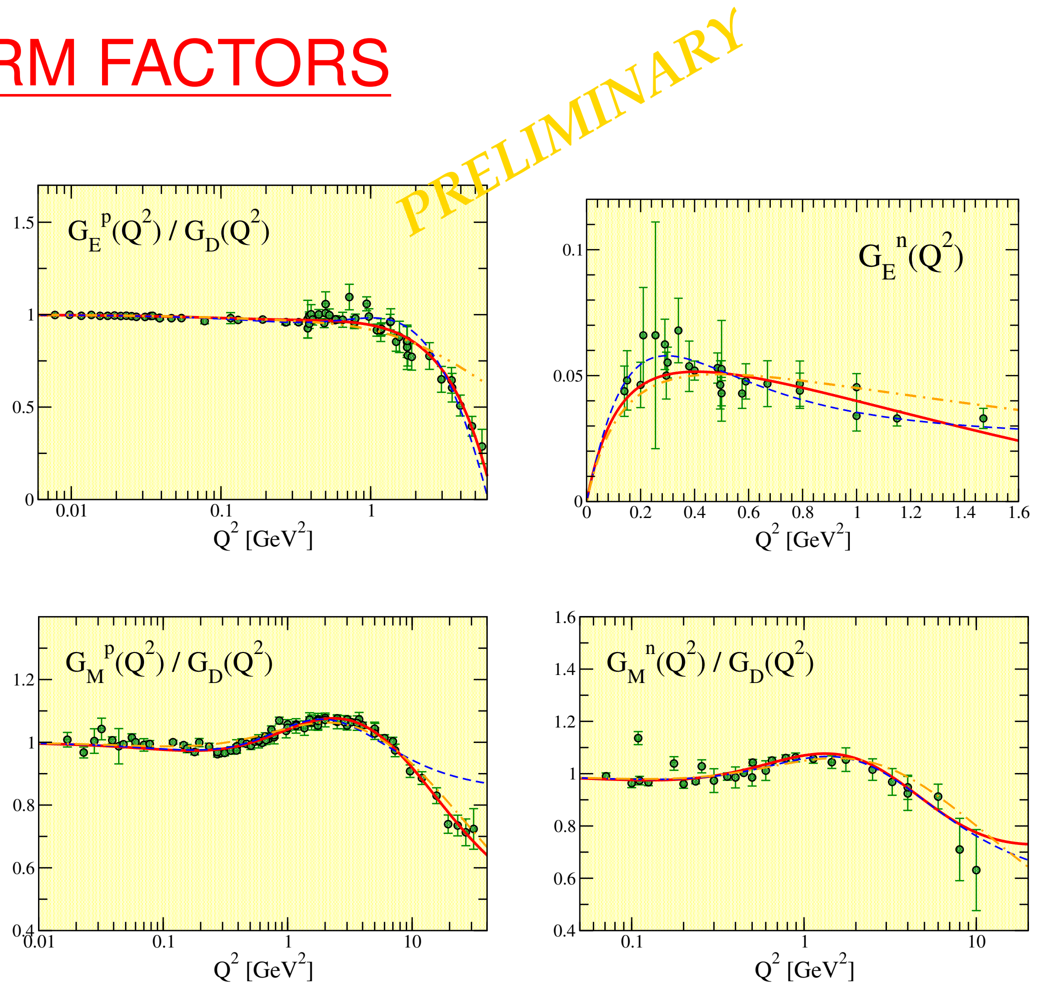
Improved description

⋆ JLab data described

⋆ higher mass poles not at physical values

MMD 96, HMD 96, HM 04 (— · —)

