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Spin rotation and oscillations for high energy particles in a crystal and storage ring

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Spin rotation of relativistic particles in a crystal

The particle wavelength is much less then typical inhomogeneities in the electric fields in a crystal, so to describe spin motion in these fields a quasi-classical approximation can be used.

Spin motion in the electromagnetic field is described by the Bargmann-Myshel-Telegdy equation

$$\frac{d\vec{P}}{dt} = \left[\vec{\Omega} \times \vec{P}\right]$$

$\vec{P}$ is the vector polarization of a particle

$$\vec{\Omega} = -\frac{e}{Mc}\left\{\left[\left(\frac{g-2}{2} + \frac{1}{\gamma}\right)\vec{B} - \frac{g-2}{2}\frac{\gamma-1}{\gamma}\frac{\left(\vec{\upsilon}\vec{B}\right)}{\upsilon^2} - \left(\frac{g-2}{2} + \frac{1}{\gamma+1}\right)\left[\vec{\beta} \times \vec{E}\right]\right]\right\}$$

is the angular velocity of particle spin precession at a particle location point at the moment of time t, $\vec{\beta} = \vec{\upsilon}/c$

A particle move in a planar channeling mode in the plane x, z in a crystal bent about the y-axis

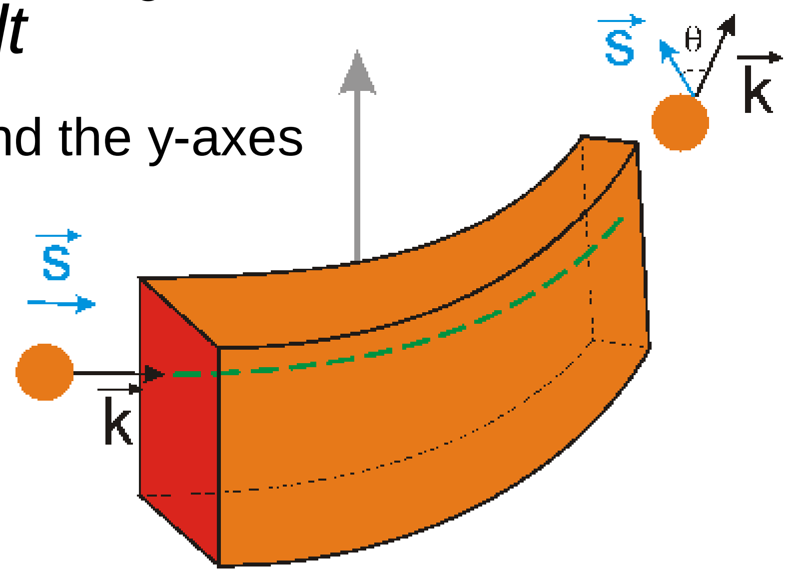
the Bargmann-Myshel-Telegdy equation

$$\frac{dP_x}{dt} = \Omega_y(t) P_z \,, \frac{dP_z}{dt} = \Omega_y(t) P_x \,, \frac{dP_y}{dt} = 0$$

where the spin precession frequency around the y-axes

$$\Omega_y(t) = \left(\frac{g-2}{2} + \frac{1}{\gamma} \right) \gamma \Omega_{0y}(t)$$

$$\left| \Omega_y(t) \right| = \left(\frac{g-2}{2} + \frac{1}{\gamma} \right) \frac{e}{Mc} E(t)$$



here E is the magnitude of electric field of a crystallographic plane at the particle location point at the moment t (in this particular case the direction of this field is practically orthogonal to the particle velocity).

During motion in a bent crystal the particle spin rotates about the direction of the momentum

V.G. Baryshevsky – JTP Letters v.5 (1979) p.182

As the magnitude of the field E, making the particle follow the bending of the crystal, is large (E ~ 10^7 -10^8 CGSE) the rotation frequencies are large

for **(g-2)~1** (particle with the spin S=1)

$$\Omega_y \approx 10^{11} \div 10^{13}\ s^{-1}$$

the angle of spin rotation by one centimeter θ_s ~ **10 – 10^3 rad/cm**

Let, for example, the curvature radius of a bent crystal be 10^2 cm , then for γ=10^2 and (g-2) ~ 1, θ_s =1 rad/cm, that is obviously observable

Relation between the spin rotation angle and the momentum rotation angle in case of planar motion

$$\theta_s = \int_0^T \Omega_y(t')dt' = \left(\frac{g-2}{2} + \frac{1}{\gamma}\right)\gamma\theta_m ,$$

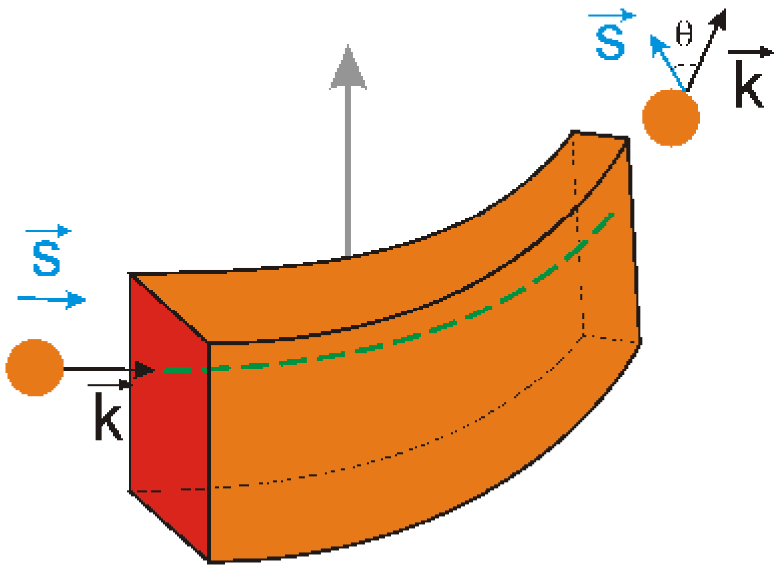
$$\theta_m = \int_0^T \Omega_{0y}(t')dt'$$

we can measure the magnitude (g-2) for a particle without making use of concrete models describing the distribution of intracrystalline fields, just by measuring θ_m and θ_s :

$$g - 2 = \frac{2}{\gamma}\frac{\theta_s - \theta_m}{\theta_m}$$

If (g-2) ~1 and γ>>1, then $(g-2) \approx 2\theta_s / \gamma\theta_m$

Experiment for measurement (g-2) rotation



E761 Collaboration

”First observation of spin precession of polarized + hyperons channeled in bent crystals”, LNPI Research Reports (1990-1991) 129.

