Entangled states and μSR

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Applications of entangled states

- Quantum cryptography
- Quantum theory of information
- Physics of quantum computation

$$\left|\Psi\right\rangle = \sum_{mF,\eta}\left(Q_{mF,\eta}\cdot\left|j,mF-1/2\right\rangle\otimes\left|1/2,1/2\right\rangle + R_{mF,\eta}\cdot\left|j,mF+1/2\right\rangle\otimes\left|1/2,-1/2\right\rangle\right)$$

$$\rho(\psi) := \left|\begin{array}{l} \text{for } m\in 0..\text{rows}(\psi)-1 \\ \quad \text{for } n\in 0..\text{rows}(\psi)-1 \\ \quad\quad \rho_{m,n}\leftarrow \psi_m\cdot\overline{\psi_n} \\ \rho \end{array}\right.$$

Pure states

$$|\psi\rangle = \sum_i c_i |\psi_i\rangle \qquad \hat{\rho} = |\psi\rangle\langle\psi| \qquad \rho_{m,n} = \psi_m \psi_n^*$$

$$|\psi\rangle = a|\uparrow\uparrow\rangle + b|\uparrow\downarrow\rangle + c|\downarrow\uparrow\rangle + d|\downarrow\downarrow\rangle$$

$$|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$$

$$\rho = \begin{pmatrix} |a|^2 & a^*b & a^*c & a^*d \\ ab^* & |b|^2 & b^*c & b^*d \\ ac^* & bc^* & |c|^2 & c^*d \\ ad^* & bd^* & cd^* & |d|^2 \end{pmatrix}$$

Entangled states

$$|\psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle \otimes \ldots \qquad \hat{\rho} = |\psi\rangle\langle\psi|$$

Examples:

EPR – state

$$|\psi_{EPR}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$$\langle\psi_{EPR}|\hat{\sigma}_\alpha^A \otimes \hat{\sigma}_\alpha^B|\psi_{EPR}\rangle = -1, \ \ \forall\alpha$$

$$\rho_{EPR} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

SC – state

$$|\psi_{SC}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle\right)$$

$$\langle\psi_{SC}|\hat{\sigma}_\alpha^A \otimes \hat{\sigma}_\alpha^B|\psi_{SC}\rangle = 1, \ \ \forall\alpha$$

$$\rho_{SC} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

Classical correlations

$$\hat{\rho} = \sum_{\lambda} w_{\lambda} \hat{\rho}_{\lambda}^{(A)} \otimes \hat{\rho}_{\lambda}^{(B)} \Rightarrow tr_A \hat{\rho} = \sum_{\lambda} w_{\lambda} \hat{\rho}_{\lambda}^{(B)} = \hat{\rho}^{(B)},$$

$$tr_A \hat{\rho} = \sum_{\lambda} w_{\lambda} \hat{\rho}_{\lambda}^{(A)} = \hat{\rho}^{(A)}$$

$$\left\langle \hat{T}^{(A)} \cdot \hat{T}^{(B)} \right\rangle = \sum_{\lambda} w_{\lambda} \, tr\left\{ \hat{T}^{(A)} \hat{\rho}_{\lambda}^{(A)} \right\} \cdot tr\left\{ \hat{T}^{(B)} \hat{\rho}_{\lambda}^{(B)} \right\} = \sum_{\lambda} w_{\lambda} \left\langle \hat{T}^{(A)} \right\rangle_{\lambda} \cdot \left\langle \hat{T}^{(B)} \right\rangle_{\lambda}$$

Examples statistical mixture

$$\left|\psi_1\right\rangle = \left|\uparrow\downarrow\right\rangle, P_1 = 1/2$$

$$\left|\psi_2\right\rangle = \left|\downarrow\uparrow\right\rangle, P_2 = 1/2$$

$$\rho_{cl} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \left\langle s_z^{(1)} \cdot s_z^{(2)} \right\rangle = \frac{-1}{4}, \quad \left\langle s_x^{(1)} \cdot s_x^{(2)} \right\rangle = \left\langle s_y^{(1)} \cdot s_y^{(2)} \right\rangle = 0$$

$$\left\langle s_x^{(1)} \cdot s_y^{(2)} \right\rangle = \left\langle s_x^{(1)} \cdot s_z^{(2)} \right\rangle = \left\langle s_y^{(1)} \cdot s_z^{(2)} \right\rangle = 0$$

Quantum correlation in pure state 1,0

$$\left|\psi_p\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle + \left|\downarrow\uparrow\right\rangle\right)$$

$$\rho_p = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \left\langle s_z^{(1)} \cdot s_z^{(2)} \right\rangle = \frac{-1}{4}, \quad \left\langle s_x^{(1)} \cdot s_x^{(2)} \right\rangle = \left\langle s_y^{(1)} \cdot s_y^{(2)} \right\rangle = \frac{1}{4}$$

$$\left\langle s_x^{(1)} \cdot s_y^{(2)} \right\rangle = \left\langle s_x^{(1)} \cdot s_z^{(2)} \right\rangle = \left\langle s_y^{(1)} \cdot s_z^{(2)} \right\rangle = 0$$