$$G_D(Q^2) = \left(1 + \frac{Q^2}{0.71\ \mathrm{GeV}^2}\right)^{-2}$$

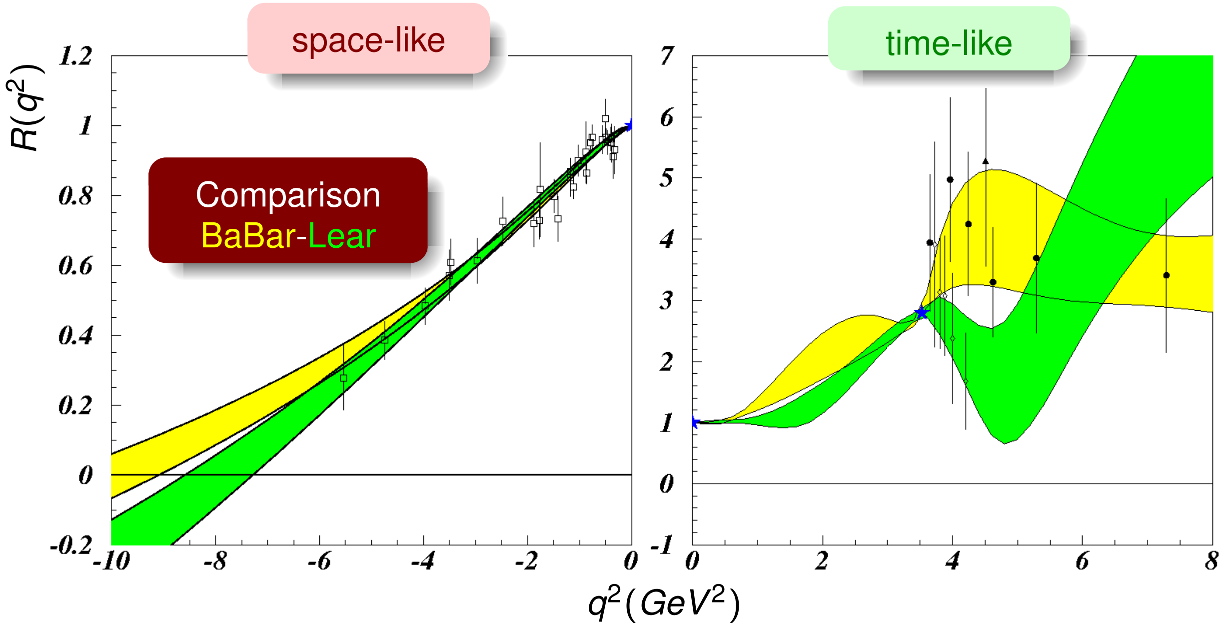


- **fitting also time-like form factors more complicated**

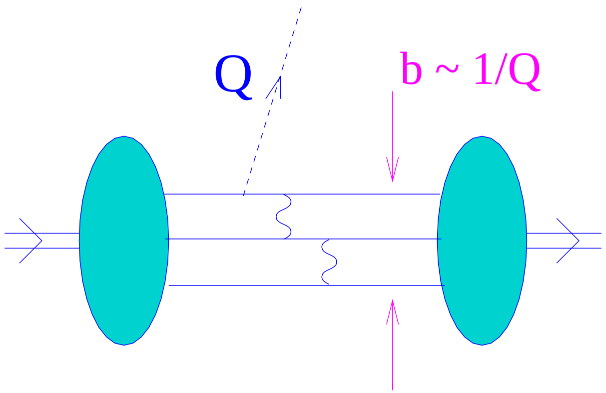
Dispersion Relations (cont.-d)

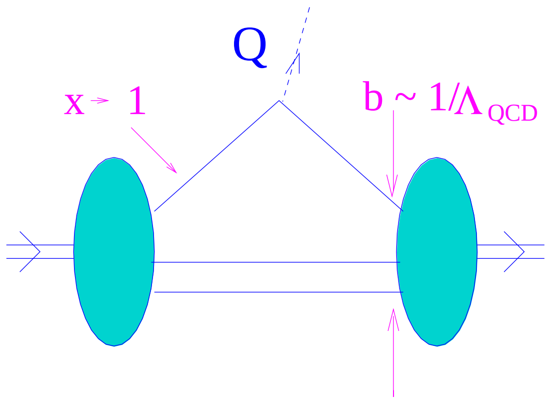
$R(q^2)$

Reconstructed R in space-like and time-like region



Paradigm: Soft vs. Hard





Hard rescattering:

Small b

Average $0 < x < 1$

Soft (Feynman):

Average b

Large $x \to 1$

- **Dominance of hard rescattering is only true for simplest reactions**
- **Soft constributions enter at the same power in $1/Q^2$ as higher-twist hard contributions**
- **Separation of soft and hard contributions is nontrivial and not unique**

- **Estimates of soft terms require a nonperturbative approach that would be explicitly consistent with perturbative QCD factorization**

Nucleon Distribution Amplitudes

Twist-3

$$\langle 0|\,\varepsilon^{ijk}\left(u_i^\uparrow(a_1 z) C \not{z} u_j^\downarrow(a_2 z)\right) \not{z} d_k^\uparrow(a_3 z)\,|P(P,\lambda)\rangle = -2pz \not{z} N^\uparrow(P) \int \mathcal{D}\xi e^{-ipz\sum \xi_i a_i} \Phi_3(\xi_i)$$

Twist-4

$$\langle 0|\,\varepsilon^{ijk}\left(u_i^\uparrow(a_1 z) C \not{z} u_j^\downarrow(a_2 z)\right) \not{p} d_k^\uparrow(a_3 z)\,|P(P,\lambda)\rangle = -2pz \not{p} N^\uparrow(P) \int \mathcal{D}\xi e^{-ipz\sum \xi_i a_i} \Phi_4^\parallel(\xi_i)$$

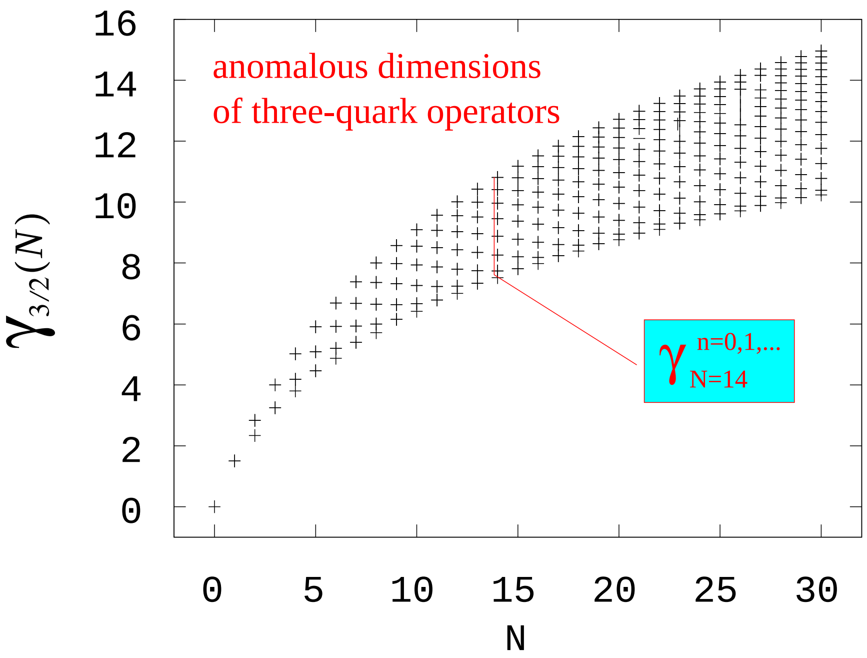
$$\langle 0|\,\varepsilon^{ijk}\left(u_i^\uparrow(a_1 z) C \gamma_\perp u_j^\downarrow(a_2 z)\right) \gamma_\perp \not{z} d_k^\downarrow(a_3 z)\,|P(P,\lambda)\rangle = 4 m_N \not{z} N^\uparrow(P) \int \mathcal{D}\xi e^{-ipz\sum \xi_i a_i} \Phi_4^\perp(\xi_i)$$