Fermi National Accelerator Laboratory

First Observation of Magnetic Moment Precession of Channeled Particles in Bent Crystals

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September 1992

First Observation Of Magnetic Moment Precession Of Channeled Particles In Bent Crystals

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September 3, 1992

Interactions contributing to the spin motion of a particle

Considering evolution of the spin of a particle (with $S \geq 1$) we should take into account several interactions:;

- interaction of the quadrupole moment with an inhomogeneous electric field;
- interaction due to birefringence effect;
- interaction of the particle with the electric field due to the tensor electric polarizability;
- interaction of the particle with the magnetic field due to the tensor magnetic polarizability;

Interaction of the particle quadrupole moment with an inhomogeneous electric field

For a particle with the quadrupole moment Q the energy of spin interaction with an inhomogeneous electric field E

$$\hat{W}_Q = \frac{1}{6}\hat{Q}_{ik}\frac{\partial E_i}{\partial x_k}, \text{ where } \quad \hat{Q}_{ik} = \frac{3Q}{2S(2S-1)}\left(\hat{S}_{ik} - \frac{2}{3}S(S+1)\delta_{ik}\right)$$

is the quadrupole interaction operator of the particle, Q is the quadrupole moment, S is the value of the particle spin, $\hat{S}_{ik} = \hat{S}_i\hat{S}_k + \hat{S}_k\hat{S}_i$ is the spin projection operator.

V.G. Baryshevsky, A.A. Sokolsky – Sov. JTP Letters, v.23 (1980) p.1419

V.G. Baryshevsky, A.G. Shechtman, "Spin oscillations and possibility of quadrupole moment measurement for W^- hyperons moving in a crystal" NIM B83 (1993)

The experiment scheme is proposed for Q measurement on the level 10^{-27} cm^2 in a set of straight tungsten crystals with total length 20 cm

Rotation and oscillation of deuteron spin in unpolarized matter (birefringence and spin dichroism) *

when a particle with the spin S≥1 passes through an unpolarized medium, the medium refraction index depends on particle spin orientation to its momentum. Therefore, the particle possesses some effective potential energy in the medium and this energy depends on the spin orientation

$$\hat{V} = -\frac{2\pi\hbar^2}{M\gamma} N(\vec{r}) \hat{f}(0) = -\frac{2\pi\hbar^2}{M\gamma} N(\vec{r}) \left(d + d_1 (\vec{S}\vec{n})^2 \right)$$

*** described theoretically** *V. Baryshevsky, Phys. Lett. A 171 (1992) 431; V.Baryshevsky, J. Phys. G 19 (1993) 273; V. Baryshevsky, F. Rathmann, Proc. COSY summer school, Vol.1 (2002).*

*** observed** *V. Baryshevsky, A. Rovba, R. Engels, F. Rathmann, H. Seyfarth, H. Stroher, T. Ullrich, C. Duweke, R. Emmerich, A. Imig, J. Ley, H. Paetz gen. Schieck, R. Schulze, G. Tenckhoff, C. Weske, M. Mikirtytchiants, A. Vassiliev "First observation of spin dichroism with deuterons up to 20 MeV in a carbon target" LANL e-print arxive: hep-ex/0501045*

*** the experiment at a storage ring is being prepared**

Interaction of the particle with the electric field due to the tensor electric polarizability

Let a real electric field $\vec{E}(\vec{r})$ acts on a particle (nucleus) ($\vec{r}$ is the particle coordinate).

$$\hat{V}_{\vec{E}} = -\frac{1}{2}\hat{\alpha}_{ik}(E_{eff})_i (E_{eff})_k$$

is the energy of particle beam in an external electric field due to the tensor of particle electric polarizability ($\hat{\alpha}_{ik}$ is the tensor of particle electric polarizability, $(E_{eff})_i$ are the components of the electric field $\vec{E}_{eff} = \vec{E} + \vec{\beta} \times \vec{B}$)

separation of scalar and tensor electric polarizabilities gives

$$\hat{V}_{\vec{E}} = \alpha_S E_{eff}^2(\vec{r}) - \alpha_T E_{eff}^2(\vec{r})(\vec{S}\vec{n}_E)^2$$

α_S is the particle scalar electric polarizability, α_T is the particle tensor electric polarizability, $\vec{n}_E$ is the unit vector along $\vec{E}_{eff}(\vec{r})$).

Interaction of the particle with the magnetic field due to the tensor magnetic polarizability

A particle with the spin S≥1 also has the magnetic polarizability which is described by the tensor of magnetic polarizability $\hat{\beta}_{ik}$ and interaction of the particle with the magnetic field due to the tensor magnetic polarizability is:

$$\hat{V}_{\vec{B}} = -\frac{1}{2}\hat{\beta}_{ik}\left(B_{eff}\right)_i\left(B_{eff}\right)_k$$

$(B_{eff})_i$ are the components of the electric field $\vec{B}_{eff} = \vec{B} - \vec{\beta}\times\vec{E}$)

separation of scalar and tensor magnetic polarizabilities gives

$$\hat{V}_{\vec{B}} = \beta_S B_{eff}^2(\vec{r}) - \beta_T B_{eff}^2(\vec{r})\left(\vec{S}\vec{n}_B\right)^2$$

β_S is the particle scalar magnetic polarizability, β_T is the particle tensor magnetic polarizability, $\vec{n}_B$ is the unit vector along $\vec{B}_{eff}(\vec{r})$.

What effects arise from the above interactions

So, when a particle moves in a bent crystal the effects of spin rotation and vector-to-tensor (tensor-to-vector) polarization conversion appear due to the interactions W_Q, V, V_E and V_B along with the spin rotation caused by (g-2).

These effects can be used for measurement Q, d_1, α_T and β_T.

It is important that the terms W_Q, V, V_E and V_B cause oscillations and rotation of spin even in a straight (non-bent) crystal, but bent crystals could provide higher electric fields E.