Entangled non-pure states

$$\left|\psi_{EPR}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right),\ P_{EPR} = 1/2$$

$$\left|\psi_{SC}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow\uparrow\right\rangle + \left|\downarrow\downarrow\right\rangle\right),\ P_{SC} = 1/2$$

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

Publications on the methods of density matrix measurement

- 1. S. Mancini, V.I. Man'ko, P. Tombesi, Symplectic tomography as classical approach to quantum systems, Phys. Lett. A 213 (1996) 1-6.
- 2. Ulf Leonhardt, Discrete Wigner function and quantum-state tomography, Physical Review A 53, N5 (1996) 2998
- 3. V.V. Dodonov, V.I. Man'ko, Positive distribution description for spin state, Phys. Lett. A 229 (1997) 335-339.
- 4. V.I. Manko, O.V. Manko Tomography of spin states, JETP, 112, #3(9), (1997) 796-804.
- 5. V.I. Manko, S.S. Safonov, Tomography of quantum states of symmetric rotator, Nuclear physics, 61, #4 (1998) 658-664.
- 6. V.A. Andreev, V.I. Manko, Tomography of two-particle spin states, JETP, 114, #2(8) (1998) 437-447.

Separability of density matrices

$$\hat{\rho} = \sum_{\lambda} w_{\lambda} \hat{\rho}_{\lambda}^{(A)} \otimes \hat{\rho}_{\lambda}^{(B)}$$

Чистые состояния

$$\left|\psi_{AB}\right\rangle = \sum_{i=1}^{M} c_i \left|uA_i\right\rangle \left|vB_i\right\rangle, \quad \text{где} \quad uA = \begin{pmatrix} uA_1 \\ \dots \\ uA_M \end{pmatrix} \Big\} M, \quad , \quad vB = \begin{pmatrix} vB_1 \\ \dots \\ vB_N \end{pmatrix} \Big\} N$$

Для определённости пусть $M \le N$, тогда $\{vB_i\}$ можно подобрать ортоганальными

$$\hat{\rho}_{AB} = \left|\psi_{AB}\right\rangle\left\langle\psi_{AB}\right| \Rightarrow \begin{cases} \hat{\rho}_A = tr_B \hat{\rho}_{AB} = \sum_i \left\langle vB_i \right| \hat{\rho}_{AB} \left| vB_i \right\rangle = \sum_i \left|uA_i\right\rangle c_i c_i^* \left\langle uA_i\right| \\ \hat{\rho}_B = tr_A \hat{\rho}_{AB} = \dots\dots\dots\dots\dots = \sum_i \left|vB_i\right\rangle c_i c_i^* \left\langle vB_i\right| \end{cases}$$

Число не равных нулю коэффициентов c_i называется Sch –
–число Шмидта

If Sch ≥ 2 $\Rightarrow$ Entangled state(Inseparability)=
=Запутанные состояния

Критерий Переса

$$\left.\begin{array}{l} \mathrm{A} \Rightarrow \mathrm{k} \Rightarrow \left(\hat{\rho}_{\mathrm{A}}\right)_{\mathrm{k};\mathrm{k}'} \\ \mathrm{B} \Rightarrow \xi \Rightarrow \left(\hat{\rho}_{\mathrm{B}}\right)_{\xi;\xi'} \end{array}\right\} \Rightarrow \left(\hat{\rho}_{\mathrm{AB}}\right)_{\mathrm{k},\xi;\mathrm{k}',\xi'} \Rightarrow \left(\hat{\sigma}_{\mathrm{AB}}\right)_{\mathrm{k},\xi;\mathrm{k}',\xi'} = \left(\hat{\rho}_{\mathrm{AB}}\right)_{\mathrm{k}',\xi;\mathrm{k},\xi'}$$

$$\min\left(\lambda\left(\hat{\sigma}_{\mathrm{AB}}\right)\right) \geq 0 \equiv \text{Необходимое условие сепарабильности (незапутанности состояния)}$$

$$\min\left(\lambda\left(\hat{\sigma}_{\mathrm{AB}}\right)\right) < 0 \rightarrow \text{подозрение на запутанность, но ???}$$

Док-во

$$\left.\begin{array}{l} \hat{\sigma} = \sum_{\lambda} w_{\lambda} \left(\hat{\rho}_{\lambda}^{(A)}\right)^{T} \otimes \hat{\rho}_{\lambda}^{(B)} \\ \left(\hat{\rho}_{\lambda}^{(A)}\right)^{T} = \hat{\tilde{\rho}}_{\lambda}^{(A)} \end{array}\right\} \begin{array}{l} \Rightarrow \sum_{\lambda} w_{\lambda} \hat{\tilde{\rho}}_{\lambda}^{(A)} \otimes \hat{\rho}_{\lambda}^{(B)} = \hat{\tilde{\rho}}^{(AB)} \Rightarrow \\ \Rightarrow \min\left\{\lambda\left(\hat{\tilde{\rho}}^{(AB)}\right)\right\} \geq 0 \end{array}$$