$$\langle 0|\,\varepsilon^{ijk}\left(u_i^\uparrow(a_1 z) C \not{p} \not{z} u_j^\downarrow(a_2 z)\right) \not{z} d_k^\uparrow(a_3 z)\,|P(P,\lambda)\rangle = 2 m_N pz \not{z} N^\uparrow(P) \int \mathcal{D}\xi e^{-ipz\sum \xi_i a_i} \Phi_4^T(\xi_i)$$

- **Momentum fraction distributions of quarks in the state with minimum number of Fock constituents at small (zero) transverse separations**
- **Complementary to parton distributions that include summation of all Fock states**
- **Largerly unknown**

Scale Dependence

The three quarks in the nucleon with the given spin J can still be in different parton states
$\Rightarrow$ a nontrivial mixing matrix:



The lowest anomalous dimension is separated from the rest by a finite 'mass gap'. This is due to 'binding' of the scalar diquark and in a different context may lead to color superconductivity

- Large parts of the evolution equations are completely integrable
- Large part of the two-loop kernel calculated (Belitsky, Korchemsky, Mueller '05)

PQCD: Collinear factorization

Classical Brosky-Lepage framework: two hard gluon exchanges

$$F_1^{p,n} \sim \frac{\alpha_s^2(Q)}{Q^4} \qquad F_2^{p,n} \sim \frac{\alpha_s^2(Q)}{Q^6} \ln^2 Q^2/\mu_{IR}^2 \equiv \frac{1}{Q^6}$$

- **Pauli form factor corresponds to the twist-3 — twist-4 inteference;**

  Explicit calculation (Belitsky, Ji, Yuan '03) confirms that collinear factorization is broken

- **The experimental behaviour $F_2(Q^2)/F_1(Q^2) \sim 1/Q$ is not supported by QCD**
- **Reason for the large difference between space-like and time-like form factors unclear**

? **Applicability at realistic values of Q^2 Wide-spread sceptizism**

? **Possible and necessary to calculate the radiative correction (new color structures)**

PQCD: Modified (k_t) factorization

Sterman, Li, '92: make use of the Sudakov suppression of large transverse separations

- **Introduce transverse-momentum dependent pion wave functions**

$$F_\pi(Q^2) = \int dx_1 dx_2 \int d^2b \phi_\pi(x_1, b)\phi_\pi(x_2, b) e^{-S(x_i,b)} T_H(x_i, Q, b)$$

- **Retain the transverse-momentum dependence of the hard kernel**

$$T_H^{LO}(x_i, Q, b) = \frac{4g^2 C_F}{x_1 x_2 Q^2 + (k_{1\perp} + k_{2\perp})^2}$$

- **Sudakov suppression factors:** $b \sim 1/Q$

$$e^{-S(x_i,b)} = \exp\left\{ -\frac{C_F}{8\pi} \sum_{i=1}^{2} \ln^2 \frac{b^2 x_i^2 Q^2}{b_0^2} + (x_i \leftrightarrow 1 - x_i) \right\}$$

? **Exponentiated π^2-terms: difference between space-like and time-like form factors**

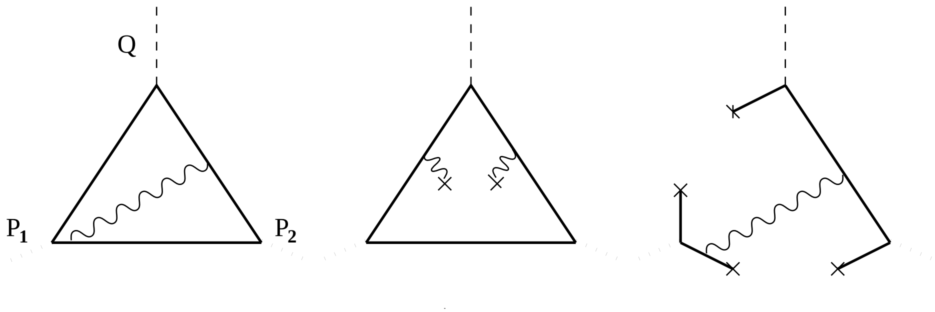
◊ **Sudakov suppression too weak: "Intrinsic" transverse-separation dependence of the wave functions cannot be ignored**

◊ **Considerable model dependence (e.g. Bolz *et al.* '95)**

Adding soft terms: Why Light-Cone Sum Rules?

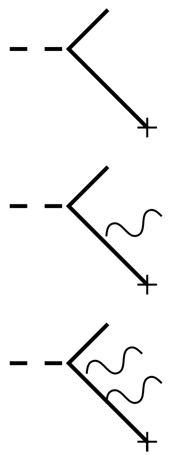
QCD Sum Rules

Nesterenko, Radyushkin '82; Ioffe, Smilga '82



$$= \left(\frac{1}{Q^4} + \frac{\alpha_s(Q)}{Q^2}\right) + \text{const}\cdot\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + Q^2 \cdot \langle \bar{q} q \rangle^2 + \ldots$$

- **Power counting in Q^2 is not consistent with OPE ?**



→ $\delta(1-x)$

→ $\delta'(1-x)$

→ $\delta''(1-x)$

Expansion goes in derivatives of the delta-function!