Multiple scattering and depolarization can be substantial for processes in crystals and complete description of these processes can be done by the density matrix formalism

How to measure α_T

$$\omega_\alpha = \frac{\alpha_T E^2}{\hbar}$$

is the typical frequency of spin rotation (oscillations) caused by the particle electric tensor polarizability

The corresponding spin rotation angle (phase of spin oscillations)

$$\varphi = \omega_\alpha \frac{L}{\upsilon} = \frac{\alpha_T E^2 L}{\hbar \upsilon}$$

The angle of momentum rotation in a bent crystal $\theta_0 = \dfrac{eEL}{Mc^2\gamma}$ ⇒ $E = \dfrac{Mc^2\gamma}{eL}\theta_0$

$$\varphi = \frac{\alpha_T \gamma^2}{\lambda_c r_0 L}\theta_0^2$$

λ_c is the Compton wavelength,

$r_0=e^2/Mc^2$ is the electromagnetic radius of the particle

Therefore measuring j , q_0, L, g we can find a_T: $\alpha_T = \dfrac{\lambda_c r_0 L}{\gamma^2}\dfrac{\varphi}{\theta_0^2}$

Let us evaluate α_T could be measured when a particle passes through a crystal

Suppose that the experimentally measured angle of spin rotation is about $\varphi \approx 10^{-4}$ rad

The strength of electric field in a bent crystal can be achieved about $E \approx 10^{9}$ CGSE

Therefore, a_T can be estimated

$$\alpha_T = \frac{\hbar c \varphi}{E^2 L} = \frac{3}{L} 10^{-39} \, cm^3$$

According to the evaluations (*Jiunn-Wei Chen, Harald W. Griesshammer, Martin J. Savage and Roxanne P. Springer, Nucl.Phys. A644 (1998) 221-234 (LANL e-print arXiv:nucl-th/9806080)*) **$a_T \sim 10^{-40}$ cm^3**

Thus, experimental measurement of the tensor electric polarizability of deuterons (nuclei) seems possible and as well as getting limits for tensor electric polarizability of elementary particles (for example, Ω-hyperon).

How to measure β_T

$$\omega_\beta = \frac{\beta_T B_{eff}^2}{\hbar} = \frac{\beta_T E^2}{\hbar}$$

is the typical frequency of spin rotation (oscillations) caused by the particle electric tensor polarizability

The corresponding spin rotation angle (phase of spin oscillations)

$$\varphi = \omega_\beta \frac{L}{c} = \frac{\beta_T E^2 L}{\hbar c}$$

The angle of momentum rotation in a bent crystal

$$\theta_0 = \frac{eEL}{Mc^2\gamma}$$

$$E = \frac{Mc^2\gamma}{eL}\theta_0$$

$$\varphi = \frac{\beta_T \gamma^2}{\lambda_c r_0 L}\theta_0^2$$

λ_c is the Compton wavelength,

$r_0 = e^2/Mc^2$ is the electromagnetic radius of the particle

Therefore measuring j , q_0, L, g we can find a_T:

$$\beta_T = \frac{\lambda_c r_0 L}{\gamma^2}\frac{\varphi}{\theta_0^2}$$

Let us evaluate β_T could be measured when a particle passes through a crystal

Suppose that the experimentally measured angle of spin rotation is about $\varphi \approx 10^{-4}$ rad

The strength of electric field in a bent crystal can be achieved about $E \approx 10^9$ CGSE

Therefore, β_T can be estimated

$$\beta_T = \frac{\hbar c \varphi}{E^2 L} = \frac{3}{L} 10^{-39}\, cm^3$$

According to the evaluations (*Jiunn-Wei Chen, Harald W. Griesshammer, Martin J. Savage and Roxanne P. Springer, Nucl.Phys. A644 (1998) 221-234 (LANL e-print arXiv:nucl-th/9806080)*) **$b_T \sim 2 \cdot 10^{-40}$ cm^3**

Thus, experimental measurement of the tensor magnetic polarizability of deuterons (nuclei) seems possible and as well as getting limits for tensor magnetic polarizability of elementary particles (for example, Ω-hyperon).

NB

All discussed phenomena reveal themselves both in bent and straight crystals. Besides they all present when a particle moves in a storage ring.

Spin rotation of proton (deuteron, antiproton) in a storage ring with a polarized target

The index of refraction and effective potential energy of particles in matter

$$n = 1 + \frac{2\pi N}{k^2} f(0)$$

when n-1<<1 →

$$n^2 = 1 + \frac{4\pi N}{k^2} f(0)$$

The wave number of the particle in vacuum is denoted *k*, *k' = kn* is the wave number of the particle in medium. The particle momentum in vacuum p=ħk is not equal to the particle momentum in medium *p'=ħkn* . The particle energy in vacuum is not equal to the particle energy in medium.

$$E = \sqrt{\hbar^2 k^2 c^2 + m^2 c^4}$$

vacuum	medium
E	E’

$$E' = \sqrt{\hbar^2 k^2 n^2 c^2 + m^2 c^4}$$

a particle in medium possesses an effective potential energy **V e f f**

$$E = E' + V_{eff}$$

$$V_{eff} = E - E' = -\frac{2\pi\hbar^2}{m\gamma} N f(0) = (2\pi)^2 N T(E),$$

whereT(E) is the T-matrix

Spin rotation of proton (deuteron, antiproton) in a storage ring with a polarized target

The amplitude of the elastic coherent scattering at the zero angle depends on the vector polarization of the target nuclei

$$\hat{f}(\vec{P}_t, 0) = A_1 \vec{S}\vec{P}_t + A_2(\vec{S}\vec{n})(\vec{n}\vec{P}_t)$$

contribution to the effective potential energy of particle interaction with matter caused by polarization of target nucleus spins looks like:

$$\hat{V}_{eff}(\vec{P}_t) = -\frac{2\pi\hbar^2}{m\gamma} N(A_1 \vec{S}\vec{P}_t + A_2(\vec{S}\vec{n})(\vec{n}\vec{P}_t))$$

$$\hat{V}_G^{nucl} \equiv \hat{V}_{eff}(\vec{P}_t) = -\vec{\mu}\vec{G} = -\frac{\mu}{S}\vec{S}\vec{G}$$

m is the particle magnetic moment, $\vec{G} = \frac{2\pi\hbar^2 S}{m\gamma\mu} N(A_1 \vec{P}_t + A_2 \vec{n}(\vec{n}\vec{P}_t))$

the energy of magnetic moment m interaction with a magnetic field $V_{mag} = -\vec{\mu}\vec{B} = -\frac{\mu}{S}\vec{S}\vec{B}.$

Particle spin rotation in a storage ring with an internal target

Time evolution of the spin and tensor polarization is described by the equations:

$$\vec{P} = \frac{\langle \Psi(t) | \frac{\vec{S}}{S} | \Psi(t) \rangle}{\langle \Psi(t) | \Psi(t) \rangle}; P_{ik} = \frac{\langle \Psi(t) | \hat{Q}_{ik} | \Psi(t) \rangle}{\langle \Psi(t) | \Psi(t) \rangle}$$

$\hat{Q}_{ik}$ is the tensor of rank two (the tensor polarization), for S=1 the tensor

$$\hat{Q}_{ik} = \frac{3}{2}(S_i S_k + S_k S_i - \frac{4}{3}\delta_{ik})$$

But it should be taken into account that multiple scattering occurring in the target brings to both change in the angular spread and depolarization of the particle beam.

