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Brane world generation by matter and gravity

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A. A., V. A., P. Giacconi, R. Soldati,
JHEP 0307, 063 (2003); JHEP 0507, 003 (2005);
TMF (2006) in press .

Сто лет со дня рождения
Матвея Петровича Бронштейна,
профессора ЛГУ
1906 -- 1938

"Будущая физика не удержит того странного и неудовлетворительного деления, которое сделало квантовую теорию "микрофизикой" и подчинило ей атомные явления, а релятивистскую теорию тяготения -- "макрофизикой", управляющей не отдельными атомами, а лишь макроскопическими телами. Физика не будет делиться на микроскопическую и космическую: она должна стать и станет единой и нераздельной."

М. П. Бронштейн, 1930

:

Why extra-dimensions ?

- Unification of gravity and gauge interactions of elementary particles.

  Kaluza-Klein idea: 4-dim.gravity + photon = 5-dim gravity

- Quantization of gravitational interactions.

  M-theory: strings in 10 or 11 dim.

- Higgs mass hierarchy problem.

  Planck scale of order few TeV is induced by extra-dim.gravity

- Cosmological constant problem.

  Large cosm.constant from extra dim. (from the “bulk”)
  Is compensating a large cosm. constant induced by the matter in our Universe

How to implement extra-dimensions?

◇ Kaluza-Klein *v.s.* Rubakov-Shaposhnikov = Brane Worlds

◇ Gravity in extra-dimensions **BUT** matter in our Universe

$\Rightarrow M_{Pl} \sim 1$ TeV

◇ Large or infinite extra dimensions ⇒ Anti-de-Sitter (AdS) space

⇒ mass hierarchy in the Standard Model

◇ Thick (= Fat) Brane generation: spontaneous breaking of translational invariance ⟹ domain wall pattern of the vacuum state

(4 + 1)-dim. fermion model with strong four-fermion interaction
$\bigoplus$ *(induced) gravity*

◇ Brane World generation in AdS_5: light fermions, scalar and massless gravitons live on a (*3+1*)-dim. brane

◇ (*3 + 1*)-dim. cosmological constant is zero! Brane World is essentially flat

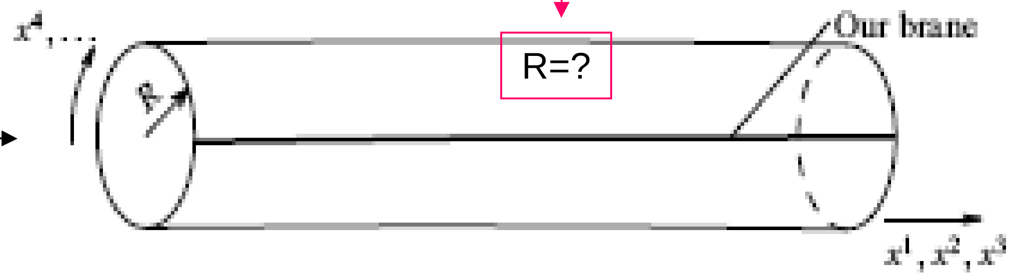
◇ Coupling of scalar matter to quarks and leptons is suppressed ⟹ scalar matter ≡ Dark Matter

Kaluza-Klein vs. Rubakov-Shaposhnikov = Brane Worlds

$R \lesssim (\mathrm{TeV})^{-1} \sim 10^{-17}$ cm

Arkani-Hamed N, Dimopoulos S, Dvali G

Factorizable geometry →



$$\left[\Box^{(4)} - \frac{\partial^2}{\partial (x^4)^2} - \ldots - \frac{\partial^2}{\partial (x^N)^2}\right] h = 0\,,$$

$$h_n = \exp\left(\mathrm{i}\omega t - \mathrm{i}\, p_i x^i\right) \exp\left(-\mathrm{i}\,\frac{x^4}{R}\, n_4\right) \ldots \exp\left(-\mathrm{i}\,\frac{x^N}{R}\, n_N\right)$$

where h is the deviation of metrics from that of flat space

$${}^{(4)}p^2 \equiv \omega^2 - {}^{(3)}\mathbf{p}^2 = \frac{\mathbf{n}^2}{R^2}\,.$$

The state with $\mathbf{n} = 0$ has zero four-dimensional mass, and it is the usual graviton. Massive gravitons with $\mathbf{n} \neq 0$ do not contribute to gravitational interactions at large distances since they lead to a Yukawa-type potential, falling off exponentially at $r \gg R$.

$$V(r) = G_* \frac{m_1 m_2}{r^{1+d}} \qquad r \ll R$$

$$V(r) = G \frac{m_1 m_2}{r}\,, \qquad r \gg R$$

$$r \sim R \qquad \frac{G_*}{R^d} \sim G\,.$$

$$G_* = \frac{1}{M_*^{2+d}} \qquad (RM_*)^d = \frac{M_{\mathrm{Pl}}^2}{M_*^2}\,.$$

! hierarchy problem $m_Z \ll M_{\mathrm{Pl}}$! choose $M_* \sim \mathrm{TeV}$

$d = 1$: $R \sim 10^{15}$ cm; No!

$d = 2$: $R \sim 0.1$ cm No!