Sum of all orders: → ***const (1-x)***

⇒ Need resummation of the OPE to all orders

SVZ Sum Rules	Light-Cone Sum Rules
• Short-distance Expansion	• LC-Expansion
• Local operators	• Light-ray operators
• Parameter: Dimension	• Parameter: twist
	• Conformal expansion
• Vacuum Condensates	• Hadron Distribution amplitudes

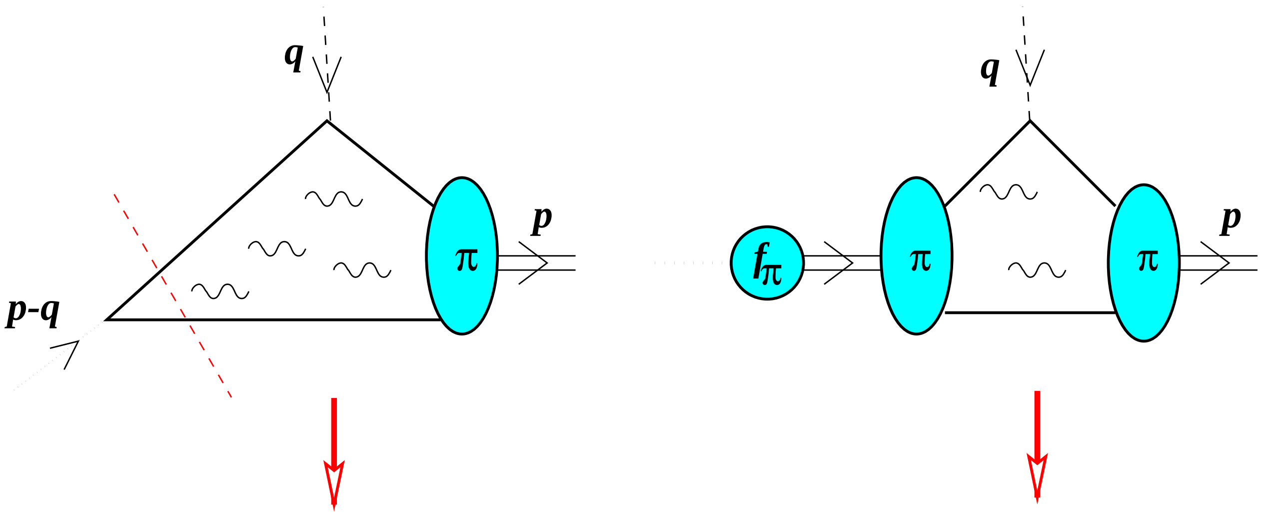
◇ Mean-field-approach (SVZ) vs. the expansion in rapidly varying background fields (LCSR)

♡

Premium: Consistent with the expansion in powers of large momentum (mass)

Example: Light-Cone Sum Rule for the Pion Form Factor

duality:



$$\int_0^{s_0} ds\, \frac{\mathrm{Disc}_{(p-q)^2} T(p,q)}{s-(p-q)^2} \overset{p^2\sim -1\mathrm{GeV}^2}{=} f_\pi \cdot \frac{1}{m_\pi^2-(p-q)^2} \cdot F_\pi(Q^2)$$

- $T(p,q)$ **is calculated in terms of pion wave functions of increasing twist**
- **No condensates!**
- **Dispersion relation in one variable**

Braun, Halperin, '94

Braun, Khodjamirian, Maul, '99

Study Case: asymptotic distribution amplitude

Expanding the sum rule at $Q^2 \gg s_0$

$$F_\pi(Q^2) = \frac{3\alpha_s C_F}{2\pi Q^2}\int_0^{s_0} ds\, e^{-s/M^2} + \frac{6}{Q^4}\int_0^{s_0} ds\, s\, e^{-s/M^2}\left\{1-\frac{\alpha_s C_F}{4\pi}\left[10-\frac{\pi^2}{3}+\ln^2\frac{Q^2}{s}\right]\right\}+\ldots$$

The leading term: $\int_0^{s_0} ds\, e^{-s/M^2} \to 4\pi^2 f_\pi^2$

$$F_\pi^{\text{as}}(Q^2) \longrightarrow \frac{8\pi\alpha_s f_\pi^2}{Q^2} = \frac{8\pi\alpha_s f_\pi^2}{9Q^2}\left|\int_0^1 du\,\frac{\varphi_\pi(u)}{\bar{u}}\right|^2$$

Soft-hard separation with explicit momentum-fraction cutoff:

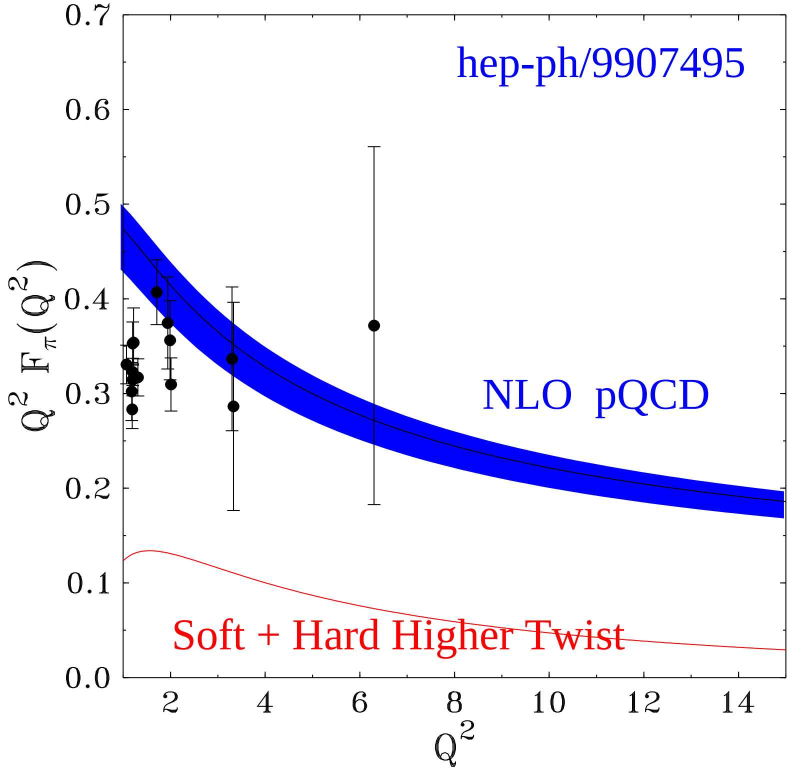
$$\int_0^1 du = \int_0^{u_0} du + \int_{u_0}^1 du; \qquad u_0 = 1 - s_0/Q^2$$

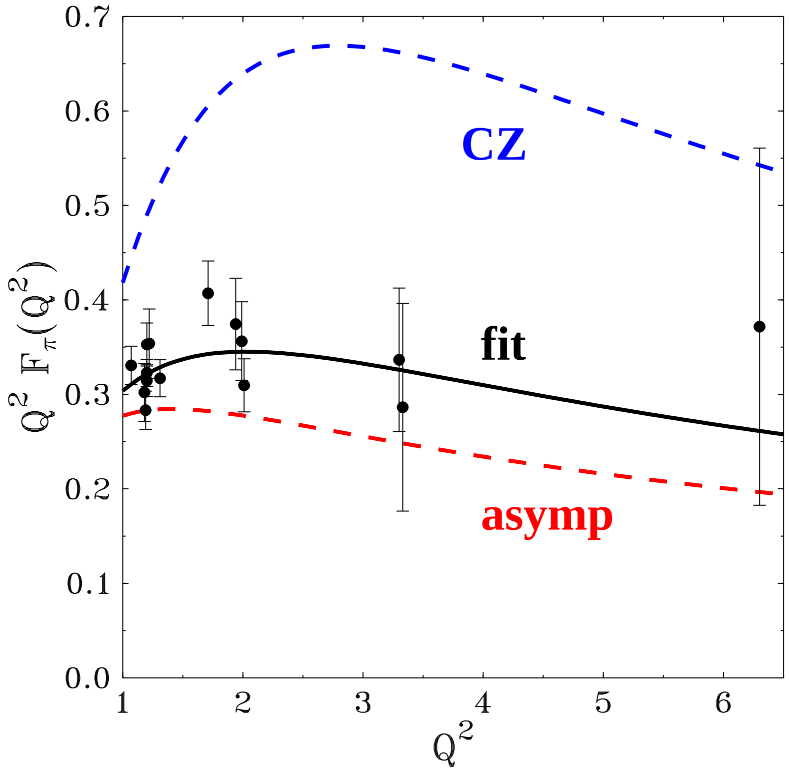
$$F_\pi^{\text{hard}}(Q^2) = \frac{3\alpha_s C_F}{2\pi Q^2} s_0 \left\{1-\frac{s_0}{Q^2}\left[\frac{13}{2}-\frac{\pi^2}{6}+\ln\frac{Q^2}{s_0}\ln\frac{\mu^2}{s_0}+\ln\frac{\mu^2}{s_0}+2\ln\frac{Q^2}{s_0}\right]\right\}$$

$$F_\pi^{\text{soft}}(Q^2) = \frac{3s_0^2}{Q^4}+\frac{3\alpha_s C_F}{4\pi Q^4}s_0^2\left\{\frac{5}{2}+\ln^2\frac{\mu^2}{s_0}-\ln^2\frac{Q^2}{\mu^2}+2\ln\frac{\mu^2}{s_0}+3\ln\frac{Q^2}{s_0}\right\}$$