So the spin density matrix formalism should be used to derive equations describing evolution of the particle spin.

Equations describing evolution of the particle spin.

The density matrix of the system "deuteron+target" $\rho = \rho_p \otimes \rho_t$,

r_d and r_t are the density matrix of the particle beam and target, respectively

The equation for the particle density matrix $$\frac{d\rho_p}{dt} = -\frac{i}{\hbar}\left[\hat{H}_p, \rho_p\right] + \left(\frac{\partial \rho_p}{\partial t}\right)_{col}$$

$$\left(\frac{\partial \rho_d}{\partial t}\right)_{col} = vN\mathrm{Sp}_t\left[\frac{2\pi i}{k}\left[F(\theta=0)\rho - \rho F^+(\theta=0)\right] + \int d\Omega F(\vec{k}')\rho(\vec{k}')F^+(\vec{k}')\right]$$

this term describes evolution of the density matrix due to collisions with target atoms (nuclei) $\vec{k}' = \vec{k} + \vec{q}$, $\vec{q}$ is the momentum transferred to a nucleus of the matter from the incident particle, v is the speed of the incident particles, *N* is the atom density in the matter, *F* is the scattering amplitude depending on the spin operators of the deuteron and the matter nucleus (atom), F^+ is the Hermitian conjugate of the operator *F*.

The first term describes coherent elastic scattering of a particle by matter nuclei, while the second term is for multiple scattering.

The first term

$$\left(\frac{\partial \rho_p}{\partial t}\right)^{(1)}_{col} = vN\frac{2\pi i}{k}\left[\hat{f}(0)\rho_p - \rho_p\hat{f}(0)^+\right]$$

The amplitude of particle scattering in a target at the zero angle $\hat{f}(0) = \mathrm{Sp}_t\hat{F}(0)\rho_t$

$$\left(\frac{\partial \rho_p}{\partial t}\right)^{(1)}_{col} = -\frac{i}{\hbar}\left(\hat{V}_{eff}\rho_p - \rho_p\hat{V}^+_{eff}\right)$$

where

$$\hat{V}_{eff} = -\frac{2\pi\hbar^2}{m\gamma}N\hat{f}(0) = -\frac{2\pi\hbar^2}{m\gamma}N(d + A_1\vec{S}\vec{P}_t + A_2(\vec{S}\vec{n})(\vec{n}\vec{P}_t) + d_1(\vec{S}\vec{n})^2)$$

$$\hat{V}_{eff} = -\frac{2\pi\hbar^2}{m\gamma}Nd - \frac{\mu}{S}\vec{S}\vec{G} - \frac{2\pi\hbar^2}{m\gamma}Nd_1(\vec{S}\vec{n})^2)$$

where $\vec{G} = \frac{2\pi\hbar^2 S}{m\gamma\mu}N(A_1\vec{P}_t + A_2\vec{n}(\vec{n}\vec{P}_t))$

The equation for the particle density matrix

$$\frac{d\rho_p}{dt} = -\frac{i}{\hbar}\left[\hat{H}_p, \rho_d\right] - \frac{i}{\hbar}\left(\hat{V_{eff}}\rho_p - \rho_p \hat{V}^+_{eff}\right) + $$
$$+ vN\mathrm{Sp}_t \int d\Omega F(\vec{k}')\rho(\vec{k}')F^+(\vec{k}')$$

When studying interaction of particles with a target inside a storage ring the density of target is chosen to make multiple scattering in target small for the observation time. In such conditions depolarization of the beam also appears small.

$$\frac{d\rho_p}{dt} = -\frac{i}{\hbar}\left[\hat{H}_p, \rho_d\right] - \frac{i}{\hbar}\left(\hat{V_{eff}}\rho_p - \rho_p \hat{V}^+_{eff}\right)$$

The equation for evolution of spin properties of a particle in a storage ring

The polarization vector $$\vec{P} = \frac{\mathrm{Sp}\ \rho_p(t)\ \frac{\vec{S}}{S}}{\mathrm{Sp}\ \rho_p(t)} = \frac{\mathrm{Sp}\ \rho_p(t)\ \vec{S}}{I(t)\ S}$$

where $I(t) = \mathrm{Sp}\ \rho_p(t)$ is the beam intensity

the differential equation providing to find the beam polarization

$$\frac{d\vec{P}}{dt} = \frac{\mathrm{Sp}\ \frac{d\rho_p(t)}{dt}\ \vec{S}}{S I(t)} - \vec{P}\frac{1}{I(t)}\ \frac{dI(t)}{dt}$$

If the particle spin is S=1 then the polarization tensor should also be found

$$P_{ik} = \frac{\mathrm{Sp}\ \rho_p\ \hat{Q}_{ik}}{I(t)}$$

$$\frac{dP_{ik}}{dt} = \frac{1}{I(t)}\mathrm{Sp}\left(\frac{d\rho_p}{dt}\hat{Q}_{ik}\right) - P_{ik}\frac{1}{I(t)}\ \frac{dI(t)}{dt}$$

is change in the tensor polarization with time

Particles with the spin S=1/2

The density matrix for such a particle $\rho_{p\frac{1}{2}} = I_{\frac{1}{2}}(t)(\frac{1}{2}\hat{I} + \vec{P}\vec{S})$

$$\frac{dI_{\frac{1}{2}}(t)}{dt} = -\frac{i}{\hbar}\mathrm{Sp}(\hat{V}_{eff}\rho_{p\frac{1}{2}} - \rho_{p\frac{1}{2}}\hat{V}^{+}_{eff}) = -(\varkappa + \frac{2\mu}{\hbar}\vec{G}_2\vec{P})I_{\frac{1}{2}}(t),$$

where $\varkappa = vN\sigma_{tot}$, σ_{tot} is the total cross-section of particle scattering by a nonpolarized nucleus and $\vec{G}_2$ is the imaginary part of the pseudomagnetic nuclear field.