Measurement of density matrix: muon

$$\hat{\rho}(t) = \frac{\hat{I}}{2} + P_\alpha(t)\,\hat{\sigma}_\alpha$$

Density matrix

$$\hat{\rho}(t) = \sum_{Lj=0}^{2\cdot j} \sum_{M_{LJ}=-Lj}^{LJ} \sum_{Ls=0}^{2\cdot s} \sum_{M_{LS}=Ls}^{Ls} \rho_{Lj,M_{Lj},Ls,M_{Ls}}(t)\ \hat{T}_{\mathrm{j}}\left(Lj, M_{Lj}\right)\ \hat{T}_{\mathrm{s}}\left(Ls, M_{Ls}\right)$$

$$\rho_{\xi}(t) = \rho_{Lj,M_{Lj},Ls,M_{Ls}}(t) = Sp\left\{\hat{T}_{\mathrm{j}}\left(Lj, -M_{Lj}\right)\ \hat{T}_{\mathrm{s}}\left(Ls, -M_{Ls}\right)\cdot\hat{\rho}(t)\right\}$$

- vector in $[(2s+1)(2j+1)]^2$ space

$$\hat{T}_{\xi} = \hat{T}_{\mathrm{j}}\left(Lj, M_{Lj}\right)\ \hat{T}_{\mathrm{s}}\left(Ls, M_{Ls}\right)$$

Time dependence

$$\rho_{\xi}(t)=\sum_{\xi'} LS_{\xi,\xi'}(t)\cdot\rho_{\xi'}(0)$$

1 $$\hat{S}(t)=e^{-i\cdot t\cdot\hat{H}/h},\quad LS_{\xi,\xi'}(t)=Sp\left\{\hat{T}_{\xi}{}^{+}\hat{S}(t)\hat{T}_{\xi'}\hat{S}^{+}(t)\right\}$$

2 $$\frac{\partial\rho_{\xi}(t)}{\partial t}=\sum_{\xi'}\left(\frac{1}{i\,h}HH_{\xi,\xi'}-\nu\nu_{\xi,\xi'}\right)\rho_{\xi}(t),$$

where $HH_{\xi,\xi'}=Sp\left\{\left[\hat{T}_{\xi'},\ \hat{T}_{\xi}\right]\ \hat{H}\right\}$

$$\nu\nu_{\xi,\xi'}=\delta_{L's,Ls}\delta_{M'_{Ls},M_{Ls}}\delta_{L'j,Lj}\delta_{M'_{Ls},M_{Ls}}\left(G_2(j,Lj)\ \nu_2+G_1(j,Lj)\ \nu_1\right)$$

Measurement

$$\rho_{0,0,1,M_{Ls}}(t)$$

$$\sum_{\xi'} \mathrm{LS}_{\xi_{\mathrm{P}\mu},\xi'}(\mathrm{t_n})\,\rho_{\xi'}(0) = \rho_{\xi_{\mathrm{P}\mu}}(\mathrm{t_n})$$

There are 15 independetn elements in matrix $\rho_{\xi'}(0)$

To measure density function it is necessary

- three counters
- five time points

Matrix of coefficient

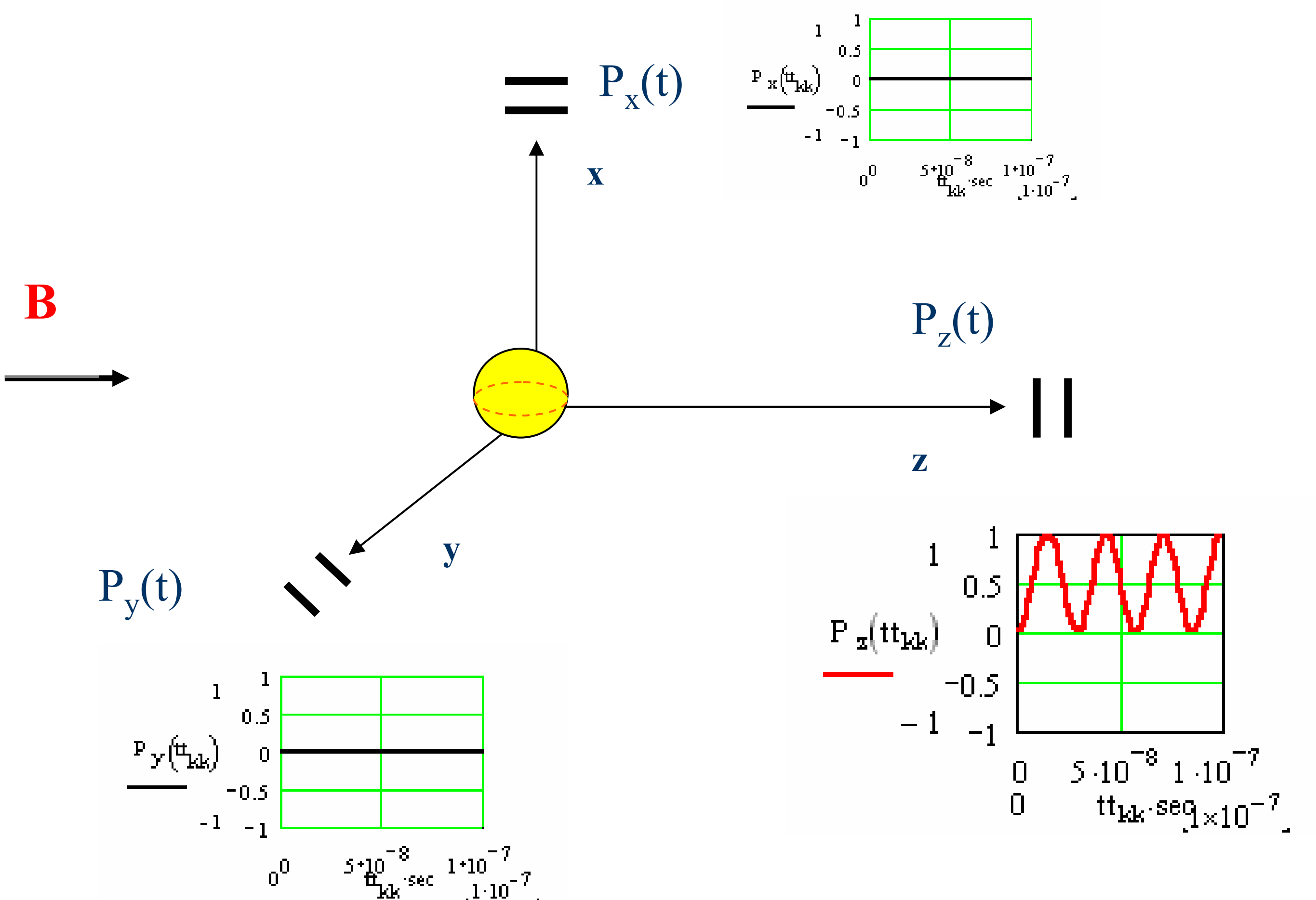
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	A	-A	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-A	A	0	0	0	0	0	0	0	0	0	0	0
0	A	-A	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-A	A	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	A	0	-A	0	0	0	0	0
0	0	0	0	0	0	0	0	-A	0	A	0	0	0	0	0
0	0	0	0	0	0	A	-A	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-A	A	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	A	-A	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-A	A	0
0	0	0	0	0	0	0	0	0	0	0	A	-A	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-A	A	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Conclusion for muonium