Successful calculations of big-bang nucleosynthesis R $< 2\ \mu m$ d = 3

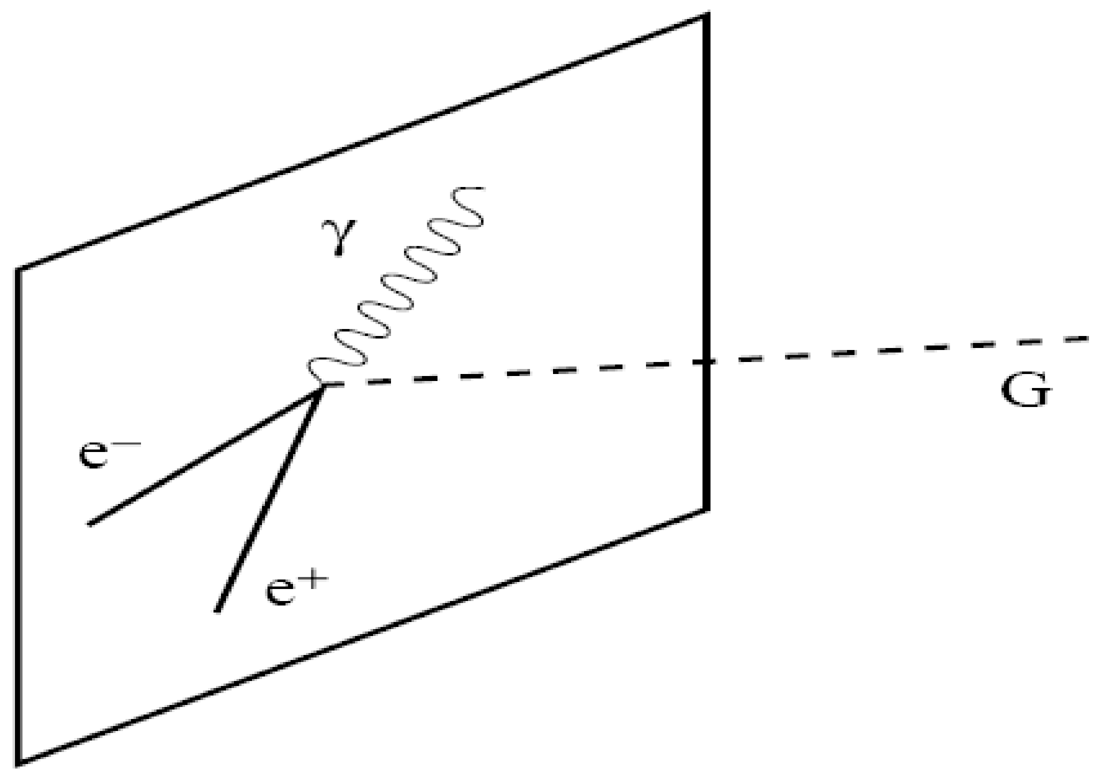
Absence of diffuse background of cosmological gamma rays R $< 0.05\ \mu m$ d = 3

Maybe!

But asymmetric extra-dim.: 1 mm + several small ?!

==

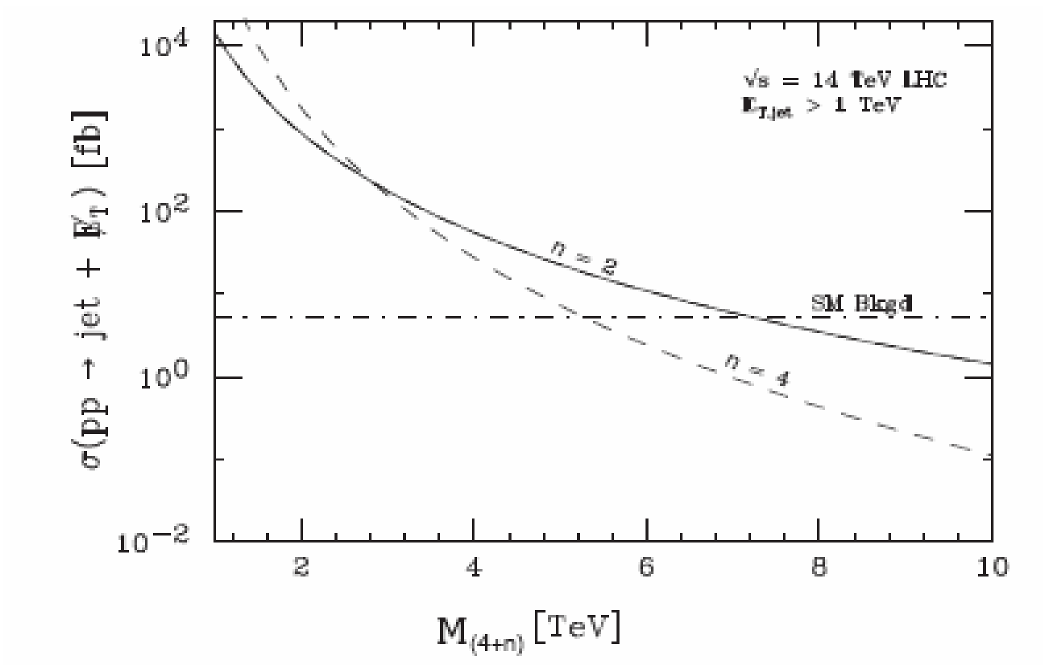
Missing energy events



$$q\bar{q} \rightarrow \text{jet} + \not{E}_T$$

$$\sigma(e^+e^- \rightarrow \gamma + \not{E}_T) \sim \frac{\alpha}{M_{Pl}^2} N(E) \sim \frac{\alpha}{E^2} \left(\frac{E}{M}\right)^{d+2}$$

$N(E)$ is the number of KK gravitons with masses below E



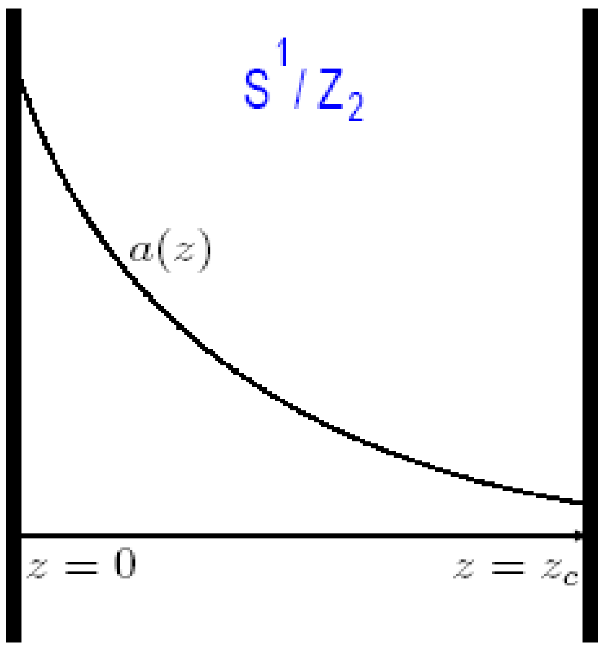
L. Randall and R. Sundrum (I) : gravity in the bulk + brane tension

$$S = M^3 \int d^4x \int_{-z_c}^{z_c} dz \sqrt{g}\,(R - 2\Lambda) \; - \sigma_0 \int_{z=0} d^4x \sqrt{h_0} \; - \sigma_c \int_{z=z_c} d^4x \sqrt{h_c}$$

where g is the bulk metric and h_0, h_c are the metrics on the branes at $z = 0$, z_c

