IS IT POSSIBLE TO STUDY NANOCRYSTAL FERROMAGNETIC FILMS BY μSR-TECHNIQUE?

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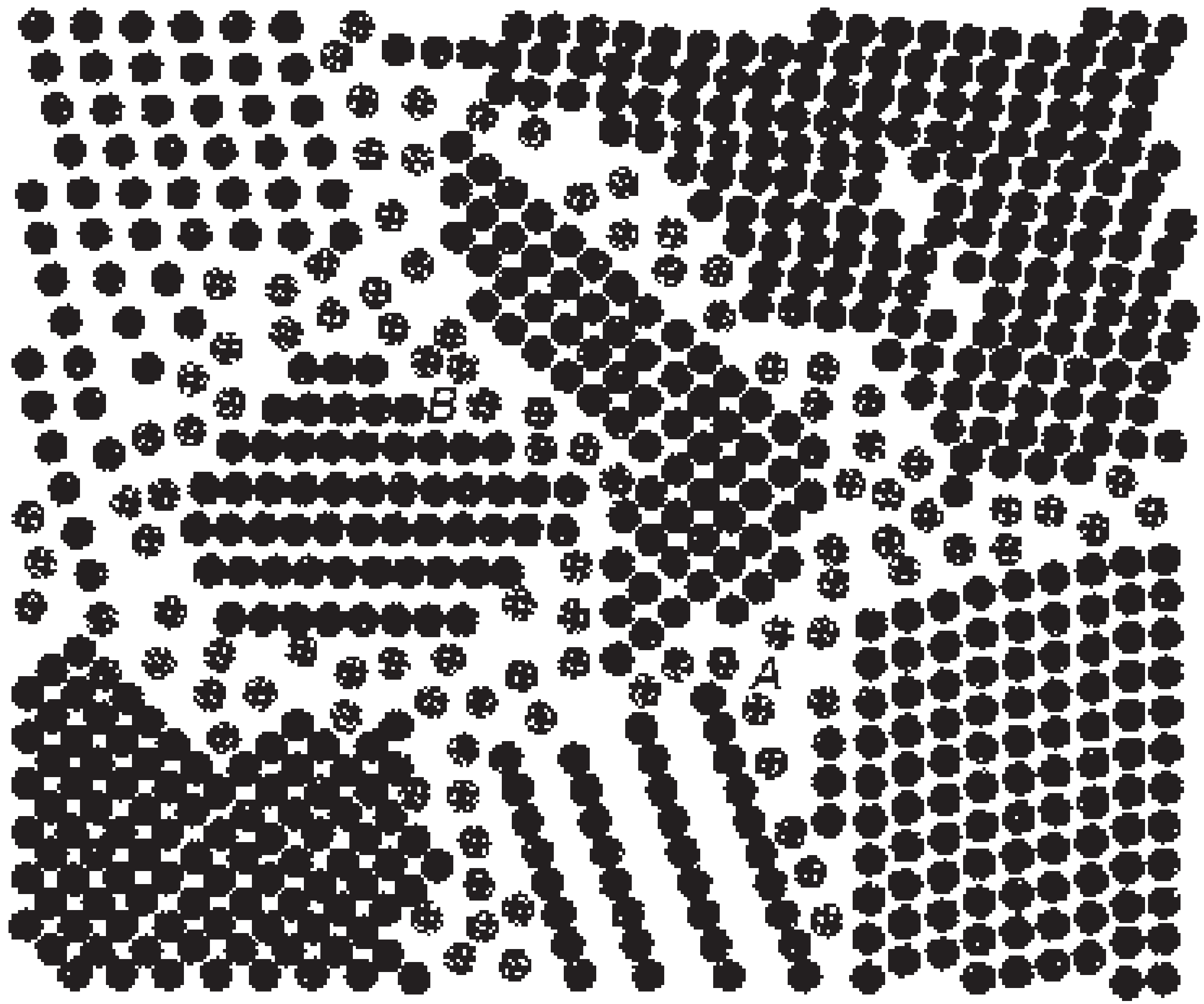
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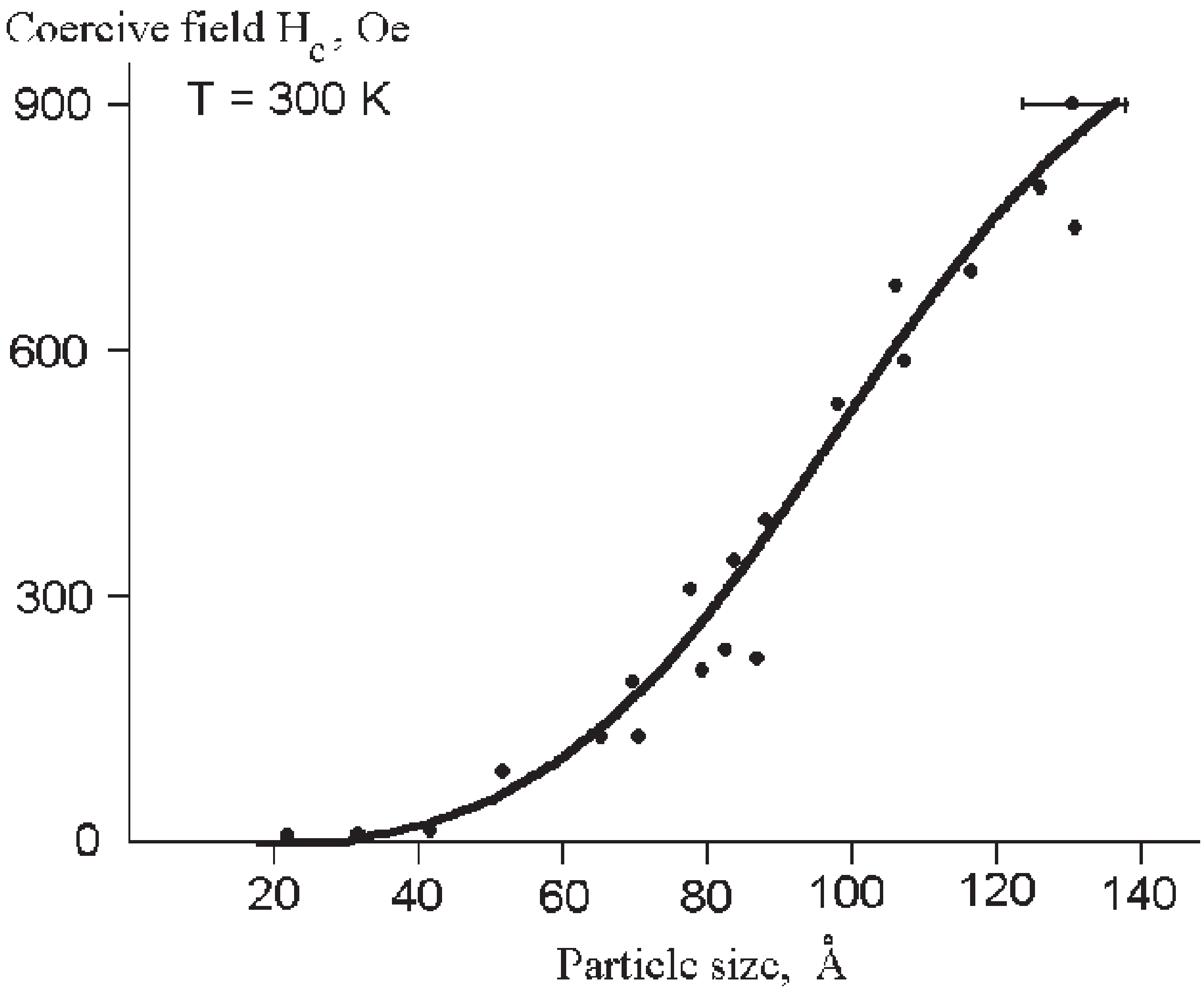