$$\frac{dI_{\frac{1}{2}}(t)}{dt} = -\varkappa_{abs}(\vec{P})I_{\frac{1}{2}}(t),$$

$$\varkappa_{abs} = vN\left[\sigma_{tot} + \tfrac{1}{2}\sigma_1(\vec{P}\vec{P}_t) + \tfrac{1}{2}\sigma_2(\vec{P}\vec{n})(\vec{n}\vec{P}_t)\right] = \varkappa + \vec{P}\vec{g}_t,\ \vec{g}_t = \tfrac{1}{2}vN(\sigma_1\vec{P}_t + \sigma_2\vec{n})(\vec{n}\vec{P}_t)$$

The cross-section $\sigma_1 = \sigma^{\uparrow\uparrow}_{tot}(\vec{n}\perp\vec{P}_t) - \sigma^{\uparrow\downarrow}_{tot}(\vec{n}\perp\vec{P}_t)$ is the difference between the cross-sections of particle scattering by a polarized nucleus with the $\vec{P}$ parallel ($\vec{P}\uparrow\uparrow\vec{P}_t$) and antiparallel ($\vec{P}\uparrow\downarrow\vec{P}_t$) to the target polarization vector $\vec{P}_t$ in conditions when the particle momentum is orthogonal to the target polarization vector ($\vec{n}\perp\vec{P}_t$).

The cross-section $\sigma_2 = \sigma^{\uparrow\uparrow}_{tot}(\vec{n}||\vec{P}_t) - \sigma^{\uparrow\downarrow}_{tot}(\vec{n}||\vec{P}_t)$ is the difference between the cross-sections of particle scattering by a polarized nucleus with the $\vec{P}$ parallel ($\vec{P}\uparrow\uparrow\vec{P}_t$) and antiparallel ($\vec{P}\uparrow\downarrow\vec{P}_t$) to the target polarization vector $\vec{P}_t$ in conditions when the particle momentum is parallel to the target polarization vector ($\vec{n}||\vec{P}_t$).

Particles with the spin S=1/2

The equation that describes evolution of the particle vector polarization under action of pseudomagnetic nuclear field ($\vec{G}_1 = \mathrm{Re}\vec{G},\ \vec{G}_2 = \mathrm{Im}\vec{G}.$)

$$\frac{d\vec{P}}{dt} = \frac{1}{SI_{\frac{1}{2}}(t)}\mathrm{Sp}\frac{d\rho_p}{dt}\vec{S} - \vec{P}\frac{1}{I_{\frac{1}{2}}(t)}\frac{dI_{\frac{1}{2}}(t)}{dt} = -\frac{2\mu}{\hbar}\left[\vec{G}_1 \times \vec{P}\right] - \frac{2\mu}{\hbar}(\vec{G}_2 - \vec{P}(\vec{G}_2\vec{P}))$$

introducing frequency of spin precession in the pseudomagnetic nuclear field

$$\vec{\Omega}_{nuc} = \frac{2\mu}{\hbar}\vec{G}_1 = \frac{2\pi\hbar}{m\gamma}N(\mathrm{Re}A_1\vec{P}_t + \mathrm{Re}A_2\vec{n}(\vec{n}\vec{P}_t))$$

$$\frac{d\vec{P}}{dt} = \left[\vec{P} \times \vec{\Omega}_{nuc}\right] - (\vec{g}_t - \vec{P}(\vec{P}\vec{g}_t))$$

Adding the contribution from the pseudomagnetic nuclear field to the BMT equation one can finally obtain the equation describing spin evolution for particles moving in a storage ring with a polarized target inside

$$\frac{d\vec{P}}{dt} = \left[\vec{P} \times (\vec{\Omega}(d) + \vec{\Omega}_{nuc})\right] - (\vec{g}_t - \vec{P}(\vec{P}\vec{g}_t)),$$

$$\frac{dI_{\frac{1}{2}}(t)}{dt} = -(\varkappa + \vec{P}\vec{g}_t)I_{\frac{1}{2}}(t), \vec{g}_t = \frac{1}{2}vN(\sigma_1\vec{P}_t + \sigma_2\vec{n}(\vec{n}\vec{P}_t))$$

The contribution from the pseudomagnetic nuclear field to spin evolution of the S=1 particle

The spin density matrix for the spin 1 particle

$$\rho_1 = I_1(t)(\frac{1}{3}\hat{I} + \frac{1}{2}(\vec{P}\vec{S}) + \frac{1}{9}P_{ik}\hat{Q}_{ik})$$

The effective potential energy of deuteron interaction with a polarized target

$$\hat{V}_{eff} = -\frac{2\pi\hbar^2}{m\gamma}Nd - \frac{\mu}{S}\vec{S}\vec{G} - \frac{2\pi\hbar^2}{m\gamma}Nd_1(\vec{S}\vec{n})^2$$

The energy $\hat{V}_{eff}$ for a deuteron (in contrast to a particle with the spin S=1/2) contains both deuteron interaction with the pseudomagnetic field and pseudoelectric field (the term proportional to d_1).

If the internal target in a storage ring is polarized

Change in beam intensity and polarization vector

$$\frac{dI_1(t)}{dt} = -\left[(\varkappa + 2\vec{g}_t\vec{P}) - \frac{\chi}{3}(2 + P_{ik}n_in_k)\right] I_1(t)$$

where $\chi = -\frac{4\pi v N}{k}\text{Im}d_1 = -vN(\sigma_{M-1} - \sigma_{M-0})$, $\varkappa = vN\sigma_{M-0}$; σ_{M-1} and σ_{M-0} are the total cross-sections of deuteron scattering by a nonpolarized nucleus for the deuteron state with the magnetic quantum number $M = 1$ and $M = 0$, respectively (the quantization axis is directed along $\vec{n}$).