Non-factorizable geometry **AdS$_5$**

Orbifold z → - z; $-z_c \leq z \leq z_c$



$+\sigma$ *Planck-brane* $M_{Pl} \sim e^{kz} M_{EW}$ $-\sigma$ *TeV-brane*

$$ds^2 = \tilde{g}_{ab} dx^a dx^b = e^{-2k|z|} \eta_{\mu\nu} dx^\mu dx^\nu + dz^2$$

$$S_{eff} = M^3 \int d^4x \, \sqrt{\bar{g}} \bar{R} \int_{-z_c}^{z_c} dz \; e^{-2k|z|} + \ldots$$

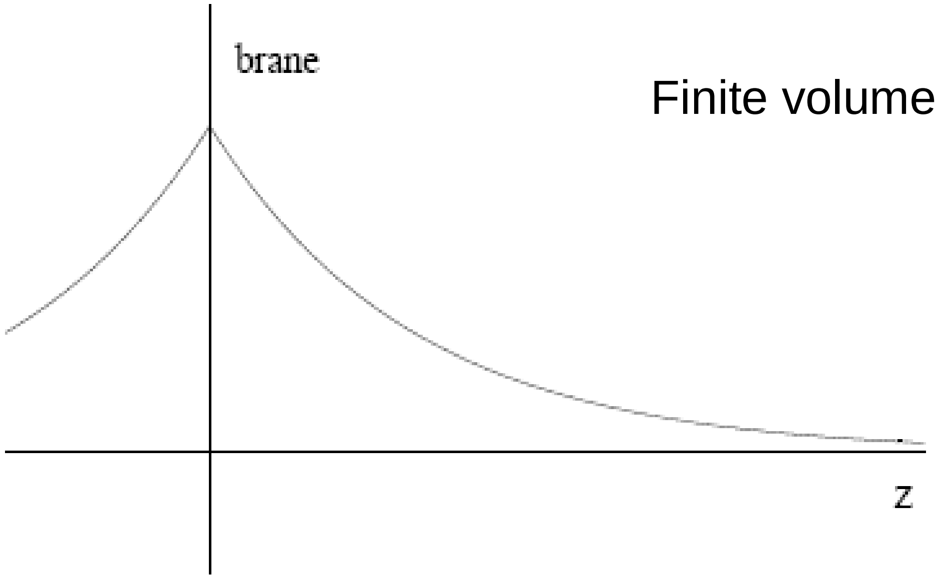
Dim-4

Lengh of 5[th] dimension

$$M_{pl}^2 = \frac{M^3}{k} \left[1 - e^{-2kz_c}\right]$$

$z_c \longrightarrow \infty$

One brane - L. Randall and R. Sundrum (II)



Finite volume of extra dimensions $\sim$ 1/k

Modified Newton law

$$V(r) = G\,\frac{m_1 m_2}{r}\left[1 + \frac{2}{3k^2r^2} + \mathcal{O}(1/r^3)\right]$$

Exp. $k > 2 \cdot 10^{-12} GeV$ = 1/10 mm

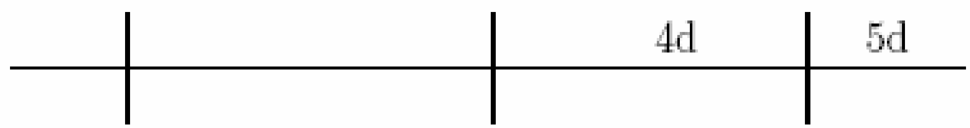
Torsion Oscillators

Infinite volume of extra-dimensions

Dvali G, Gabadadze G, Porrati M
Rubakov V.

Gravity in the bulk + gravity on the brane

$$S^{\rm eff} = M_*^{N+2}\int d^{N+1}x\,\sqrt{{}^{(N+1)}g}\;{}^{(N+1)}R + M_{\rm Pl}^2\int_{\rm brane} d^4x\,\sqrt{{}^{(4)}g}\;{}^{(4)}R + \dots$$



$$w \ll R_c \sim M_*^{-1} \ll r_c \sim \frac{M_{\rm Pl}}{M_*^2} \sim 10^{28}\ {\rm cm}$$

UV cutoff (w)

5d or strong coupling

Size of the Universe (10^{28} cm)

Deviation from Newton law at very large and very small distances

Trapping of fermions on (3 + 1)-dim. brane

Dim-5 fermion bi-spinor $\psi(X)$, $(X_\alpha) = (x_\mu, z)$ coupled to a scalar field $\Phi(X)$,

$$[\, i\gamma_\alpha \partial^\alpha - \Phi(X)\,]\psi(X) = 0 \ , \quad \gamma_\alpha = (\gamma_\mu, -i\gamma_5) \ , \quad \{\gamma_\alpha, \gamma_\beta\} = 2g_{\alpha\beta}$$

Trapping of light fermions on a four-dimensional layer
== domain wall == thick brane, localized, say, at $z = 0$
is promoted by a *topological*, background configuration of scalar field

$$\langle \Phi(X) \rangle_0 = \varphi(z),$$

owing to zero-modes in fermion spectrum:

$$(-\partial_\mu \partial^\mu - \widehat{m}_z^2)\psi(X) \ = 0;$$

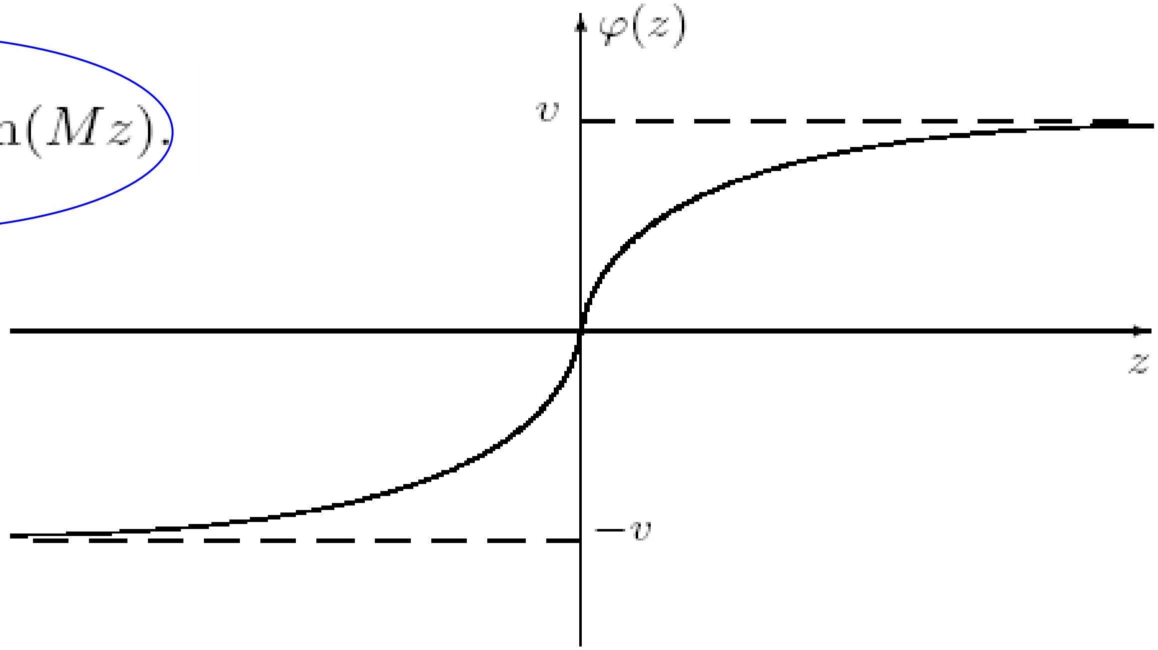
$$\widehat{m}_z^2 = -\partial_z^2 + \varphi^2(z) - \gamma_5 \varphi'(z) = \widehat{m}_+^2 P_L + \widehat{m}_-^2 P_R$$