Coercive field H_c, Oe
T = 300 K
900
600
300
0
20
40
60
80
100
120
140
Particle size, Å

Main features of the muon spin polarization behavior in nanostructured films

A behaviour of a muon spin polarization strongly depend on a behaviour of a muon itself. Note first of all that a muon can stop both in some interstitial site of a grain and in an intergrain area. So, we can write a muon spin polarization in a form of a sum

$$\mathcal{P}(t) = \mathbf{P}_{\rm cr}(t) + \mathbf{P}_{\rm nc}(t), \tag{28}$$

where $\mathbf{P}_{\rm cr}(t)$ and $\mathbf{P}_{\rm nc}(t)$ are fraction of a polarization in a grain and outside of it respectively.

For non-diffusion muons in disordered media we have the well-known formula

$$P_i(t) = \mu_{ik}(t) P_k(0)$$

where the polarization tensor is determined as

$$\mu_{ik}(t) = n_i n_k + (\delta_{ik} - n_i n_k) \cos \gamma b_\mu t + e_{ikl} n_l \sin \gamma b_\mu t$$

Here

$\mathbf{b}_\mu$ is the local field at the muon position

$\mathbf{n} = \mathbf{b}_\mu / |\mathbf{b}_\mu|$ - the unit vector and

$\gamma = 13.554 \text{ kHz/G}$ - gyromagnetic ratio for the muon

Hierarchy of Fields

$\mathcal{B}$, - an external field to the whole sample (film);

macroscopic fields $\mathbf{B}$ and $\mathbf{H}$

and a magnetization $\mathbf{M}$ inside grains.

Averaged fields $\langle\mathbf{B}\rangle$, $\langle\mathbf{H}\rangle$ and a magnetization $\langle\mathbf{M}\rangle$. in a film.

$$\mathcal{B}_i = \langle H_i \rangle + 4\pi N_{ik} \langle M_k \rangle, \quad \langle\mathbf{B}\rangle = \langle\mathbf{H}\rangle + 4\pi\langle\mathbf{M}\rangle. \tag{1}$$

$\langle\mathbf{B}\rangle$, $\langle\mathbf{H}\rangle$ and $\langle\mathbf{M}\rangle$ fields and magnetization in an intergrain volume;

a field $\langle\mathbf{B}\rangle$ acts on a muon stopped outside a grain.

$$N_{zz} \equiv N_{\perp} \approx 1, \quad N_{||} \sim N_{xx} \sim N_{yy} \sim N_{xy} \ll 1.$$

$$N_{zx} \sim N_{zy} \ll 1.$$

$\langle\mathbf{B}\rangle$ is the “external” field for grains.