- Impossible to estimate all elements of density matrix for isotropic muonium
- Anysotropic muonium density matrix could be measured by μSR

ЭПР - состояние

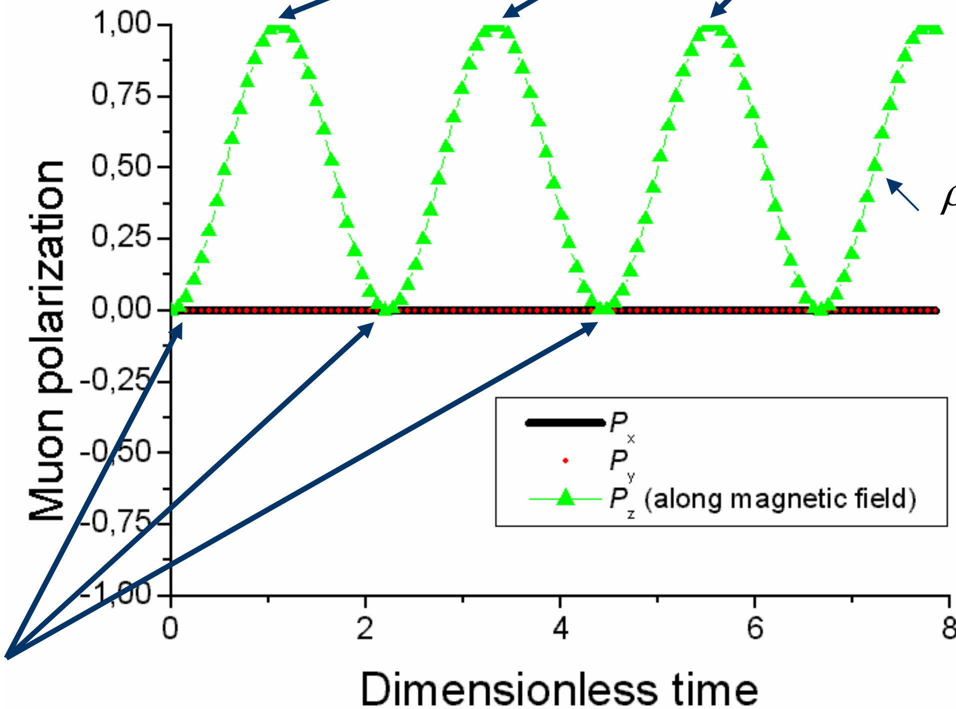
$$\left|\psi_{EPR}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right) \Leftrightarrow \rho_{EPR} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \left\langle \hat{s}_x \right\rangle = \left\langle \hat{s}_y \right\rangle = \left\langle \hat{s}_z \right\rangle = 0$$



μSR photo of EPR-state

$$|\psi_2\rangle = |\downarrow\uparrow\rangle$$

$$\rho_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



$$\rho_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & -\frac{1}{4}+\frac{1}{2\sqrt{2}} & 0 \\ 0 & -\frac{1}{4}-\frac{1}{2\sqrt{2}} & \frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$|\psi_{EPR}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$$\rho_{EPR} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P_\alpha(t) = 2\cdot Sp\left\{\hat{s}_\alpha \cdot \hat{S}(t,x)\,\hat{\rho}(0)\,\hat{S}^{+}(t,x)\right\}$$