where $P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$.

From the viewpoint of dim-4 Minkowski space-time $\psi(X)$ assembles an infinite set of dim-4 fermions.

An important example, a "kink" background,

$$\varphi(z) = M \tanh(Mz).$$

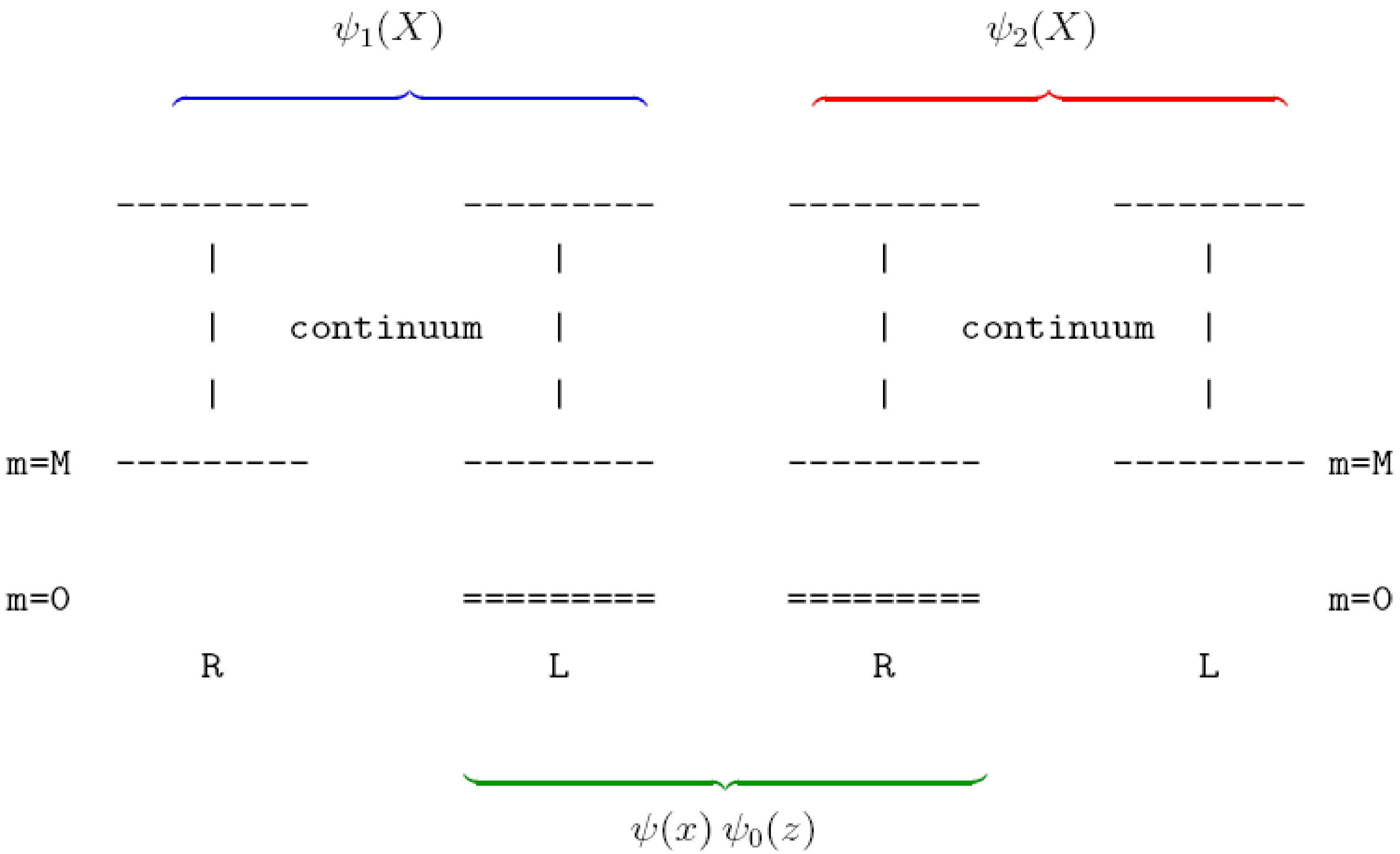


Normalizable zero mode appears in the mass operator $\widehat{m}_+^2$

$$\psi_0^+(x,z) = \psi_L(x)\,\psi_0(z) \ , \qquad \psi_0(z) \equiv \sqrt{M/2}\ \text{sech}(Mz)$$

Weyl fermion arises !

Fermion spectrum pattern



Heavy Dirac fermions have masses $m \geq M$, being de-localized in the extra-dimension. For light particle scattering with energies $\ll M$ all physics interplays on the brane with thickness $\sim 1/M$.

