Nucleon form factors from lattice QCD

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Outline

QCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \sum_q \overline{\psi}_q \left(\imath \not{D} - m_q\right) \psi_q,$$

Main properties

- Asymptotic freedom : the coupling vanish at large momentum
- Confinement : quarks bound into hadrons
- Spontaneous chiral symmetry breaking

Parameters : N_f quark masses

$\rightsquigarrow$ two different regimes refered as "perturbative" and "non perturbative"

Non perturbative approaches

Effective field theory

- Systematic expansion that allow analytical computation.
- Number of coupling constant grows with the accuracy required

Lattice QCD

- Allow ab initio numerical calculation
- Systematic errors can be controled BUT numerically expensive

$\rightsquigarrow$ strong interplay between the two approaches.

Path integral formulation of QCD

- Correlation functions in euclidean space given by

$$\langle \mathcal{O}[\mathcal{A}, \psi, \overline{\psi}] \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\overline{\psi} e^{-S[\mathcal{A},\psi,\overline{\psi}]} \mathcal{O}[\mathcal{A}, \psi, \overline{\psi}]$$

- Contains information needed to compute observables
- Aim : numerical estimation of the functional integral

Asymptotic behaviour of correlators

Strategy

Extract observable from asymptotic behaviour of suitable combination of correlation function.

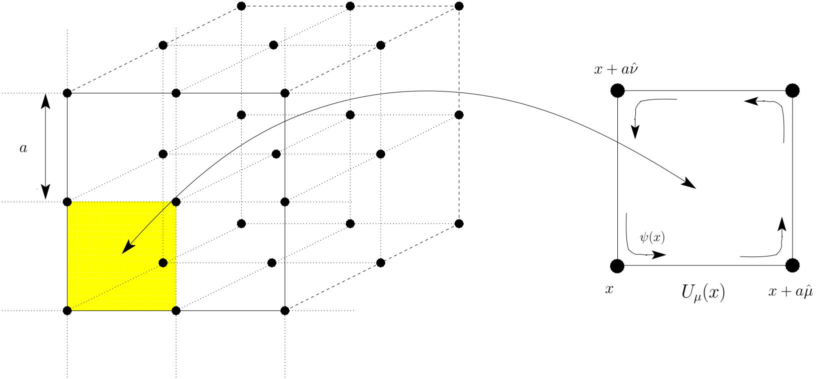
J : interpolating field of a hadron X with definite quantum numbers
Masses :

$$C^X_{\rm 2pts}(t) = \sum_{\vec{x}} \langle J(x) J^\dagger(0) \rangle \propto e^{-M_X t} + \mathcal{O}(e^{-\Delta M t})$$

$\rightsquigarrow m_{\rm eff}(t) = \log \frac{C^X(t)}{C^X(t+1)} = M_X + \ldots$ with $\Delta M = M_{X^*} - M_X$
Matrix elements

$$R(t, t_s) = \frac{\sum_{\vec{x},\vec{y}} \langle J(x) O(y) J^\dagger(0) \rangle}{C^X_{\rm 2pts}(t_s)} = \langle X | O(0) | X \rangle + \mathcal{O}(e^{-\Delta M_X (t - t_s)}) + \mathcal{O}(e^{-\Delta M_X t_s})$$

QCD discretization



- Discretize the QCD action on hypercubic lattice of lattice spacing a, and Volume $V = L^3 \times T$
- The fermionic part can be written : $S_{\text{fermion}} = \sum_x \overline{\psi}(x) D \psi(x)$
- Many choice possible for the Dirac operator D

Fermionic integration

- QCD in Euclidean space :

$$\langle O[\overline{\psi}, \psi, U]\rangle = \int \mathcal{D}\psi\mathcal{D}\overline{\psi}\mathcal{D}U\, e^{-S[\overline{\psi},\psi,U]} O[\overline{\psi}, \psi, U]$$

exact integration of the fermionic fields

$$\rightsquigarrow \langle O[\overline{\psi}, \psi, U]\rangle = \int \mathcal{D}U\, P[U] O[D^{-1}, U]$$

where $P[U] = e^{-S_{\text{gluon}}[U]} \det D[U]$

- Use a supercomputer to generate $\{U_1, \ldots, U_N\}$
- Estimator

$$\langle O[\overline{\psi}, \psi, U]\rangle = \frac{1}{N}\sum_i O[D^{-1}[U_i], U_i] + \mathcal{O}(1/\sqrt{N})$$