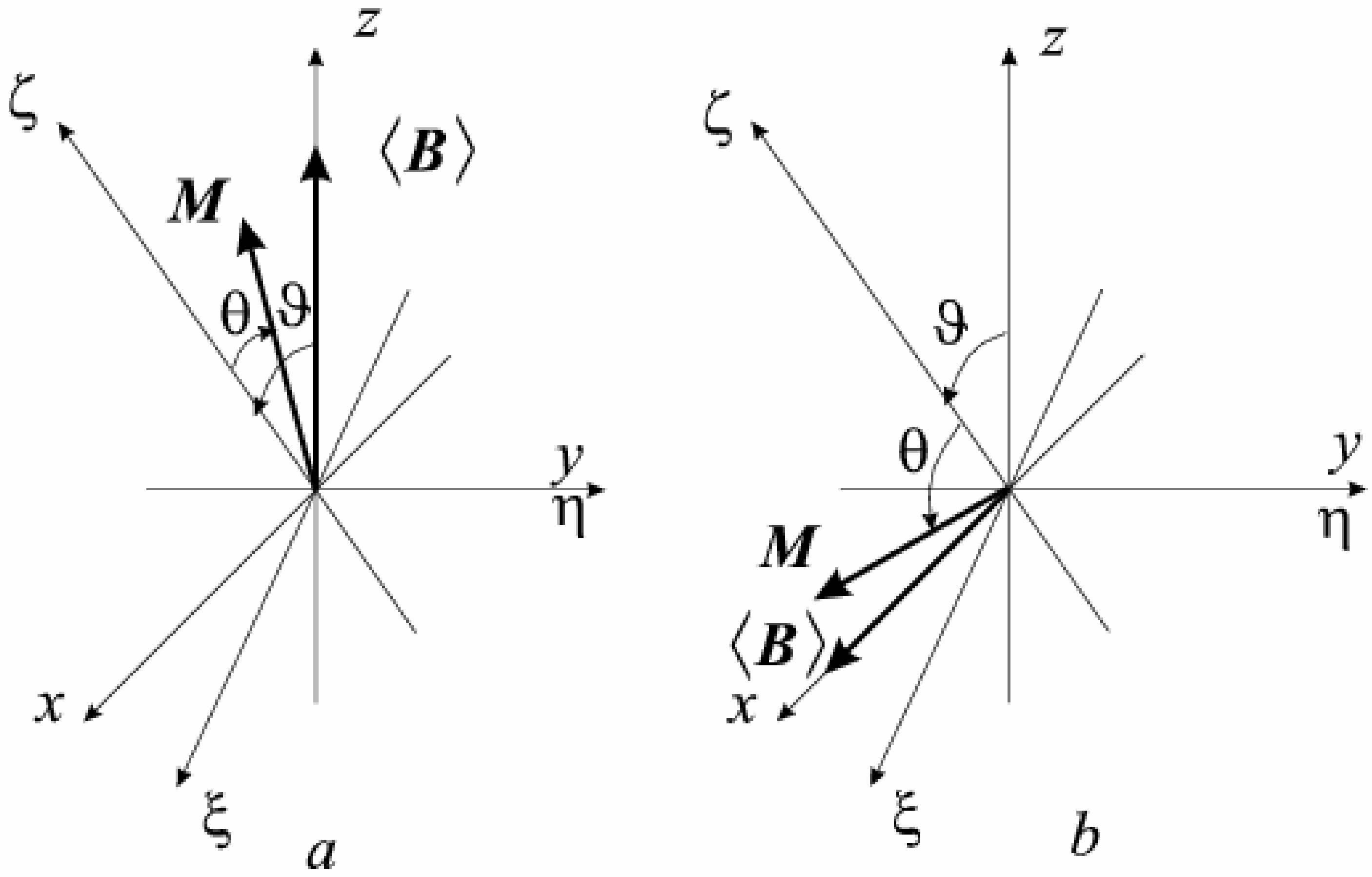

Рис. 1:

External field is directed perpendicular to the plane $\mathcal{B} \| z$

$$\mathcal{B}_z \equiv \mathcal{B} = \langle H_z \rangle + 4\pi \langle M_z \rangle, \quad \langle H_{\rm pl} \rangle \approx 0,$$

$$\langle B_z \rangle = \mathcal{B}, \quad \langle B_{\rm pl} \rangle \approx 4\pi \langle M_{\rm pl} \rangle.$$

External field is parallel to the plane $\mathcal{B} \perp z$.

$$\langle H_z \rangle + 4\pi \langle M_z \rangle = 0, \quad \langle H_z \rangle = -4\pi \langle M_z \rangle, \quad \langle H_{\rm pl} \rangle \approx \mathcal{B}.$$

$$\langle B_z \rangle = 4\pi \langle M_z \rangle, \quad \langle B_{\rm pl} \rangle = \mathcal{B} + 4\pi \langle M_{\rm pl} \rangle.$$

For the every grain $\langle B_i \rangle = H_i + 4\pi n_{ik} M_k,$

$$n_{\|} = n_{zz} \approx 1, \quad n_{\perp} = 1 - n_{\|} \ll 1,$$

External field is directed perpendicular to the plane $\mathcal{B}\| z$

$$\langle B_z\rangle \approx \mathcal{B} = H_z + 4\pi M_z, \quad \text{или} \quad H_z = \mathcal{B} - 4\pi M_z.$$

$$H_{\rm pl} = \langle B_{\rm pl}\rangle = 4\pi\langle M_{\rm pl}\rangle, \quad B_{\rm pl} = 4\pi\big(\langle M_{\rm pl}\rangle + M_{\rm pl}\big).$$

External field is parallel to the plane $\mathcal{B}\perp z,$

$$H_z = -4\pi M_z, \quad H_{\rm pl} \approx \langle B_{\rm pl}\rangle = \mathcal{B} + 4\pi\langle M_{\rm pl}\rangle.$$

$$B_z = 0, \quad B_{\rm pl} = \mathcal{B} + 4\pi\big(\langle M_{\rm pl}\rangle + M_{\rm pl}\big).$$

We need to determine **M** inside grains and <**M**> inside the film.