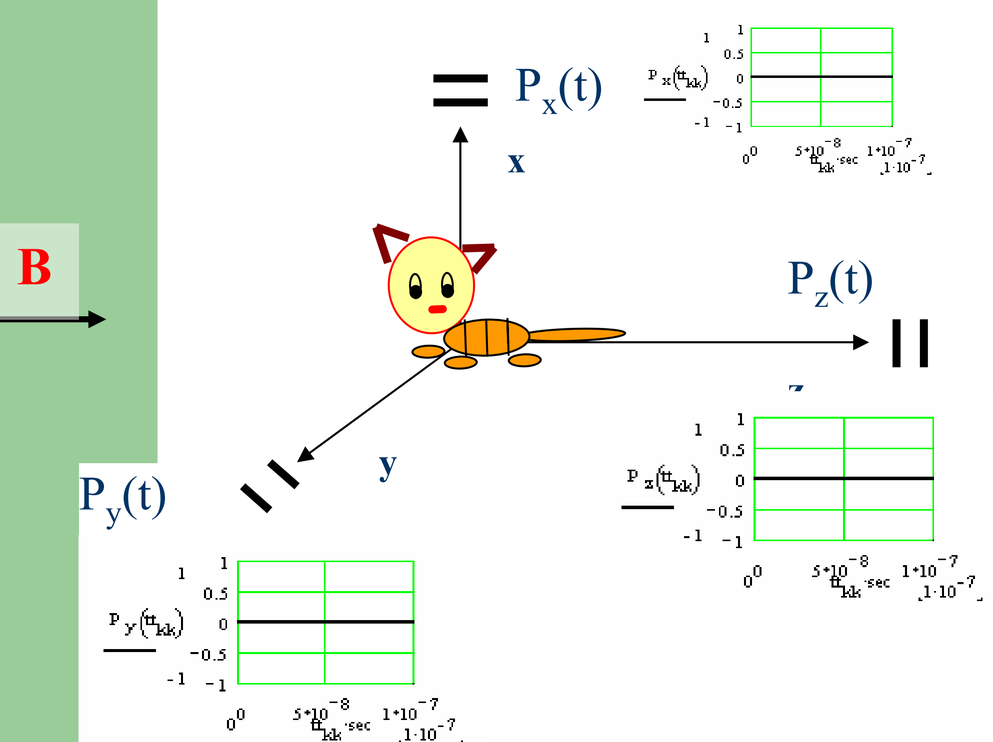
Ш Кот в анфас и сверху

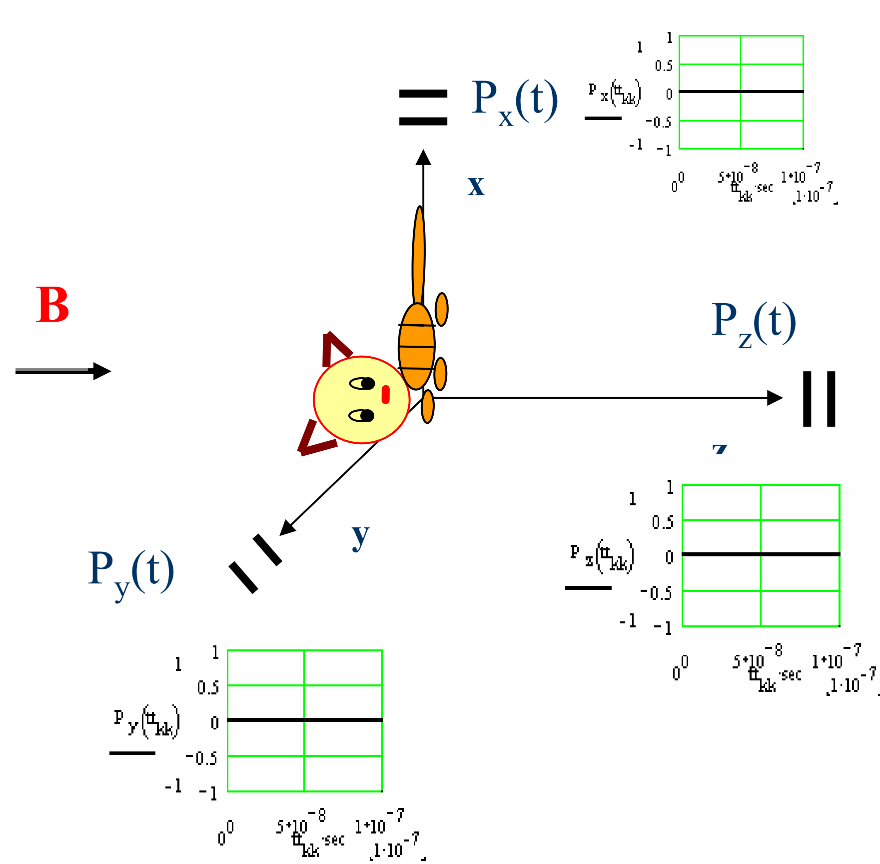
$$\rho_{ShC} = \begin{pmatrix} 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 \end{pmatrix} \Leftrightarrow |\psi_{SC}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle\right)$$

$$nC\phi := \frac{1}{1}\cdot\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \phi := \frac{\pi}{2}$$

$$\rho_{ShC\phi} := O(\phi, nC\phi)^{-1} \cdot \rho_{ShC} \cdot O(\phi, nC\phi)$$

$$\rho_{ShC\phi} = \begin{pmatrix} 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 \end{pmatrix}$$