Generating configurations

Hybrid Monte Carlo algorithm

- idea : Starting from a configuration $U^{(n)}$
- Molecular dynamics $\rightarrow U^{(n+1)}$
- Metropolis acceptance test to guarantee the correct probability distribution

Expensive part : need to solve equation of motion for $O(V)$ degrees of freedom

Our laboratory

Cost

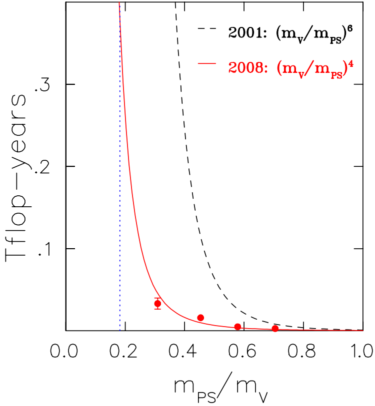


Figure: Simulation cost of TMF using $L_S = 2.1$ fm, $a = 0.089$ fm as a function of the pion mass to the ρ-meson mass [Jansen,2009]. The physical point is showed by the dotted horizontal line.

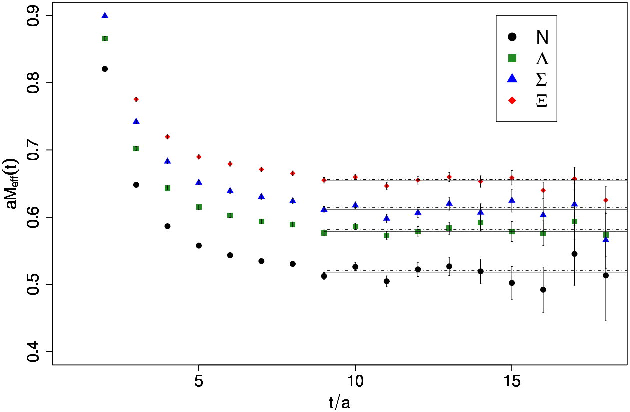
$$C_{\mathrm{sim}} \propto \left(\frac{300\ \mathrm{MeV}}{m_\pi}\right)^{c_m} \left(\frac{L}{2\ \mathrm{fm}}\right)^{c_L} \left(\frac{0.1\ \mathrm{fm}}{a}\right)^{c_a},$$

Systematic effects

- Lattice QCD allows to compute *ab initio* :
  - ⋆ Moments of parton distribution function
  - ⋆ Form Factors (Electroweak, Generalized)
- Needs control over statistical errors :
  - ⋆ Finite statistics : $\sigma \sim 1/\sqrt{N}$
- Needs control over systematic errors :
  - ⋆ Finite Size effects : $V \longrightarrow \infty$
  - ⋆ Finite lattice spacing effects : $a \longrightarrow 0$
  - ⋆ "Chiral" limit : $m_q \longrightarrow m_q^{\mathrm{phys}} \sim 0$
- Dynamical simulation with strange quark are now common ($N_f = 2 + 1$)
- First simulation with a doublet of non strange and charm quarks ($N_f = 2 + 1 + 1$)

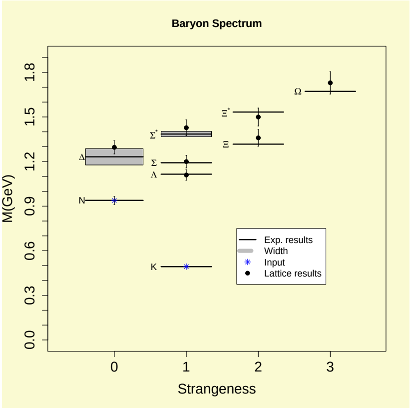
[ETM setup]