For a single-axis ferromagnet

$$U_{\rm an} = K_{ik} m_i m_k,$$

Minimization of free-energy potential for a grain

$$\widetilde{F} = \frac{1}{2}(4\pi n_{ik} + \beta_{ik}) M_i M_k - \langle \mathbf{B} \rangle \mathbf{M},$$

For $\mathcal{B} || z$

$$\widetilde{F}(\theta) = \frac{\beta}{2} M^2 \sin^2\theta + 2\pi M^2 \cos^2(\vartheta - \theta) - M\mathcal{B}\cos(\vartheta - \theta).$$

Minimum condition

$$\frac{1}{2}\beta M \sin 2\theta - 2\pi M \sin 2(\vartheta - \theta) + \mathcal{B}\sin(\vartheta - \theta) = 0.$$

Analytical solution in a limit of strong external fields

$\mathcal{B} \gg M,$ or $|\langle \mathbf{B} \rangle| \gg M$

Define $\theta = \vartheta - \delta$;

$$\delta \approx \frac{1}{2}\beta\frac{M}{\mathcal{B}}\sin 2\vartheta.$$

Inside grains

$$M_z = M\cos\delta \approx M\left(1 - \frac{\beta^2}{8}\frac{M^2}{\mathcal{B}^2}\sin^2 2\vartheta\right),$$

$$M_{\rm pl} = M\sin\delta \approx \frac{1}{2}\beta\frac{M^2}{\mathcal{B}}\sin 2\vartheta.$$

Respectively

$$\langle M_z \rangle = M\left(1 - \frac{\beta^2}{15}\frac{M^2}{\mathcal{B}^2}\right), \quad \langle M_{\rm pl} \rangle = 0.$$

For $\mathcal{B} \perp z,$ we have:

$$M_z = M \sin\delta, \quad \langle M_z \rangle = 0, \quad M_{\rm pl} = M\cos\delta, \quad \langle M_{\rm pl}\rangle = M\left(1 - \frac{\beta^2}{15}\frac{M^2}{\mathcal{B}^2}\right).$$

For cubic ferromagnet of easy-axis type

$$\widetilde{F} = -\frac{1}{4}\beta M^2\left(m_\xi^4 + m_\eta^4 + m_\zeta^4\right) + 2\pi n_{ik} M_i M_k - \langle \mathbf{B}\rangle \mathbf{M}.$$

Direction of the magnetization vector

$$\delta \approx \frac{1}{4}\beta\frac{M}{\mathcal{B}}\sin 4\vartheta.$$

For the external field $\mathcal{B} \| z$

$$M_z = M\cos\delta \approx M\left(1 - \frac{\beta^2}{32}\frac{M^2}{\mathcal{B}^2}\sin^2 4\vartheta\right),$$

$$M_{\rm pl} = M\sin\delta \approx \frac{1}{4}\beta\frac{M^2}{\mathcal{B}}\sin 4\vartheta.$$

$$\langle M_z \rangle = M\left(1 - \frac{\beta^2}{63}\frac{M^2}{\mathcal{B}^2}\right), \quad \langle M_{\rm pl} \rangle = 0.$$

If the external field $\mathcal{B} \perp z$,

$$M_z = -M\sin\delta, \quad \langle M_z \rangle = 0, \quad M_{\rm pl} = M\cos\delta, \quad \langle M_{\rm pl} \rangle = M\left(1 - \frac{\beta^2}{63}\frac{M^2}{\mathcal{B}^2}\right).$$

Local field at a muon

Local field at a muon determines a precession frequency of its spin polarization. A local field at a muon in any grain is determined by the formula [14, 15]

$$\mathbf{b}_\mu = \mathbf{B} - \frac{8\pi}{3}\mathbf{M} + \mathbf{b}_{\text{dip}} + \mathbf{B}_{\text{cont}}, \tag{18}$$

where $\mathbf{B}_{\text{cont}}$ is the field created by conduction electrons, and $\mathbf{b}_{\text{dip}}$ is the microscopic field of all dipoles inside the Lorentz sphere. A contact field could always written in the form

$$B_{i\,\text{cont}} = K_{ik} B_k. \tag{19}$$

For cubic crystals is valid a relation $K_{ik} = \delta_{ik} K$ and contact field is reduced to the well-known isotropic Knight shift.