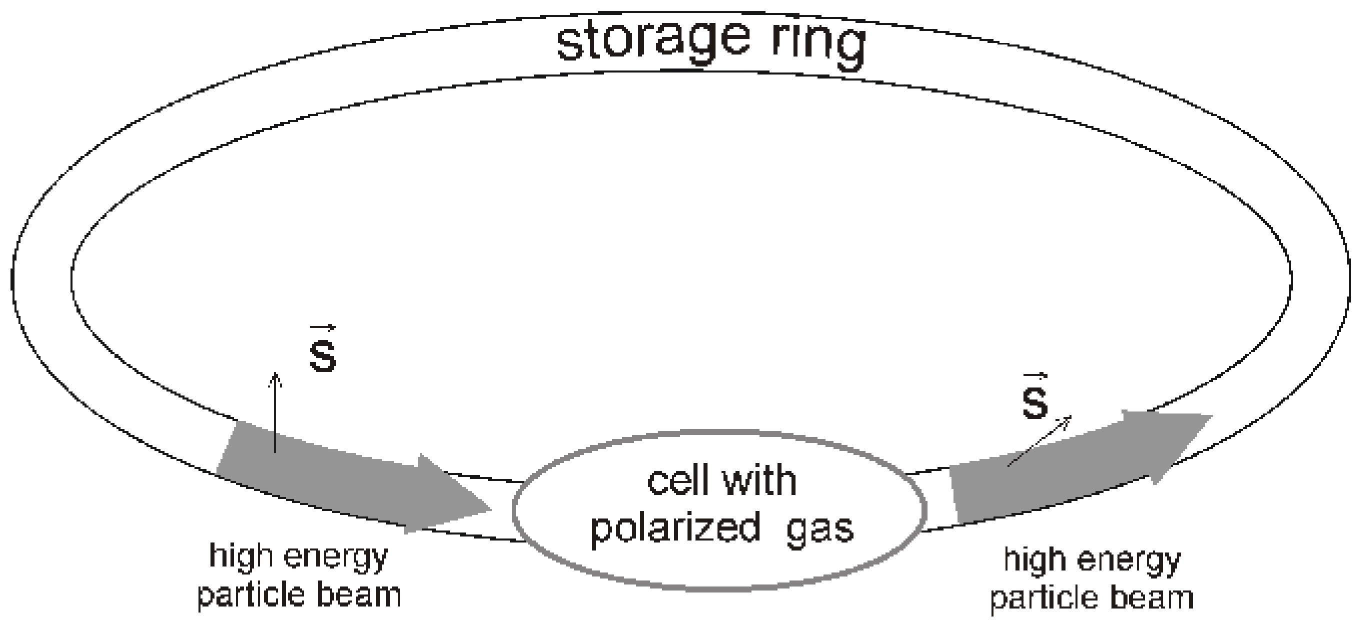
$$\frac{d\vec{P}}{dt} = \left[\vec{P} \times \vec{\Omega}_{nuc}\right] - \frac{2}{3}P_{ik}g_{tk} - \frac{4}{3}\vec{g}_t + 2\vec{P}(\vec{P}\vec{g}_t) + \frac{\eta}{3}[\vec{n} \times \vec{n}^{\prime}] + \\ + \frac{\chi}{2}(\vec{n}(\vec{n} \cdot \vec{P}) + \vec{P}) - \frac{2\chi}{3}\vec{P} - \frac{\chi}{3}(\vec{n} \cdot \vec{n}^{\prime})\vec{P}$$

where $\eta = -\frac{4\pi N}{k}\text{Re } d_1$, $n_i^{\prime} = P_{ik}n_k$.

the nuclear pseudomagnetic field causes deuteron spin precession with the frequency $\vec{\Omega}_{nuc}$

When particle beam absorption in the target is low all changes in the equation for deuteron spin behavior can be made by $\vec{\Omega}(d)$ replacement with $\vec{\Omega}(d) + \vec{\Omega}_{nuc}$.

Note that as the target inside the storage ring is of the finite size then the particle moving in the storage ring interacts with the target at times (the expressions for $\vec{\Omega}_{nuc}$ and $\vec{g}_t$ contain the density *N*, which means the density in the point of particle location i.e. *N=N(t)*). This is the reason for $\vec{\Omega}_{nuc}$ and $\vec{g}_t$ oscillating with the frequency of particle rotation in the storage ring Ω_0 .



For further analysis

Let us expand $\vec{\Omega}_{nuc}$ and $\vec{g}_t$ into Fourier series and be confined with the zero harmonics. In this case $\vec{\Omega}_{nuc}$ and $\vec{g}_t$ appears to be constants

$$\begin{aligned}\vec{\Omega}_{nuc} &= \frac{2\pi\hbar}{m\gamma}N(\mathrm{Re}A_1\vec{P}_t + \mathrm{Re}A_2\vec{n}(\vec{n}\vec{P}_t))\frac{l}{vT} = \\ &= \frac{2\pi\hbar}{m\gamma}\frac{j_t}{L}(\mathrm{Re}A_1\vec{P}_t + \mathrm{Re}A_2\vec{n}(\vec{n}\vec{P}_t)) = \\ &= \frac{2\pi\hbar}{m\gamma}\frac{j_t\nu}{v}(\mathrm{Re}A_1\vec{P}_t + \mathrm{Re}A_2\vec{n}(\vec{n}\vec{P}_t)),\end{aligned}$$

The zero harmonics for $\vec{g}_t$

$$\begin{aligned}\vec{g}_t &= \frac{1}{2}vN(\sigma_1\vec{P}_t + \sigma_2\vec{n}(\vec{n}\vec{P}_t))\frac{l}{vT} = \\ &= \frac{1}{2}j_t\nu(\sigma_1\vec{P}_t + \sigma_2\vec{n}(\vec{n}\vec{P}_t)).\end{aligned}$$

Let us evaluate now the effect magnitude

For protons, antiprotons and deuterons with the energy from MeV to GeV

$\mathrm{Re}\, A_1 \sim 10^{-12} - 10^{-13}$ cm.

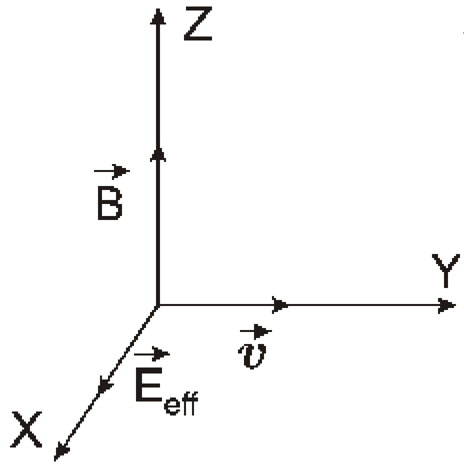
for $\nu \sim 10^6\ s^{-1}$ → the nuclear precession frequency $\Omega_{nuc} = 10^{-4} - 10^{-5}\ s^{-1}$

the ratio $\frac{1}{\gamma}\mathrm{Re}A_{1(2)}$ is proportional to the T-matrix, therefore, Ω_{nuc} depends on the particle energy only due to possible dependence of T-matrix on energy

The obtained estimation for Ω_{nuc} allows to expect, for example for COSY, to observe the spin rotation angle $\theta = \Omega_{nuc} t \approx 10^{-1} - 10^{-2}$ rad in the observation time $t \sim 10^3$ s. This value is quite observable.