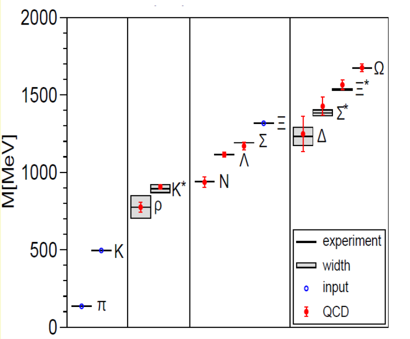
Exemple : mass of the baryon octet



$N_f = 2$, $m_{PS} \approx 300$ MeV, $L = 24a$ [ETM, 2009]

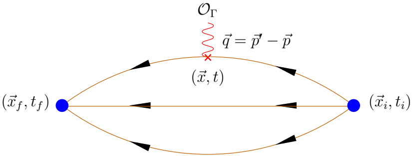
Low lying spectrum





ETM, spectrum $N_f = 2$ [ETM,2009] BMW, spectrum $N_f = 2 + 1$ [BMW, Science (2008)]

Correlators



Nucleon matrix elements are extracted from a suitable ratio of correlation function which involve 2 time scales t and t_s :

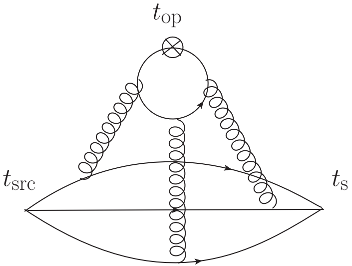
$$\begin{aligned} \langle N(t_s)\mathcal{O}(t)\bar{N}(0)\rangle\rangle &\propto \langle N|\mathcal{O}|N\rangle e^{-m_N t_s} + \ldots \\ \langle N(t_s)\bar{N}(0)\rangle &\propto e^{-m_N t_s} + \ldots \end{aligned}$$

$$R(t, t_s) = \frac{\langle N(t_s)\mathcal{O}(t)\bar{N}(0)\rangle}{\langle N(t_s)\bar{N}(0)\rangle} = \langle N|\mathcal{O}|N\rangle + \text{terms that vanish in the limit } t, t_s \longrightarrow \infty$$

Disconnected diagrams

- (quark)-disconnected diagrams contribute to singlet quantities : for instance :$O = \bar{u}\gamma^{\mu} u$
- Class of diagrams that is extremely noisy

$$R_{\rm full}(t_{\rm op}, t_s) = R_{\rm connected}(t_{\rm op}, t_s) + R_{\rm disconnected}(t_{\rm op}, t_s)$$



Form factors and related observables

Goal : describe the vertex $\gamma N \longrightarrow$ the relevant matrix element is :

$$\langle N(p',s')\, |j^\mu|\, N(p,s)\rangle = \left(\frac{m_N^2}{E_N(\mathbf{p}')\, E_N(\mathbf{p})}\right)^{1/2} \bar{u}(p',s')\mathcal{O}^\mu u(p,s) \qquad (1)$$

$j^\mu = \overline{\psi}\gamma^\mu\tau^3\psi = \bar{u}\gamma^\mu u - \bar{d}\gamma^m ud$: vector current (conserved)
$\rightsquigarrow$ NO disconnected contributions if $m_u = m_d$!
(Continuum) form factor decomposition :

$$\mathcal{O}^\mu = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_N}F_2(q^2), \quad q^2 = (p'-p)^2 \qquad (2)$$

$F_1(0)$: nucleon charge
$F_2(0)$: anomalous magnetic moment

Sachs form factors :

$$\begin{aligned} G_E(q^2) &= F_1(q^2) + \frac{q^2}{(2m_N)^2}F_2(q^2) \\ G_M(q^2) &= F_1(q^2) + F_2(q^2) \quad . \end{aligned} \qquad (3)$$

Isovector operator : $G_{E,M} = G^{p-n}_{E,M}$ (using isospin symmetry)