Microscopic field $\mathbf{b}_{\rm dip}$ inside the Lorentz sphere one can represent in a form [14, 15])

$$b_{i\,\rm dip} = -\frac{4\pi}{3}M_i + a_{ik}M_k. \qquad (20)$$

A tensor a_{ik} depends on an interstitial site where a muon stopped. It can be calculated by the well-known Evald method. Calculations shown (see e.g. [14, 15]) that for the FCC-lattice (Ni) the tensor is determined as $a_{ik} = \delta_{ik}4\pi/3$, hence, a microscopic dipole field is equal to zero:

$$\mathbf{b}_{\rm dip}(fcc) = 0. \qquad (21)$$

Thus, for the FCC-lattice the local field at a muon doesn't depend on a type of an interstitial site and is equal to

$$\mathbf{b}_\mu(fcc) = \mathbf{B} - \frac{8\pi}{3}\mathbf{M}. \qquad (22)$$

Microscopic dipole field for the HCP-lattice (Co) is small too, but has different values for different crystallographically unequivalent interstitial sites:

$$a_{ik}(hcp) = \frac{4\pi}{3} + \delta a_{ik}. \tag{23}$$

If the z-axes is directed along the hexagonal axes components of the tensor a_{ik} can be written in the form:

$$\begin{aligned} \delta a^h_{xx} &= \delta a^h_{yy} = \delta/2, \quad \delta a^h_{zz} = -\delta \quad \text{in an octahedral site,} \\ \delta a^t_{xx} &= \delta a^t_{yy} = -\delta, \quad \delta a^t_{zz} = 2\delta \quad \text{in a tetrahedral site.} \end{aligned} \tag{24}$$

We can consider $b_{\rm dip} \ll M$ because $\delta \approx 0.104$. The local field at a muon in the HCP-lattice can be written in the form:

$$b_{\mu\, i}(hcp) = B_i - \frac{8\pi}{3} M_i + \delta a_{ik} M_k. \tag{25}$$

The more complicated picture is observed in the BCC-lattice (Fe) where a dipole field have a large value and depend both on a type of an interstitial site and on a direction of a magnetization vector $\mathbf{M}$. Components of a tensor $a_{ik}(bcc)$ in the main axis are equal to

$$\begin{aligned} a^h_{xx} &= a^h_{yy} = -1.165, \quad a^h_{zz} = 14.9 \quad \text{in an octahedral site,} \\ a^t_{xx} &= a^t_{yy} = \quad 5.707, \quad a^t_{zz} = 1.152 \quad \text{in a tetrahedral site.} \end{aligned} \tag{26}$$

The local field at a muon in the BCC-lattice can be written in the form:

$$b_{\mu\, i}(bcc) = B_i - 4\pi M_i + a_{ik}(bcc) M_k. \tag{27}$$

Fast diffusion

No difference between HCP and BCC lattices.

Strong fields, $\mathcal{B}\|z$ Local field:

$$b_z = (1+K)\mathcal{B} - \frac{8\pi}{3}M_z \approx \mathcal{B} - \frac{8\pi}{3}M\left(1 - \frac{1}{2}\delta^2\right),$$

$$b_{\rm pl} = (1+K)B_{\rm pl} - \frac{8\pi}{3}M_{\rm pl} \approx \frac{4\pi}{3}M\delta,$$

Two items for the local field: $b = b_0 + b(\vartheta).$

$$b_0 = \mathcal{B} - \frac{8\pi}{3}M + \frac{1}{2}\left(\frac{4\pi}{3}\right)^2\frac{M^2}{\mathcal{B}}, \quad b(\vartheta) = \frac{4\pi}{3}M\delta^2.$$

Transverse polarization

$$\mathcal{P}_\perp = \langle \frac{b_z}{b} \mathrm{e}^{-\mathrm{i}\gamma_\mu b t} \rangle \approx \mathrm{e}^{-\mathrm{i}\omega_0 t} \langle \mathrm{e}^{-\mathrm{i}\omega(\vartheta) t} \rangle,$$

$$\omega_0 = \gamma_\mu b_0, \; \omega(\vartheta) = \gamma_\mu b(\vartheta).$$

$$\mathcal{P}_\perp^{\mathrm{diff}} = \mathrm{e}^{-\mathrm{i}(\omega_0 + \Delta\omega)t} \mathrm{e}^{-\sigma_{\mathrm{diff}}^2 t^2},$$

$$\Delta\omega = \gamma_\mu \frac{1}{63} \frac{2\pi}{3} M \left(\frac{M}{\mathcal{B}}\right)^2 \quad \sigma_{\mathrm{diff}}^2 = \frac{1}{2} \left(\frac{4\pi}{3} \gamma_\mu M\right)^2 \langle \delta^2 \rangle$$