Lessons to be learnt

♡ **LCSR are fully consistent with pQCD:**





- **Observe a complicated interplay of soft and hard contributions;**

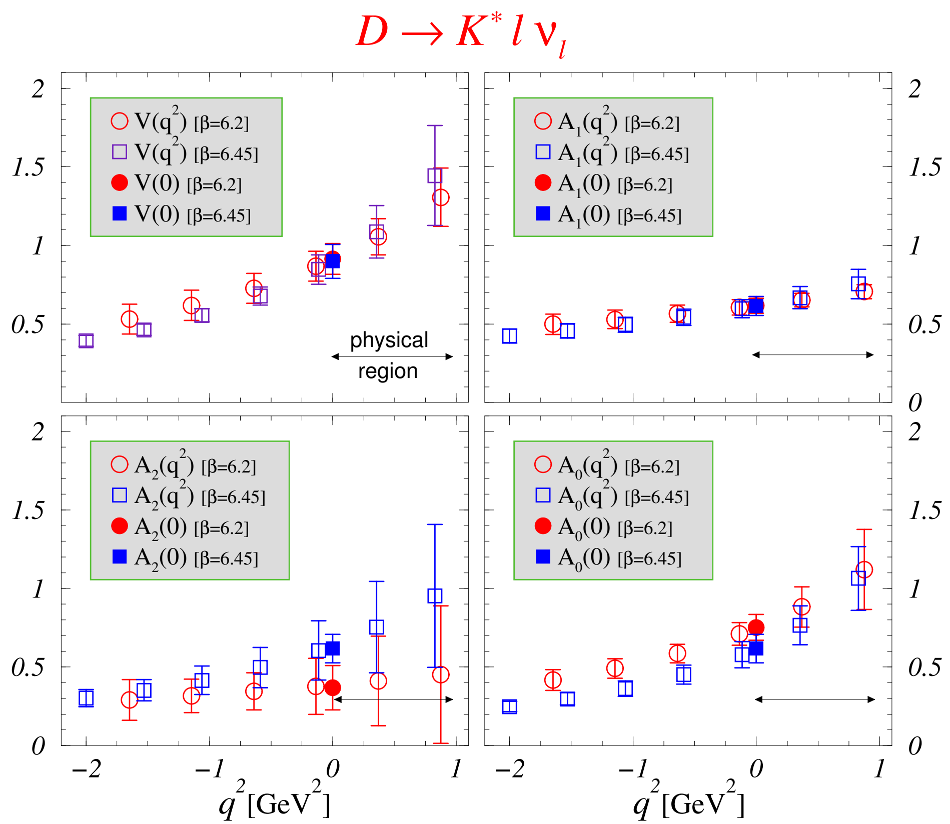
  perturbation theory might be rescued by the cancellation between soft and hard higher-twist corrections

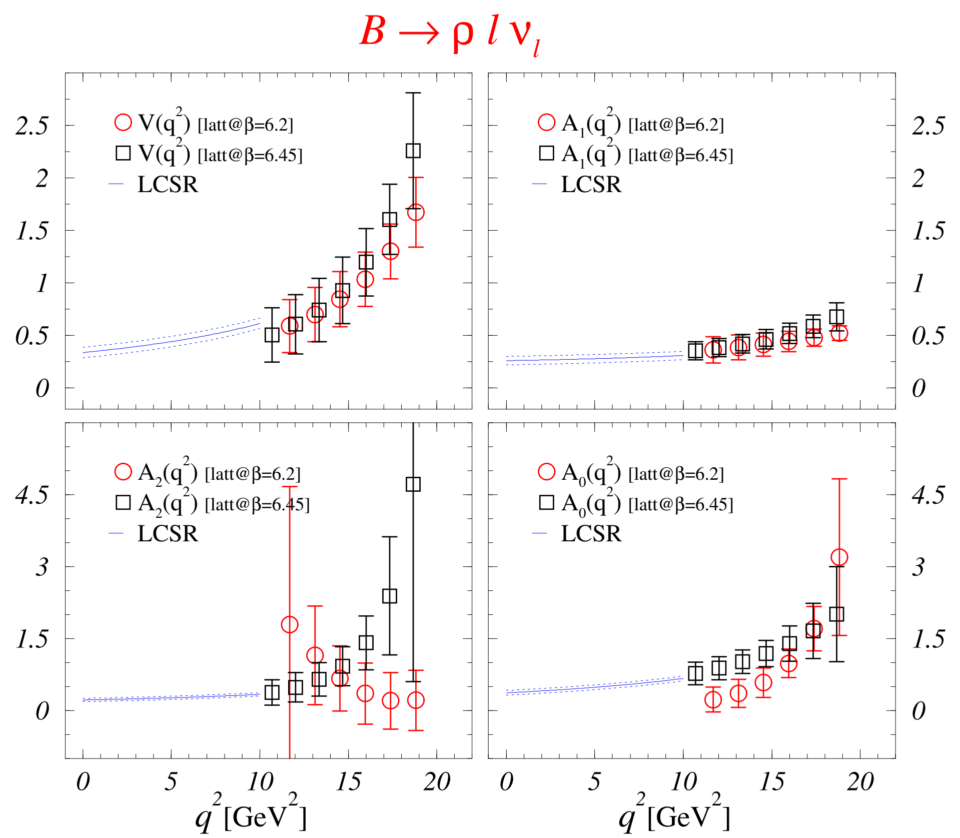
- **Further theoretical progress possible**