To prevent suppression of the the spin precession in the pseudomagnetic nuclear field by the storage ring magnetic field $\vec{B}$ the polarized target should be placed in the straight section of the storage ring, where the field $\vec{B}=0$ and the particle moves along the straight trajectory.

The particular example



Suppose the axis OZ is orthogonal to the orbit plane (OS || B).

The pseudomagnetic field is directed along the axis OX.

In this case for the particle in the straight section the storage ring the vertical spin component rotates around the direction of the pseudomagnetic field in the ZOY plane.

Just change of the vertical spin component should be observed.

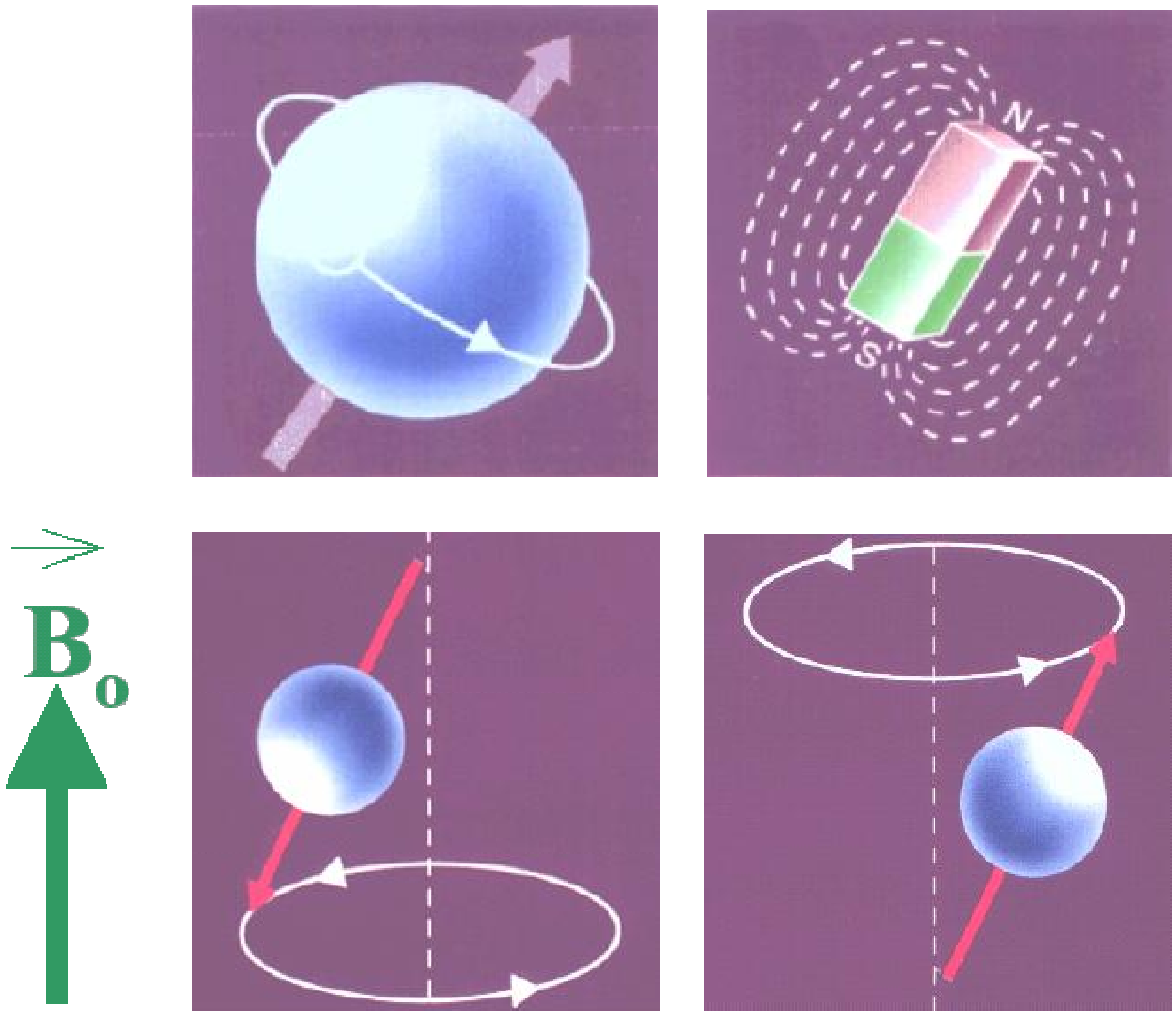
As the typical spin rotation angle $\theta = \Omega_{nuc} t \ll 1$, then change of the vertical spin component with time

$$P_3(t) = P_3(0) - \frac{1}{2}\vartheta^2 t^2 = P_3(0) - \Omega_{nuc}^2 t^2.$$

The effect can be strengthened when adding a magnetic field $\vec{b} \gg \vec{G}_1$ directed along the pseudomagnetic field $\vec{G}_1$ (along the axis OX in the case under consideration). In this case, the angle of rotation is expressed as $\vartheta = \vartheta_{mag} + \vartheta_{nuc} = (\Omega_{mag}(\vec{b}) + \Omega_{nuc})t$ and the vertical component $P_3(t) = P_3(0) - \frac{1}{2}\vartheta_{mag}^2 t^2 - \vartheta_{mag}\vartheta_{nuc}$ (the terms $\sim \vartheta_{nuc}^2$ are neglected comparing with the previous terms).

A lot of experiments could be carried out at a storage ring, but a common challenge exists: the particle spin rotates during its motion in the storage ring. To cancel this interfering effect let us recollect a well-known ….