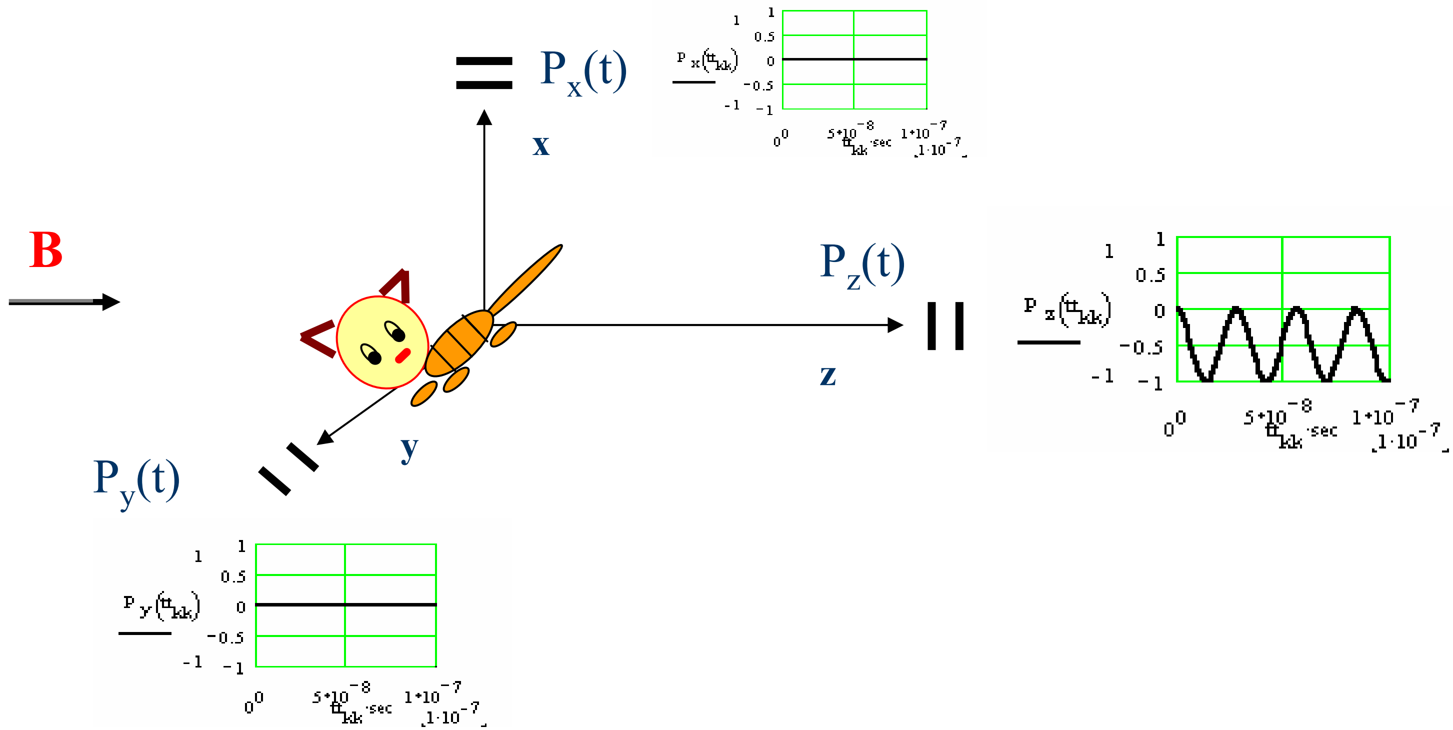


ШК в профиль

$$n_x = \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \phi = \frac{\pi}{2}$$

$$O(\phi, nC\phi) = \begin{pmatrix}0.5 & 0.5i & 0.5i & -0.5\\ 0.5i & 0.5 & -0.5 & 0.5i\\ 0.5i & -0.5 & 0.5 & 0.5i\\ -0.5 & 0.5i & 0.5i & 0.5\end{pmatrix} \qquad \rho\ ShC\phi = \begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0.5 & 0.5 & 0\\ 0 & 0.5 & 0.5 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

$O(n_x, \pi/2)$



3-μSR photo of EPR-state and ShC

$$nC\phi := \frac{1}{\sqrt{3}} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \phi := \frac{\pi}{3}$$