LCSR for Heavy meson decays

Lattice calculations: **A. Abada *et al.*, hep-lat/0209116**

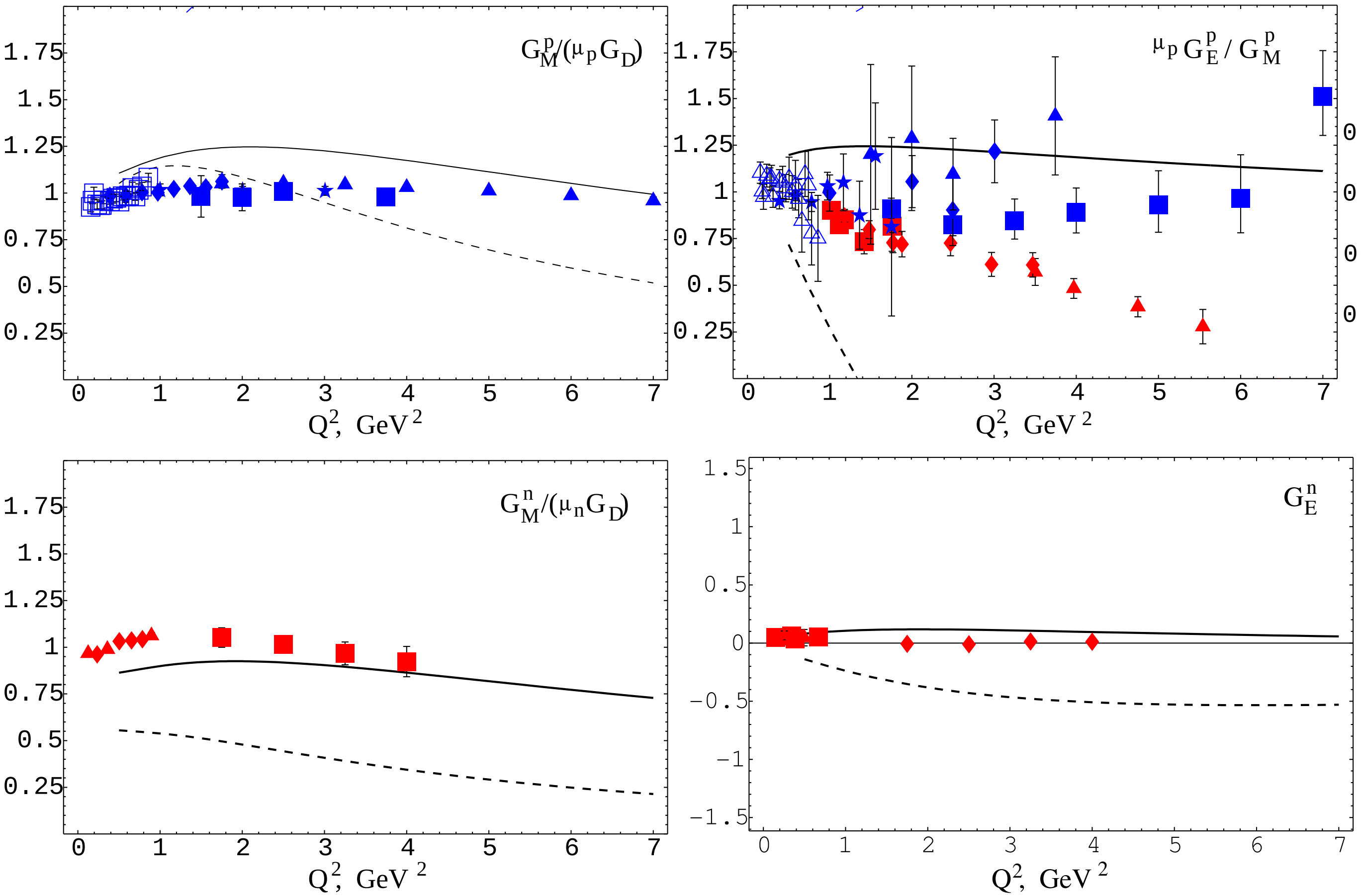
LCSR: **P. Ball, V. Braun, PRD 58 (1998) 094016**





Nucleon electromagnetic form factors

- **Tree-level light-cone sum rules:**



- **solid curves:**

  asymptotic distribution amplitudes
- **dashed curves:**

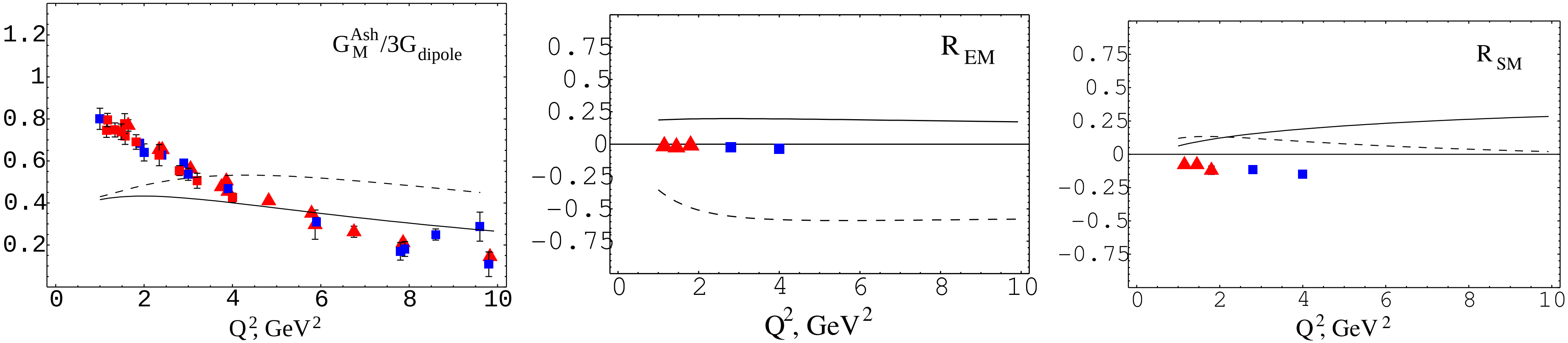
  extended CZ model

V. Braun, A. Lenz, M. Wittmann

paper in preparation

$N\Delta\gamma$ transition form factors

- **Tree-level light-cone sum rules:**



- **solid curves:** *asymptotic distribution amplitudes*
- **dashed curves:** *extended CZ model*