Quarks and leptons of the Standard Model are mainly massive. Therefore, for each light fermion in the Brane World one needs two five-dimensional proto-fermions $\psi_1(X), \psi_2(X)$ to generate left- and right-handed parts of a four-dimensional Dirac bi-spinor as zero modes. Those fermions have clearly to couple with opposite charges to the scalar field $\Phi(X)$, in order to produce the required zero modes with different chiralities,

$$[\, i \not\partial - \tau_3 \Phi(X)\,]\Psi(X) = 0 \ , \quad \not\partial \equiv \hat{\gamma}_\alpha \partial^\alpha \ , \quad \Psi(X) = \begin{pmatrix} \psi_1(X) \\ \psi_2(X) \end{pmatrix} \ ,$$

where $\hat{\gamma}_\alpha \equiv \gamma_\alpha \otimes 1_2$ and "Pauli matrices" $\tau_a \equiv 1_4 \otimes \sigma_a, \ a = 1, 2, 3.$

In this way one obtains a *massless Dirac particle on the brane*.

$$\Psi(X) = \begin{pmatrix} \psi_L(x)\,\psi_0(z) + \psi_{1L}^{(M)}(X) \\ \psi_{1R}^{(M)}(X) \\ \psi_{2L}^{(M)}(X) \\ \psi_R(x)\,\psi_0(z) + \psi_{2R}^{(M)}(X) \end{pmatrix} = \Psi^{(M)}(X) \bigoplus \{\psi(x)\,\psi_0(z)\}$$

The next task is to supply it with a light mass.

As the mass operator

$$\bar{\psi}_(x)\psi_(x) = \bar{\psi}_R(x)\psi_L(x) + \bar{\psi}_L(x)\psi_R(x)$$

mixes left- and right-handed components of dim-4 fermion it is embedded in the dim-5 Dirac operator with the mixing matrix $\tau_1 m_f$ of the fields $\psi_1(X)$ and $\psi_2(X)$,

$$\bar{\Psi}(X)\tau_1 m_f \Psi(X) = m_f\Big(\bar{\psi}_1(X)\psi_2(X) + \bar{\psi}_2(X)\psi_1(X)\Big)$$

For dynamical fermion mass generation one introduces the second scalar field $H(X)$ to make this job, replacing the bare mass,

$$\tau_1 m_f \longrightarrow \tau_1 H(X)$$

5-dim fermion self-interaction generates composite scalars $\Phi(X)$ and $H(x)$

$$\mathcal{L}^{(5)}(\overline{\Psi},\Psi) = \overline{\Psi}\; i\,\partial\!\!\!/\Psi + \frac{g_1}{4N\Lambda^3}\left(\overline{\Psi}\tau_3\Psi\right)^2 + \frac{g_2}{4N\Lambda^3}\left(\overline{\Psi}\tau_1\Psi\right)^2$$

$$\Longrightarrow \overline{\Psi}(i\,\partial\!\!\!/ - \tau_3\Phi - \tau_1 H)\Psi - \frac{N\Lambda^3}{g_1}\Phi^2 - \frac{N\Lambda^3}{g_2}H^2$$

where $N = 2\times 3\times N_c + (1.5 \div 2)\times 3 \simeq 22.5 \div 24$ is the total number of 5-dim fermion species related to the Standard Model.

1.5 = 1 Dirac f. + 1 Weyl f. 2 = 2 Dirac f.

Composite scalar fields: $\Phi \sim \overline{\Psi}\tau_3\Psi$; $H \sim \overline{\Psi}\tau_1\Psi$

τ-symmetry:

$$\Psi \longrightarrow \tau_1\Psi\ ; \quad \Phi \longrightarrow -\Phi\ ;\ \text{and}\ \Psi \longrightarrow \tau_2\Psi\ ; \quad \Phi, H \longrightarrow -\Phi, -H\ ;$$

Λ is a compositeness scale for scalar bosons emerging after the breakdown of the τ-symmetry.

Proceed to the Euclidean space and integrate out the high-energy part of the fermion spectrum, $\Psi_h(p) \equiv \Psi(p)\vartheta(|p|-\Lambda_0)\vartheta(\Lambda-|p|)$.

Low-energy lagrangian:

$$\mathcal{L}^{(5)}_{\rm low} = \overline{\Psi}_l(X)i\Big[\partial\!\!\!/ + \tau_3\Phi(X) + \tau_1 H(X)\Big]\Psi_l(X)$$

$$+\frac{N\Lambda}{4\pi^3}\left[\Big(\partial_\alpha\Phi(X)\Big)^2+\Big(\partial_\alpha H(X)\Big)^2-2\Delta_1\Phi^2(X)-2\Delta_2 H^2(X)+\Big(\Phi^2(X)+H^2(X)\Big)^2\right].$$

Critical point to generate spontaneous symmetry breaking!

$$\Delta_i = \frac{2\Lambda^2}{9g_i}\left(g_i - 9\pi^3)\right) \ll \Lambda^2.$$

Second critical point of τ-symmetry breaking

$$\Delta_1 \equiv M^2 > \Delta_2 \equiv \frac{1}{2}(M^2 \pm \mu^2); \quad \mu^2 \ll M^2$$

Stationary point (vacuum state) conditions

$$\left[2(\Delta_1 - \Phi^2 - H^2) + \partial_\alpha^2\right]\Phi = 0, \qquad \left[2(\Delta_2 - H^2 - \Phi^2) + \partial_\alpha^2\right]H = 0.$$

Vacuum state for $+\mu^2$:

$$\langle\Phi(X)\rangle_0 = M\tanh(\beta z)\ , \quad \langle H(X)\rangle_0 = \mu\,\mathrm{sech}(\beta z)$$

with $\beta = \sqrt{M^2 - \mu^2}$

In this phase the vacuum state breaks τ-symmetries and translational invariance.

! for $-\mu^2$ one finds $\langle H(X)\rangle_0 = 0,\ \beta \to M$

Ultra-low energy physics

Scalar field and fermion zero-modes for Standard Model multiplets with number of fermions $N = 21.5 \div 24$ and number of colors $N_c = 3$

$$\Phi(X) \simeq \langle \Phi(X) \rangle_0 + \phi(x)\phi_0(z) \ ; \quad \phi_0(z) \simeq \mathrm{sech}^2(Mz)\sqrt{\frac{3M\pi^3}{2\Lambda N}} \ ;$$
$$H(X) \simeq \langle H(X) \rangle_0 + h(x)h_0(z) \ ; \quad h_0(z) \simeq (\mathrm{sech}(Mz))^{1-2\epsilon}\sqrt{\frac{M\pi^3}{\Lambda N_c}} \ ; \quad \epsilon \equiv \frac{\mu^2}{M^2};$$
$$\Psi_j(X) \simeq \psi_j(x)\,\psi_{0,j}(z) \ ; \quad \psi_{0,j}(z) \simeq \mathrm{sech}(Mz)\sqrt{\frac{M}{2}} \ .$$