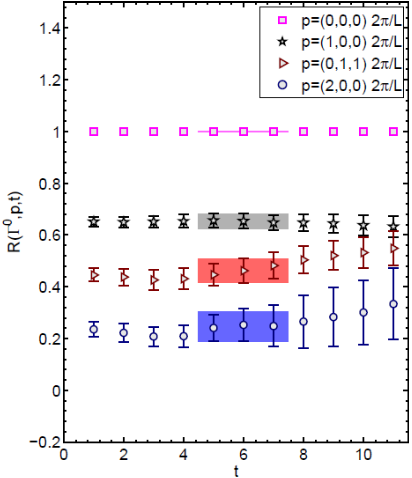
$$\langle p|(\frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d)|p\rangle - \langle n|(\frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d)|n\rangle = \langle p|(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d)|p\rangle. \qquad (4)$$

radii :

$$< r_i^2 >= -\frac{6}{F_i(Q^2)}\frac{dF_i(Q^2)}{dQ^2}|_{Q^2=0} \qquad i = 1,2 \qquad (5)$$

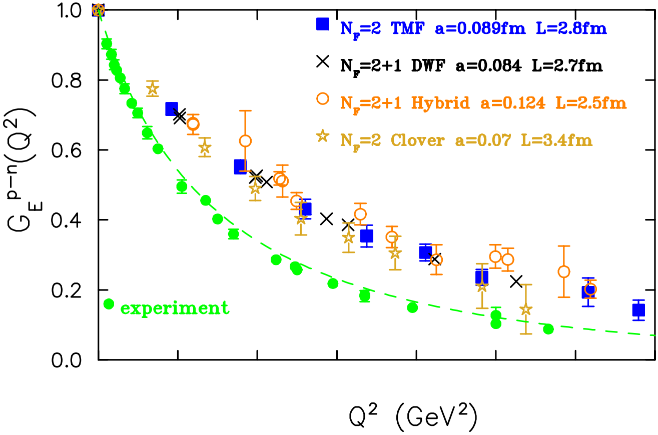
Exemple of plateau

Results extracted from a suitable ratio of correlators that cancel the leading time dependence and the overlap factors.



⇝ Noise increase with momentum transfer [T. Korzec]

Electric Sachs form factor

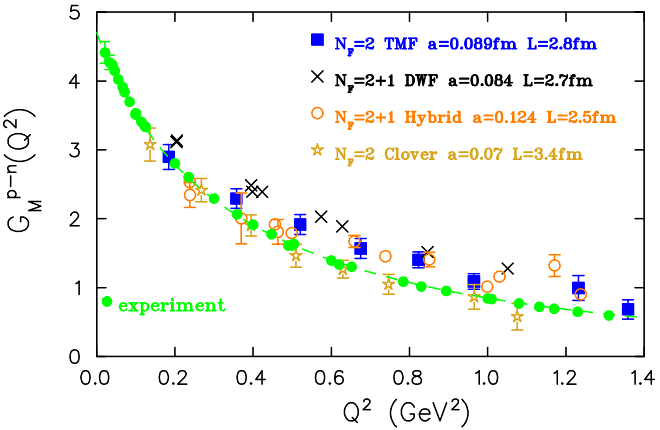


Pion mass : $\sim$ 300 MeV

Mild dependence as a function of m_π [ETM, 1102.2208]

Dashed line : Parametrization of the experimental data [J.J. Kelly, Phys. Rev. C70 068202 (2004)]

Magnetic Sachs form factor



Pion mass : $\sim$ 300 MeV

Mild dependence as a function of m_π [ETM, 1102.2208]

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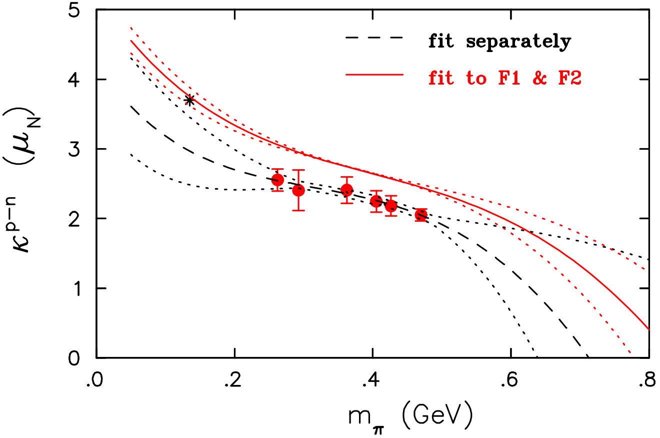
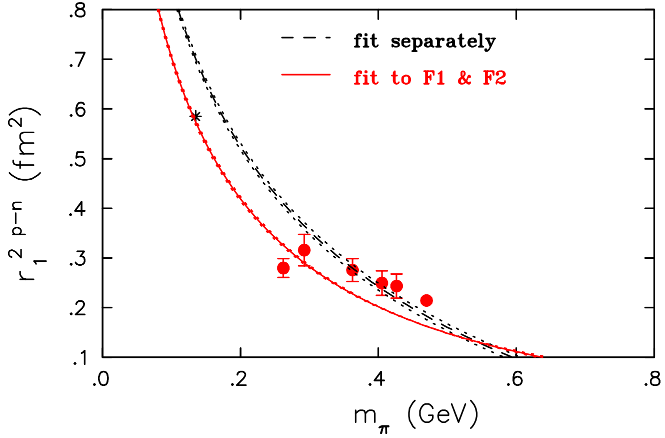
Magnetic moment