Longitudinal polarization

$$\mathcal{P}_\parallel^{\mathrm{diff}} = \mathrm{e}^{-\mathrm{i}\omega_0 t} \langle \frac{b_{\mathrm{pl}}}{b} \mathrm{e}^{-\mathrm{i}\omega(\vartheta) t} \rangle = 0.$$

Non-diffusion muons

BCC lattice, $\mathcal{B} \| z$

Components of the dipolar tensor

$$a_{zz} = a + \delta a \cos 2\vartheta, \quad a_{xx} = a - \delta a \cos 2\vartheta, \quad a_{zx} = a_{xz} = \delta a \sin 2\vartheta,$$

where

$$a = (a_{\|} + a_{\perp})/2, \; \delta a = (a_{\|} - a_{\perp})/2, \; a_{\|} = a_{\zeta\zeta}, \; a_{\perp} = a_{\xi\xi}.$$

Components of the local field:

$$\begin{aligned} b_z =& \mathcal{B} - 4\pi M_z + a_{zz} M_z + a_{zx} M_{\text{pl}} = \\ =& \mathcal{B} - \frac{1}{2}(4\pi + a_{\perp})M + \delta a M \cos 2\vartheta + \frac{1}{4}\delta a \beta \frac{M^2}{\mathcal{B}} \sin 4\vartheta \cos 2\vartheta, \\ b_{\text{pl}} =& \delta a M \sin 2\vartheta - \left[\frac{1}{2}(4\pi + a_{\perp}) + \delta a \cos 2\vartheta\right] \frac{1}{4}\beta \frac{M^2}{\mathcal{B}} \sin 4\vartheta. \end{aligned}$$

Two items of the local field:

$$b_0 = \mathcal{B} - \frac{1}{2}(4\pi + a_\perp)M,$$

$$b(\vartheta) \approx \delta a M \cos 2\vartheta + \frac{1}{2}\delta a \left(\frac{1}{4}\beta \cos 2\vartheta + \delta a\right) \frac{M^2}{\mathcal{B}} \sin^2 2\vartheta.$$

Transverse component of a polarization

$$\mathcal{P}_\perp = \langle \frac{b_z}{b} \mathrm{e}^{-\mathrm{i}\omega(\vartheta)t} \rangle \mathrm{e}^{-\mathrm{i}\omega_0 t}.$$

represented by the gaussian exponent

$$\mathcal{P}_\perp = \mathrm{e}^{-\mathrm{i}(\omega_0 + \Delta\omega)t} \mathrm{e}^{-\sigma_{\mathrm{nd}}^2 t^2},$$

$$\omega_0 = \gamma_\mu \mathcal{B} \left(1 - (2\pi + \frac{1}{2} a_\perp) \frac{M}{\mathcal{B}} \right),$$

$$\Delta\omega = -\frac{1}{3} \gamma_\mu \delta a M \left[1 - \frac{2}{15} \left(2\delta a - \frac{41}{28} \beta \right) \frac{M}{\mathcal{B}} \right],$$

$$\sigma^2_{\mathrm{nd}} = \frac{7}{30} \left(\gamma_\mu \delta a M \right)^2.$$

Longitudinal polarization is equal approximately to zero

$$\mathcal{P}_{||} = \langle \frac{b_{\mathrm{pl}}}{b} \mathrm{e}^{-\mathrm{i}\omega(\vartheta)t} \rangle \mathrm{e}^{-\mathrm{i}\omega_0 t} = 0.$$

For the external field parallel to the plane we have the same formulas.

In the case of single-axes ferromagnets we have a similar behavior:

1. No oscillations in the longitudinal polarization
2. Gaussian exponent for the transverse component of polarization with parameters

$$\omega_0^{hcp} = \gamma_\mu \mathcal{B}\left[1 - \left(\frac{8\pi}{3} - \frac{1}{2}\delta a^{hcp}\right)\frac{M}{\mathcal{B}}\right],$$

$$\Delta\omega^{hcp} = -\frac{1}{2}\gamma_\mu \delta a^{hcp} M\left[1 - \frac{4}{5}(\beta + \frac{3}{2}\delta a^{hcp})\frac{M}{\mathcal{B}}\right],$$

$$\sigma_{\rm nd}^2 = \frac{21}{40}\left(\gamma_\mu \delta a^{hcp} M\right)^2.$$

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