generate ultralow-energy effective Lagrange density on the Minkowski brane at the critical point $\mu = 0$:

$$\mathcal{L}^{(4)}|_{\mu=0} = \sum_{j=1}^{N_f} \overline{\psi}_j(x)\left(i\,\not{\partial} - g_j^{(Y)}\,h(x)\right)\psi_j(x) + \frac{1}{2}(\,\partial_\mu\phi(x)\,)^2 + \frac{1}{2}(\,\partial_\mu h(x)\,)^2$$
$$- \ \lambda_1\phi^4(x) - \lambda_2\phi^2(x)h^2(x) - \lambda_3 h^4(x) \ ,$$

with the ultra-low energy effective couplings given by

$$g_j^{(Y)} = \frac{\pi}{4}\bar{g}_j\sqrt{\frac{N\zeta}{N_c}}\,, \quad \lambda_1 = \frac{18}{35}\zeta\,, \quad \lambda_2 = \frac{4}{5}\zeta\,, \quad \lambda_3 = \frac{N}{3N_c}\zeta\,, \quad \zeta \equiv \frac{M\pi^3}{\Lambda N} = \frac{\pi^3}{3N}\sqrt{\kappa}.$$

In the vicinity of critical point $\mu \ll M$ the "Higgs" particle and fermion masses as well as scalar self-interaction is induced,

$$\Delta\mathcal{L}_{\mu}^{(4)} = -\frac{1}{2}\, m_h^2\, h^2(x) - \sum_{j=1}^{N_f} m_j^{(f)}\, \overline{\psi}_j(x)\psi_j(x) - \lambda_4\, h^3(x) - \lambda_5\, \phi^2(x) h(x)\, ;$$

$$m_h^2 = \mu^2 \left(4 - \frac{2N_c}{N}\right)\ ; \quad m_j^{(f)} = \frac{\pi}{4}\, \bar{g}_j \mu\ ; \quad \lambda_4 = \frac{4}{3}\mu\sqrt{\zeta\frac{N}{N_c}}; \quad \lambda_5 = \frac{8}{5}\mu\sqrt{\zeta\frac{N_c}{N}}\ .$$

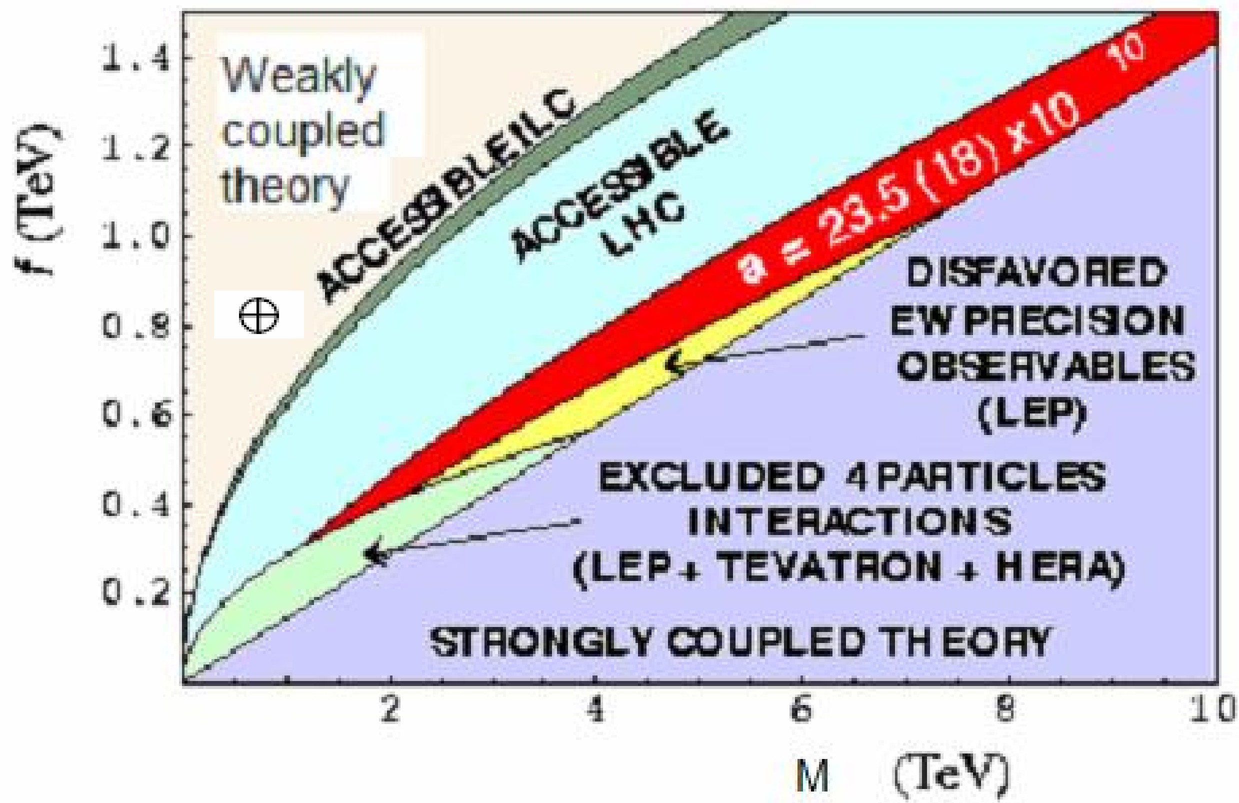
Higgs scalar decay into two branons

Few intermediate conclusions:

a) The masses of h-scalar and fermions are controlled by the ultralow scale μ independently on ζ. Thus one expects $\mu \sim m_{top} \sim 200 GeV$, of order of the Electroweak Symmetry Breaking scale.

b) ϕ-particle are massless being Goldstone bosons of spontaneous breaking of translational invariance, they are called "branons" and describe fluctuations of the brane shape.

c) All interaction vertices are governed by the parameter $\zeta \sim M/\Lambda$, if $\zeta \ll 1$ the scalar matter decouples from the fermion one and does not interact without gravity! Two candidates for the dark matter. However this parameter is not fixed without gravity and is subject to experimental bounds.



Discovery potential for branons (A.Dobado et al.)