Systematic effects under control for $m_\pi > 270$ MeV

Chiral extrapolation rely heavily on Heavy baryon chiral perturbation theory (HBχPT)

Charge radius



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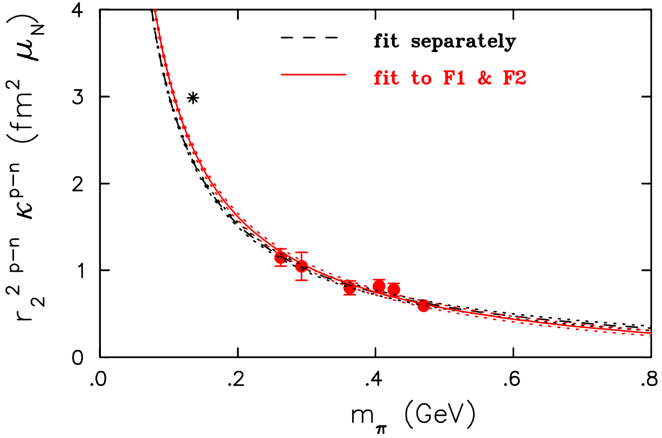
Chiral extrapolation rely heavily on Heavy baryon chiral perturbation theory

(HBχPT) and singular in the chiral limit.
$\rightsquigarrow$ extrapolation need to be checked with simulation at lower pion mass

Charge radius



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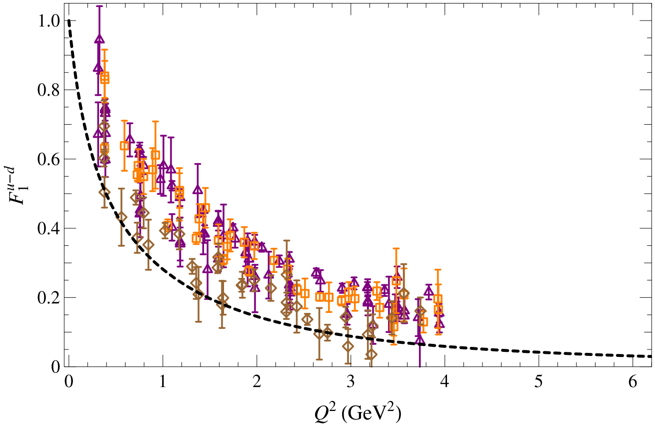
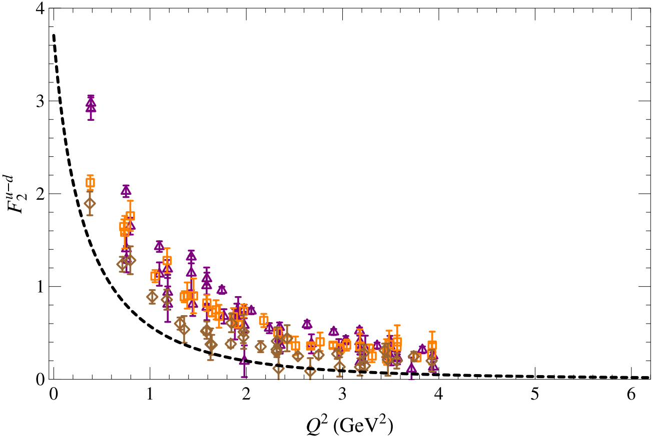
$\rightsquigarrow$ extrapolation need to be checked with simulation at lower pion mass