Paramagnetic resonance



Two energy states

$\Delta E = E_1 - E_0$

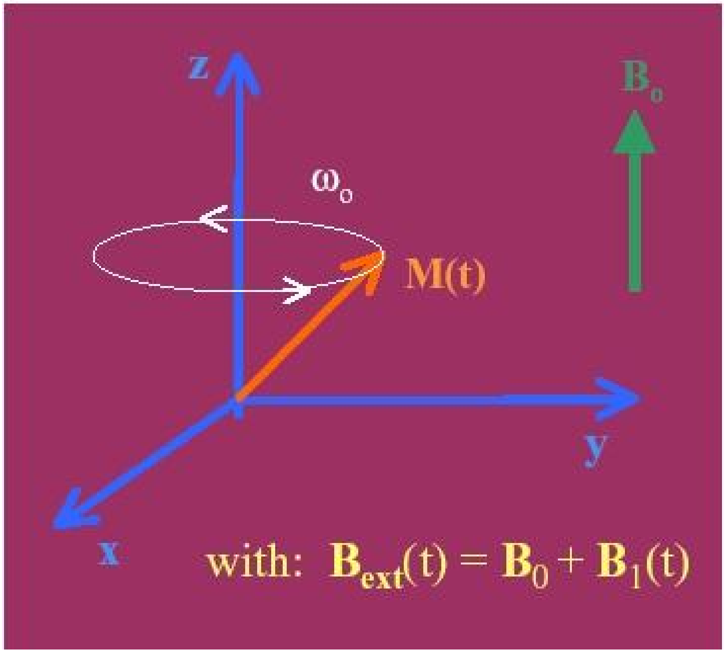
$\Delta E = \hbar\omega_0$

Precession frequency

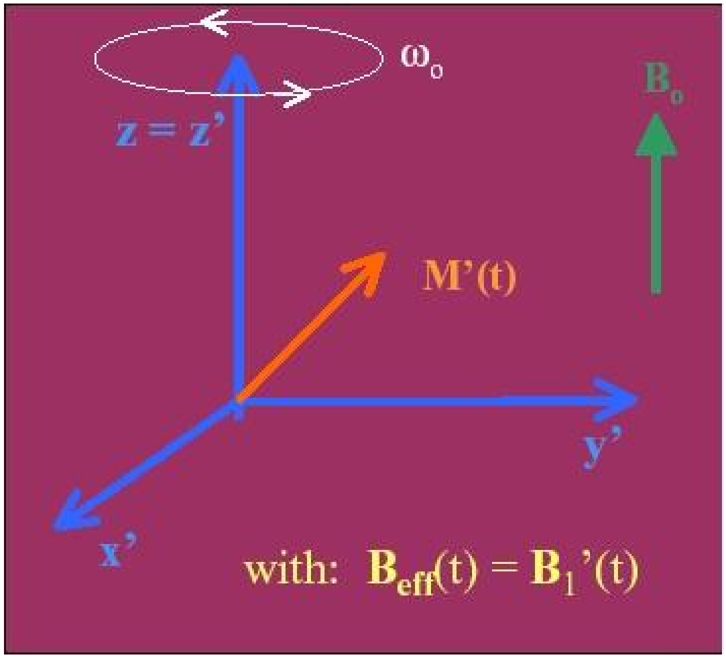
$\omega_0 = gB_0$

Paramagnetic resonance

Laboratory coordinate system



Rotating coordinate system



if we have the strong field orthogonal to the weak one (in our case $\vec{B} \perp \vec{G}$) and $\vec{G}$ either rotates or oscillates with the frequency corresponding to the splitting, caused by the field $\vec{B}$, the resonance occurs. In our case this leads to the conversion of horizontal spin component to the vertical one with the frequency determined by the frequency of spin precession in the field $\vec{G}$

If either vector or tensor polarization of a target rotates

If either vector or tensor polarization of a target rotates then the effects provided by $\vec{G}_s$, $\vec{G}_w$ periodically depend on time

$$i\hbar\frac{\partial\Psi(t)}{\partial t} = (\hat{H}_0 + \hat{V}_{EDM} + \hat{V}_{\vec{B}} + \hat{V}_{\vec{E}} + \hat{V}_E^{nucl}(t) + \hat{V}_G^{nucl}(t))\Psi(t)$$

This equation coincides with the equation for the paramagnetic resonance

The effective potential energy of a particle in the pseudomagnetic nuclear field

$$\hat{V}_G^{nucl} = -\vec{\mu}\vec{G},$$

here $\vec{\mu}$ is the magnetic moment of the particle

this interaction looks like the interaction of a magnetic moment with a magnetic field, thus the field $\vec{G}$ contributes to the change of the particle polarization similar a magnetic field does. It should be especially mentioned that $\hat{V}_G^{nucl}$ contains both the real part, which is responsible for spin rotation, and imaginary part, which contributes to spin dichroism (i.e. beam absorption dependence on spin orientation).

Interaction of a particle with the pseudomagnetic nuclear field

$$\hat{V}_G^{nucl} = -\vec{\mu}\vec{G},$$

$$\vec{G} = \vec{G}_s + \vec{G}_w, \quad \vec{G}_s = \frac{2\pi\hbar^2}{\mu m}\rho[A_1\langle\vec{J}\rangle + A_2\vec{n}(\vec{n}\langle\vec{J}\rangle) + ...],$$

$$\vec{G}_w = \frac{2\pi\hbar^2}{\mu m}\rho[b\vec{n} + b_1[\langle\vec{J}\rangle \times \vec{n}] + b_2\vec{n}_1 + b_3\vec{n}(\vec{n}\vec{n}_1) + b_5[\vec{n} \times \vec{n}_1] + ...]$$

where $\vec{n} = \vec{v}/v$, $\vec{J}$ is the spin of nuclei of matter, $\langle\vec{J}\rangle = \mathrm{Sp}\rho_{nucl}\vec{J}$ is the average value of nuclear spin, $\vec{n}_1$ has the components $n_{1j} = \langle Q_{ij}\rangle n_j$, where $\langle Q_{ij}\rangle = \mathrm{Sp}\rho_{nucl}Q_{ij}$ is the polarization tensor

$$Q_{ij} = \frac{1}{2J(2J-1)}\left\{J_iJ_j + J_jJ_i - \frac{2}{3}J(J+1)\delta_{ij}\right\}$$

Interaction with the field $\vec{G} = \vec{G}_s + \vec{G}_w,$ contains two summands: the first $\vec{G}_s$ corresponds to the strong interaction, which is T,P-even, while the second $\vec{G}_w$ describes spin rotation by the weak interaction, which has both T,P-odd (the term containing the constant b1) and T-odd, P-even (the term containing the constant b5) terms.

A lot of experiments can be carried out with polarized high-energy particles in a crystal and at a storage ring with a polarized target.

A part of these experiments are included in the project ANKE for FAIR (COSY, Germany, 2005)