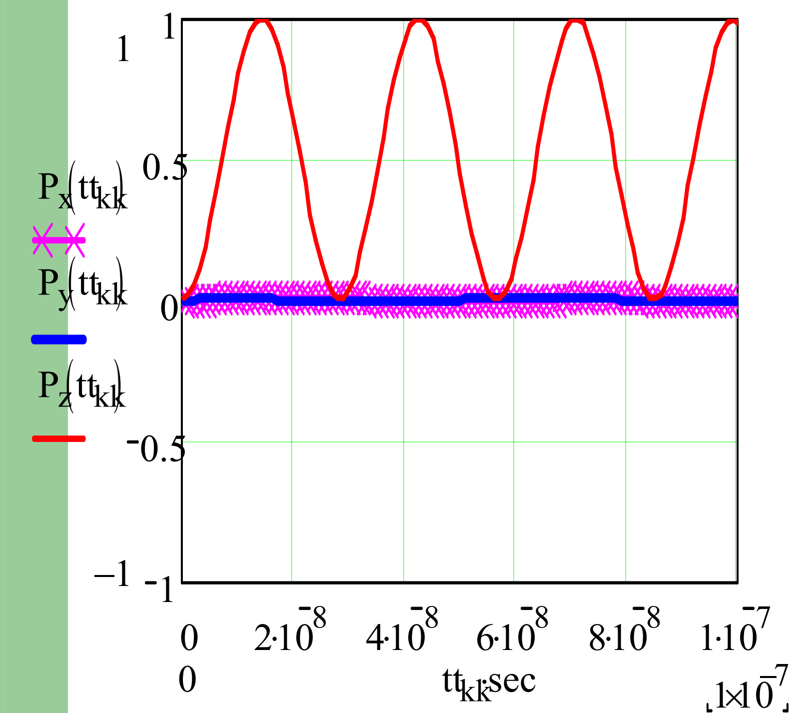


Рис.3 MuSR-Фотография ЭПР-с

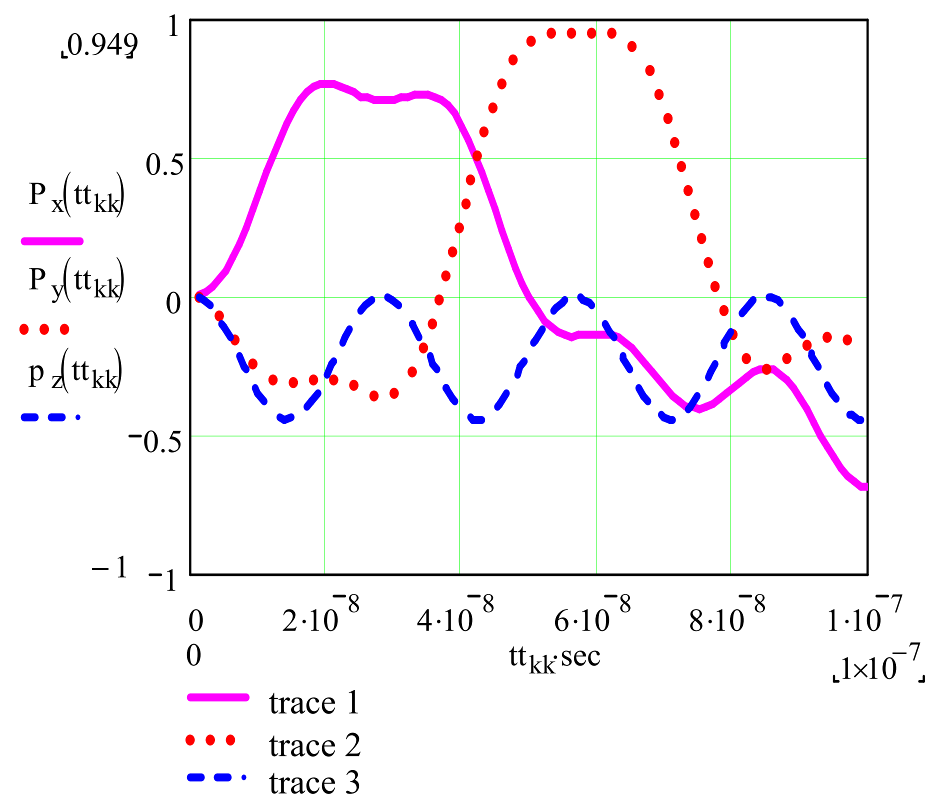


Рис.7 mSR-фотография Ш. кота, развернуто

Dimensionless time

$$t^* = t \cdot \frac{A(2J+1)}{4}$$

$$B^* = \frac{B}{B_C}, \qquad B_C = \frac{\hbar A(2J+1)}{2(g\mu_B - 2\mu_\mu)}$$

Separability criterion

A. Peres, ‘Separability crterion for density matrices”, Phys. Rev. Lett., vol. 77(e-print archive quant-ph/9604005)

$$\hat{\sigma}(t) = \sum_{Lj=0}^{2\cdot j} \sum_{M_{LJ}=-Lj}^{LJ} \sum_{Ls=0}^{2\cdot s} \sum_{M_{LS}=Ls}^{Ls} \rho_{Lj,M_{Lj},Ls,-M_{Ls}}(t)(-1)^{M_{Ls}} \hat{T}_{\mathrm{j}}(Lj, M_{Lj}) \cdot \left(\hat{T}_{\mathrm{s}}(Ls, M_{Ls})\right)$$