M is a cutoff = threshold to leave the brane,

f is a brane tension (in our model f $\sim$ M, see the point $\oplus$)

d) the tree-level coupling of light fermions to the massless scalar $\overline{\psi}(x)\psi(x)\phi(x)$ does not appear: it is suppressed by additional powers of μ^2/M^2 (heavy fermion exchange). Thereby the low-energy Standard Model matter is essentially stable.

e) ϕ-particles may obtain a mass if the Brane World generation is triggered by a very small manifest defects $\mathrm{F}_\Phi(z)$ and $\mathrm{F}_H(z)$ in the dim-5 vacuum:

$$\mathcal{L}^{(5)}_{\mathrm{F}} = \; - \; \mathrm{F}_\Phi(z)\,\overline{\Psi}(X)\tau_3\Psi(X) \; - \; \mathrm{F}_H(z)\,\overline{\Psi}(X)\tau_1\Psi(X)$$

In units comparable with the low energy effective action,

$$\mathrm{F}_\Phi(z) \equiv \frac{g_1\mu^3}{4\pi^3\Lambda^2}\,\mathrm{f}_\Phi(z) \ , \qquad \mathrm{F}_H(z) \equiv \frac{g_2\mu^3}{4\pi^3\Lambda^2}\,\mathrm{f}_H(z) \ ,$$

For a defect of topological type:

$$\mu\,\mathrm{f}_\Phi(z) = M\,\gamma\tanh(\bar{\beta}z); \quad \mathrm{f}_H(z) = 2\xi\,\mathrm{sech}(\bar{\beta}z) \ ,$$

for $\mu/M \ll 1, \gamma \ll 1; \xi \ll 1$.

In the presence of a small defect:

The branon mass happens to be triggered entirely by the topological background,

$$(m_\phi)^2 \approx \gamma\mu^2 \ .$$

the h-particle mass

$$(m_h)^2 \approx \mu^2 \left(2 + 6\xi + \frac{1}{2}\gamma\right) \ .$$

Fermion mass and the constant λ_4:

$$m_f \approx m_f^{(0)}(1+\xi),\ \ \lambda_4 \approx \lambda_4^{(0)}(1+\xi)$$

The "Higgs" mass ratio to the fermion ($\sim$ top-quark) mass can be substantially reduced with an appropriate choice of a non-topological part of the defect $\sim \xi < 0$, for instance, to a phenomenologically acceptable value ~ 135 GeV for a defect $\xi \sim -0.4$.

$\oplus$ (*4 + 1*)-dim. (induced) gravity

Gravity is described by the metric field $g_{AB}(X)$. The action,

$$S(\Phi, H, \overline{\Psi}_l, \Psi_l, g) = \int_{\mathcal{M}_5} d^5X \sqrt{g} \left[\mathcal{L}^{(5)}_{\text{fermion}} + \mathcal{L}^{(5)}_{\text{boson}}\right] \; ; \qquad g \equiv \det(g_{AB}).$$

Invariant fermion Lagrange density

$$\mathcal{L}^{(5)}_{\text{fermion}} = i\overline{\Psi}_l \left[\,\widehat{\gamma}_k e^A_k \,(\partial_A + \omega_A) + \tau_3\Phi + \tau_1 H\,\right]\Psi_l = i\overline{\Psi}_l\,(\not\nabla + \tau_3\Phi + \tau_1 H)\,\Psi_l$$

Invariant bosonic (Euclidean) Lagrange density

$$\mathcal{L}^{(5)}_{\text{boson}} = N\Lambda^3 \left(\frac{\Phi^2}{g_1} + \frac{H^2}{g_2}\right) - \frac{\Lambda}{\mathcal{G}}\left(\epsilon\frac{R}{2} - \lambda_0\right) \; .$$

where $\epsilon = \pm 1, 0$.

Induced gravity $\Leftrightarrow \epsilon = 0$**!!**

After integration over high-energy fermions one obtains the low-energy Lagrange density

$$\mathcal{L}^{(5)}_{\rm low} \equiv= i\overline{\Psi}_l(X)\,[\,\nabla\!\!\!\!/ + \tau_3\Phi(X) + \tau_1 H(X)\,]\,\Psi_l(X)$$

$$+\frac{N\Lambda}{4\pi^3}\left\{\partial_A\Phi(X)\partial^A\Phi(X) + \partial_A H(X)\partial^A H(X) - 2\Delta_1\Phi^2(X) - 2\Delta_2 H^2(X)\right\}$$

$$-\frac{\Lambda}{\mathcal{G}}\left\{\frac{R(X)}{2\kappa} - \lambda\right\}$$

$$+\frac{N\Lambda}{4\pi^3}\left[\Phi^2(X) + H^2(X)\right]\left\{\Phi^2(X) + H^2(X) + \frac{R(X)}{6}\right\}$$

Irrelevant for
Weak gravity

$$+\frac{N\Lambda}{2880\pi^3}\{5R^2(X) - 8R_{AB}(X)R^{AB}(X) - 7R_{ABCD}(X)R^{ABCD}(X)\}$$

where R_{ABCD}; R_{BD}; R are the Riemann curvature tensor, the Ricci tensor and the scalar curvature respectively.

Interplay between classical and fermion induced vertices

$$\lambda = \lambda_0 + \frac{N\Lambda^4}{75\pi^3}\mathcal{G}\ ; \qquad \kappa = \frac{1}{\epsilon + N\Lambda^2\mathcal{G}/54\pi^3}\ .$$

Brane generation

τ-symmetry and translational invariance breaking by $< \Phi(X) >= \phi_0(z); \quad < H(X) >= h_0(z)$ is accompanied by a geometry generation, also breaking translational invariance,

$$ds^2 = g_{AB}(X)\, dX^A\, dX^B = \exp\{-2\rho(z)\}\, dx_\mu dx_\mu + dz^2$$

Search for solutions of classical equations in the gravitational strong coupling regime in which $|\rho'(z)|/M = \emptyset(\kappa)\,,\ |\rho''(z)|/M^2 = \emptyset(\kappa)$ all along the large extra–dimension.

Equations of motion in this regime:

$$R_{AB} - \frac{1}{2}\, g_{AB}\,(R - 2\kappa\lambda) \ \equiv G_{AB} \ + \ \kappa\lambda\, g_{AB} \ = \ \frac{N\kappa\mathcal{G}}{2\pi^3}\Big\{\partial_A\Phi\,\partial_B\Phi + \partial_A H\,\partial_B H$$
$$- \ \frac{1}{2}\, g_{AB}\Big[\partial_C\Phi\,\partial^C\Phi + \partial_C H\,\partial^C H - 2\Delta_1\Phi^2 - 2\Delta_2 H^2 + \big(\Phi^2 + H^2\big)^2\Big]$$
$$+ \ \frac{1}{6}\Big(R_{AB} - \frac{1}{2}\, g_{AB}\, R + g_{AB}\, D^C\partial_C - D_B\,\partial_A\Big)\big(\Phi^2 + H^2\big)\Big\}$$

Terms quadratic in curvature are subdominant in κ and omitted.