V. Braun, A. Lenz, G. Peters, A. Radyushkin

hep-ph/0510237

Electroproduction of soft pions

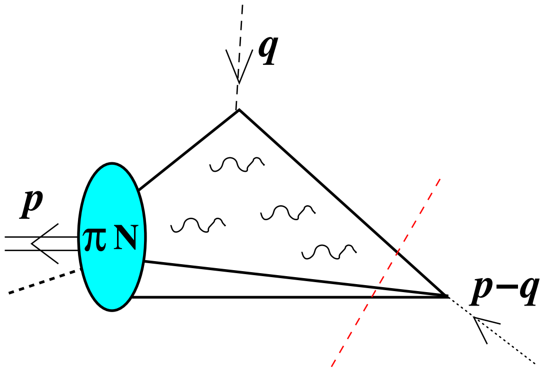
- $Q^2 \to \infty$ **does not commute with the chiral limit** $m_\pi \to 0$**:** $Q^2 \ll \Lambda^3/m_\pi$ **vs.** $Q^2 \gg \Lambda^3/m_\pi$

at the threshold

$$\langle N\pi(P-q)|j_\mu^{\rm em}|N(P)\rangle = \frac{i}{f_\pi}\bar{N}(P-q)\left[\gamma_\mu\gamma_5 G_A^{\pi N}(Q^2) - \frac{1}{2m}\gamma_5 q_\mu G_P^{\pi N}(Q^2)\right]N(P)$$

chiral rotation:

P. Pobylitsa, M. Polyakov, M. Strikman '01



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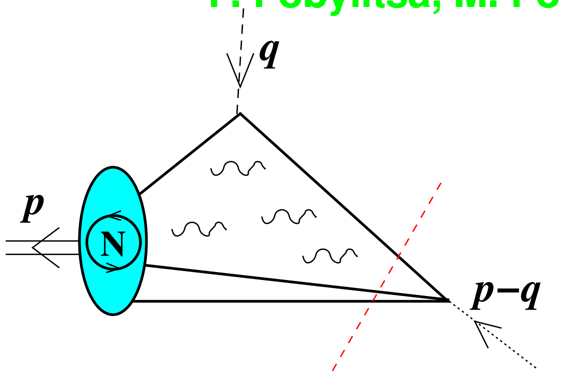


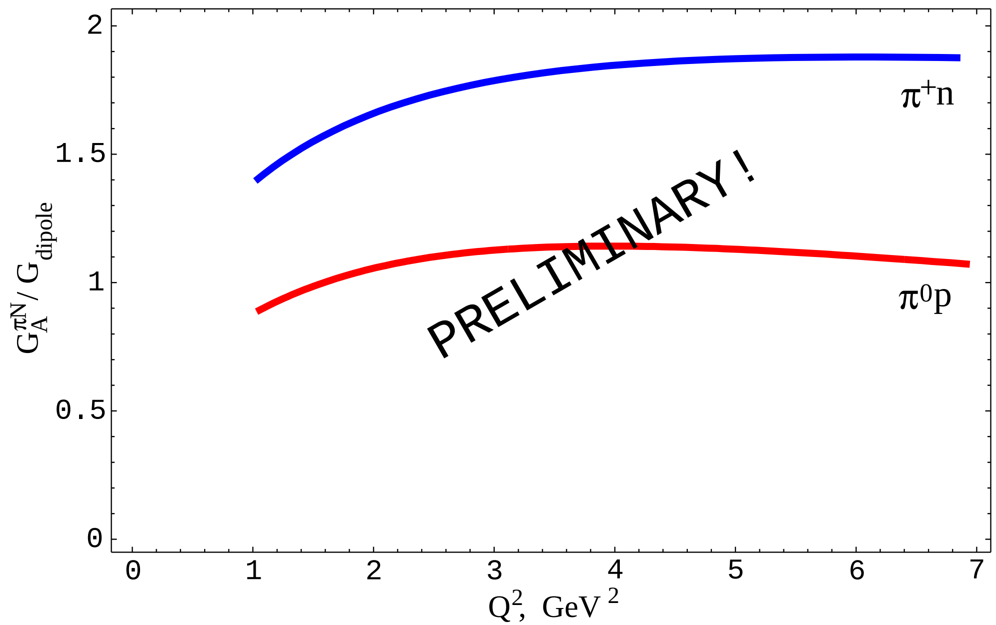
$$|p\uparrow\rangle = \frac{\phi_s(x)}{\sqrt{6}}|2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle + \frac{\phi_s(x)}{\sqrt{2}}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

$$|p\uparrow\pi^0\rangle = \frac{\phi_s(x)}{2\sqrt{6}f_\pi}|6u_\uparrow d_\downarrow u_\uparrow + u_\uparrow u_\downarrow d_\uparrow + d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_s(x)}{2\sqrt{2}f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

$$|n\uparrow\pi^+\rangle = \frac{\phi_s(x)}{\sqrt{12}f_\pi}|2u_\uparrow d_\downarrow u_\uparrow - 3u_\uparrow u_\downarrow d_\uparrow - 3d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_s(x)}{2f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

V. Braun, A. Lenz, A. Peters *work in progress*

- **Tree-level light-cone sum rules, preliminary:**



- **solid curves:** *asymptotic distribution amplitudes*

V. Braun, A. Lenz, A. Peters; work in progress

Outlook

- many new experimental data; more to come
- Numerical experiment increasing in importance

- Understanding form factors:
  - ? Interplay of soft and hard mechanisms
  - ? Time-like vs. Space-like

- To be done:
  - Two-loop PQCD
  - radiative corrections to LCSR
  - soft-pion emission in hard processes

- Mid-term goal: information on nucleon distribution amplitide, in particular average momentum fraction carried by the three valence quarks to 5-7% precision

	$u^{\uparrow}$	$u^{\downarrow}$	$d^{\uparrow}$
asymptotic	33%	33%	33%
CZ model	58%	19%	23%