$$\hat{\rho}(t) = \sum_{Lj=0}^{2\cdot j} \sum_{M_{LJ}=-Lj}^{LJ} \sum_{Ls=0}^{2\cdot s} \sum_{M_{LS}=Ls}^{Ls} \rho_{Lj,M_{Lj},Ls,M_{Ls}}(t)\ \hat{T}_{\mathrm{j}}(Lj, M_{Lj})\ \hat{T}_{\mathrm{s}}(Ls, M_{Ls})$$

$$\mathrm{T}_{\mathrm{L,M}}{}^{+} = (-1)^{\mathrm{M}}\ \mathrm{T}_{\mathrm{L,-M}}$$

$$\mathrm{T}_{\mathrm{L,M}}{}^{\mathrm{T}} = (-1)^{\mathrm{M}}\ \mathrm{T}_{\mathrm{L,-M}}; \quad \mathrm{T}^{*}_{\mathrm{L,M}} = \mathrm{T}_{\mathrm{L,M}}$$ в циклическом базисе

Создание запутонности

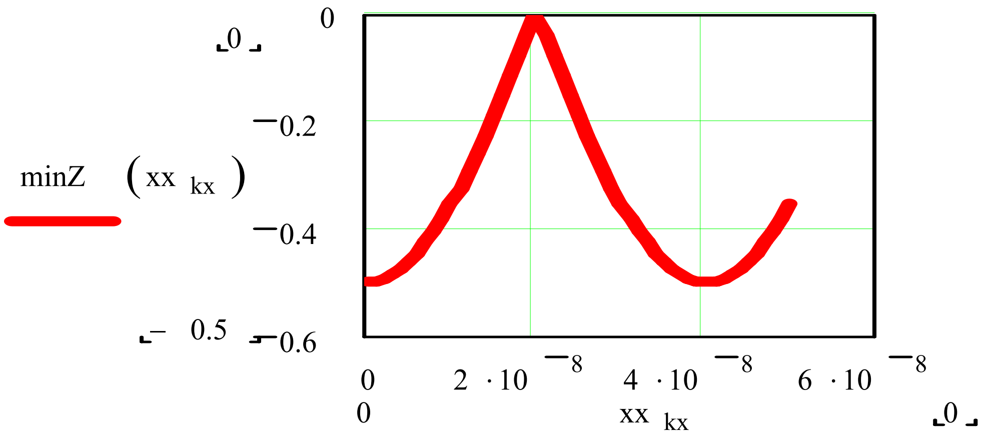
$$\rho_{ShC} = \begin{pmatrix} 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 \end{pmatrix}$$

$$\rho_{ShC\phi} := O(\phi, nC\phi)^{-1} \cdot \rho_{ShC} \cdot O(\phi, nC\phi) \quad nC\phi := \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \phi := \frac{\pi}{4}$$

$$\rho 0M = \begin{pmatrix} 0.25 & 0.25i & 0.25i & 0.25 \\ -0.25i & 0.25 & 0.25 & -0.25i \\ -0.25i & 0.25 & 0.25 & -0.25i \\ 0.25 & 0.25i & 0.25i & 0.25 \end{pmatrix}$$

$$Z(t) := \text{eigenvals}\left[\sigma M\left[\rho M\left[t \cdot \left[\left[\frac{A}{2} \cdot \left(J + \frac{1}{2}\right)\right] \cdot \sec\right], x, \rho 0M\right]\right]\right]$$

$$\text{minZ}(t) := \text{Re}(\min(Z(t)))$$



$$\rho 0M := Mux\left(\frac{\delta sM}{2\cdot s + 1} + -1\cdot s_y, \frac{\delta jM}{2\cdot j + 1} + -1\cdot Jy\right) \qquad 2\cdot tr(JyM\cdot\rho 0M) = -1 \qquad 2\cdot tr(SyM\cdot\rho 0M) = -1$$

$$\rho sjSh := \rho M\left[2\cdot 10^{-8}\cdot\left[\left[\frac{A}{2}\cdot\left(J + \frac{1}{2}\right)\right]\right]\cdot sec\right], x, \rho 0M\right]$$

$$\rho sjSh = \begin{pmatrix} 0.25 & 0.012+0.25i & 0.016+0.249i & 0.248-0.028i \\ 0.012-0.25i & 0.25 & 0.25-3.927i\times 10^{-3} & -0.016-0.249i \\ 0.016-0.249i & 0.25+3.927i\times 10^{-3} & 0.25 & -0.012-0.25i \\ 0.248+0.028i & -0.016+0.249i & -0.012+0.25i & 0.25 \end{pmatrix} \qquad \rho\ ShC\phi = \begin{pmatrix} 0.25 & 0.25i & 0.25i & 0.25 \\ -0.25i & 0.25 & 0.25 & -0.25i \\ -0.25i & 0.25 & 0.25 & -0.25i \\ 0.25 & 0.25i & 0.25i & 0.25 \end{pmatrix}$$

$$Z(t) := eigenvals\left[\sigma M\left[\rho M\left[t\cdot\left[\left[\frac{A}{2}\cdot\left(J + \frac{1}{2}\right)\right]\right]\cdot sec\right], x, \rho 0M\right]\right]\right]$$

$$mZ(t) := Re(min(Z(t)))$$