F_1^{u-d} : High Q^2



Novel techniques : investigation large Q^2 (up to 6 GeV^2) [H.-W. Lin *et al.*, 1005.0899]
⇝ Large discretization effects ? ($aq >> 1$)

F_2^{u-d} :High Q^2



Summary

Results

- Qualitative agreement
- systematic effects (V, a, excited states) : good control for $m_\pi > 250$ MeV
- Comparison with experiment rely on delicate extrapolation

Perspectives

- Large q^2
- Twisted boundary condition to improve resolution at low q^2 (magnetic moment determination)
- Lower pion masses
- singlet form factor : proton and neutron electromagnetic FFs (including disconnected contribution)

Definitions

Matrix element that describes β-decay :

$$\langle N(p', s')|A^3_\mu|N(p,s)\rangle = i\left(\frac{m_N^2}{E_N(\mathbf{p}')E_N(\mathbf{p})}\right)^{1/2} \bar{u}_N(p',s')\left[G_A(q^2)\gamma_\mu\gamma_5 + \frac{q_\mu\gamma_5}{2m_N}G_p(q^2)\right]\frac{1}{2}u_N(p,s) \tag{6}$$

where:

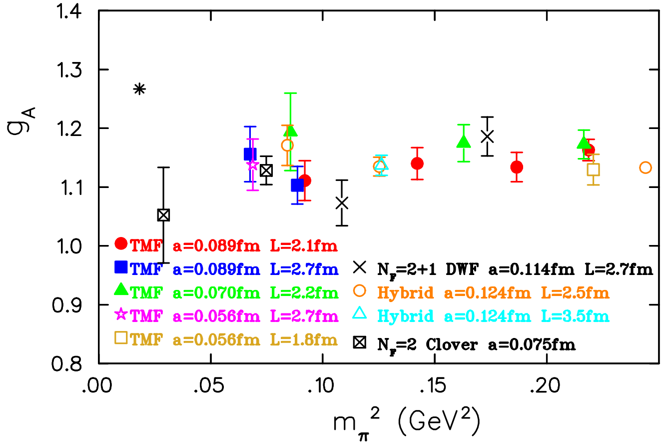
$$A^a_\mu(x)=\bar{\psi}(x)\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x) \tag{7}$$

$\tau^3 \rightsquigarrow$ NO disconnected contributions

$G_A(q^2)$: axial form factor ($G_A(0) = g_A$)
$G_P(q^2)$: induced pseudoscalar form factor

Similarly to the EM case, one can define axial radii.

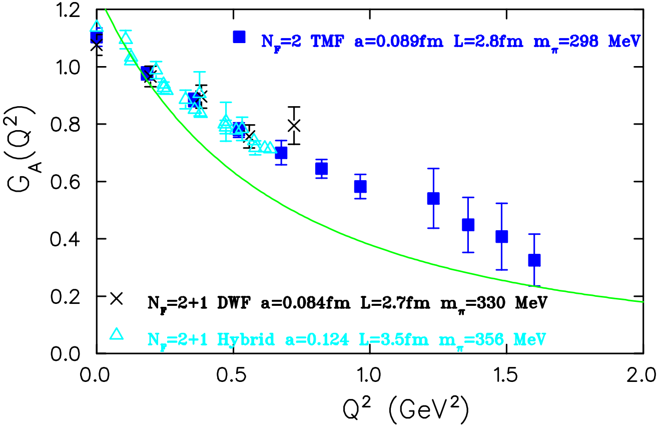
Axial charge puzzle



[1111.5960]

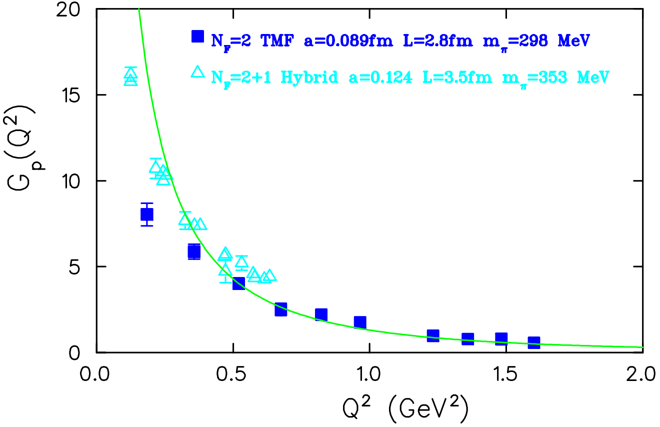
⟶ chiral extrapolation is NOT the problem ?