Dimensionless strength of gravitation,

$$\overline{\kappa} \;\equiv\; \frac{N\kappa}{6\pi^3} M^2 \mathcal{G} \;\ll\; 1$$

redefinition of the cosmological constant

$$\frac{1}{3}\kappa\lambda \;\equiv\; \overline{\kappa}\,\lambda_{\rm eff}$$

Leading order in $\overline{\kappa} \ll 1$.

$$\frac{\rho''}{M^2} = \frac{\overline{\kappa}}{M^4}\left\{\Phi'^{\,2} + H'^{\,2} - \frac{1}{6}\frac{d^2}{dz^2}\left(\Phi^2 + H^2\right)\right\} + \varnothing(\overline{\kappa}^2)\ ,$$

whereas the cosmological constant

$$\frac{\lambda_{\rm eff}}{M^2} \;= \frac{1}{2M^4}\left\{\Phi'^{\,2} + H'^{\,2} + 2\Delta_1\Phi^2 + 2\Delta_2 H^2 - \left(\Phi^2 + H^2\right)^2\right\} + \varnothing(\overline{\kappa})\ .$$

Equations of motion for scalar fields,

$$2\left[\Delta_1 - \Phi^2 - H^2\right]\Phi \;=\; \left(\frac{R}{6} \;-\; \frac{1}{\sqrt{g}}\partial_C\sqrt{g}\; g^{CD}\partial_D\right)\Phi\ ,$$

$$2\left[\,\Delta_2 - H^2 - \Phi^2\,\right]H \;=\; \left(\frac{R}{6} \;-\; \frac{1}{\sqrt{g}}\partial_C\sqrt{g}\; g^{CD}\partial_D\right)H$$

For $\overline{\kappa} \ll 1$ kink–like solutions remain in the flat space and therefore are the same,

$$\Phi'' + 2\Phi\left(\Delta_1 - \Phi^2 - H^2\right) = \varnothing(\overline{\kappa})\ ,$$

$$H'' + 2H\left(\Delta_2 - \Phi^2 - H^2\right) = \varnothing(\overline{\kappa})\ .$$

To this order cosmological constant in dim-5

$$\lambda = \frac{N\mathcal{G}M^4}{4\pi^3}$$

Conformal factor approaches the Anti-de-Sitter (AdS_5) metric for large z

$$\rho(z) \simeq \frac{2\overline{\kappa}}{3}\ln\cosh(Mz) \stackrel{|z|\to\infty}{\sim} k|z|\ ; \quad k \;\approx\; \frac{2}{3}\overline{\kappa}M\ .$$

Newton's constant and other scales

Relation between the five dimensional and brane gravity constants from the factorized Riemannian metric

$$ds^2 = \exp\{-2\rho(z)\}\, g_{\mu\nu}(x) dx^\mu dx^\nu + dz^2 \ .$$

The gravitational action

$$S[\,g\,] = -\frac{\Lambda}{2\kappa\mathcal{G}} \int d^5X \sqrt{g(X)} R(X) \simeq -\frac{\Lambda}{2\kappa\mathcal{G}} \int d^4x \sqrt{g(x)}\, R(x) \int_{-\infty}^{+\infty} dz \ \exp\{-2\rho(z)\}$$

$$\equiv -\frac{1}{16\pi G_N} \int d^4x \sqrt{g(x)} R(x)$$

Thus

$$G_N \ \simeq \ \frac{\pi^2\,\overline{\kappa}^2}{2N\Lambda M};$$

Let us adopt the induced gravity relations

$$\kappa \simeq \frac{54\pi^3}{N\Lambda^2 \mathcal{G}}; \qquad \bar{\kappa} \simeq \frac{9M^2}{\Lambda^2} \simeq \zeta^2$$

Then

$$kM_P^2 = \frac{4N}{27\pi^3}\Lambda^3; \qquad k^5 M_P^4 = \frac{128N^2}{27\pi^6} M^9.$$

For instance, lower experimental bound

$k > 2 \cdot 10^{-12} GeV = 1/10\ mm \Longleftrightarrow V(r) \propto \frac{\mathcal{G}_{(4)}}{r}\left(1 + \frac{const}{(kr)^2}\right)$

is reached

for $M \sim 100 GeV, \quad \Lambda \sim 10^9 GeV; \quad \zeta \sim M/\Lambda \sim 10^{-7}; \quad \bar{\kappa} \sim 10^{-13}$

Other options: ← Too low to be true!

for $M \sim 1 TeV$ (accepted by experimental data):

$k \sim 10^{-10} GeV; \quad \Lambda \sim 10^{10} GeV; \quad \zeta \sim M/\Lambda \sim 10^{-6.5}; \quad \bar{\kappa} \sim 10^{-12}$

for $k \sim 2 \cdot 100 GeV$ (EW breaking scale):

$M \sim 10^{10} GeV, \quad \Lambda \sim 10^{14} GeV; \quad \zeta \sim M/\Lambda \sim 10^{-4}; \quad \bar{\kappa} \sim 10^{-8}$

Thus $k \ll M$ and brane is thin, $\bar{\kappa} \ll 1$ and gravity is very weak;

ζ is very small $\Rightarrow$ branons belong to the Dark side of our Universe

Induced cosmological constant on the brane

$$\Lambda_{\text{cosmo}} \equiv \frac{N\Lambda}{2\pi^3 G_N} \int_{-\infty}^{+\infty} dz \;\; \exp\{-4\rho(z)\}\Big\{-4\frac{M^2}{\bar{\kappa}}[\,\rho'(z)\,]^2 \qquad \textit{[gravity]}$$

$$+(\phi'(z))^2 + (h'(z))^2 + \frac{2}{3}((\phi(z))^2 + (h(z))^2)(2\rho''(z) - 5[\,\rho'(z)\,]^2)\Big\} \qquad \textit{[matter]}$$

$$= 0$$

It holds exactly !! for any choice of parameters and supports the Minkowski geometry on the brane.

! Translational invariance of the Minkowski world enforces to vanish the cosmological constant. The compensation mechanism is competing SUSY !