G_A



Mild dependence as a function of m_π [ETM,1012.0857]

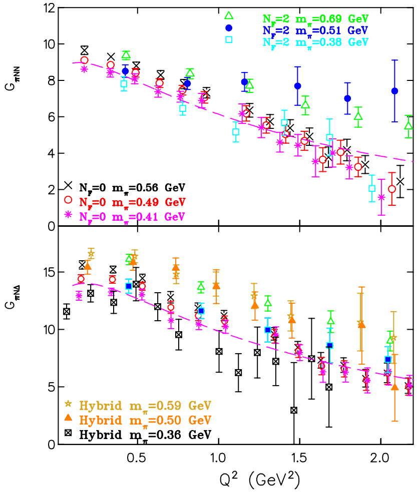
G_P



Mild dependence as a function of m_π [ETM,1012.0857]

Pseudoscalar form factor

$$\langle N(0)|\bar{u}\gamma_5 u - \bar{d}\gamma_5 d|N(0)\rangle \quad (8)$$

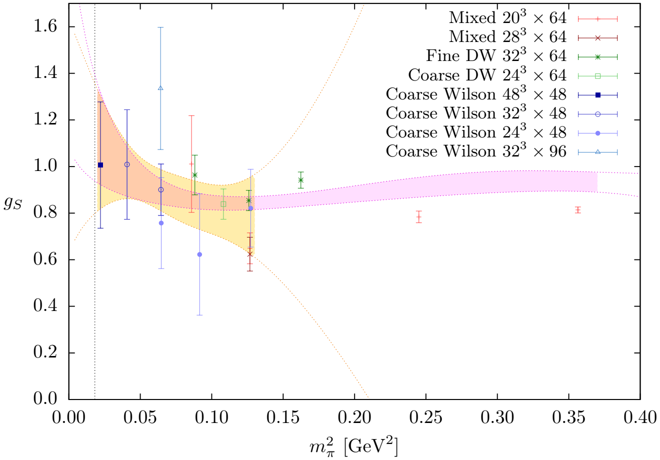


Scalar charge

Useful to constraint interaction beyond the standard model or for effective theory describing interaction of the Nucleon :

$$\langle N(0)|\bar{u}u + \bar{d}d|N(0)\rangle \sim g_S \quad (9)$$

comment disconnected ?



$g_S = 1.08 \pm 0.28$

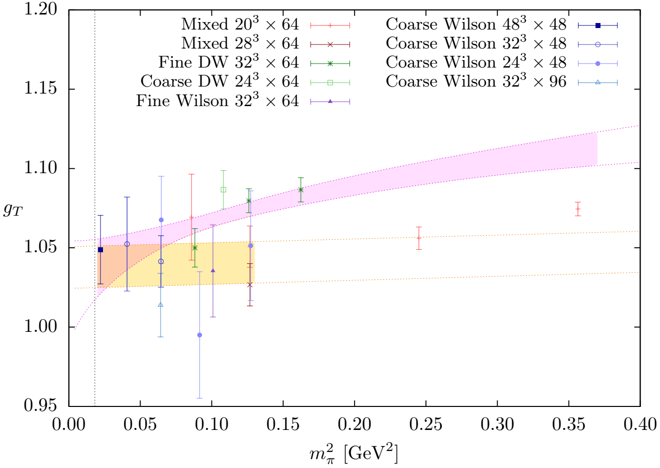
Negele et al. [arxiv:1206.4527]

Tensor charge

Useful to constraint interaction beyond the standard model or for effective theory describing interaction of the Nucleon :

$$\langle N(0)|\bar{u}\sigma^{\mu\nu}u + \bar{d}\sigma^{\mu\nu}d|N(0)\rangle \sim g_T\sigma^{\mu\nu} \quad (10)$$

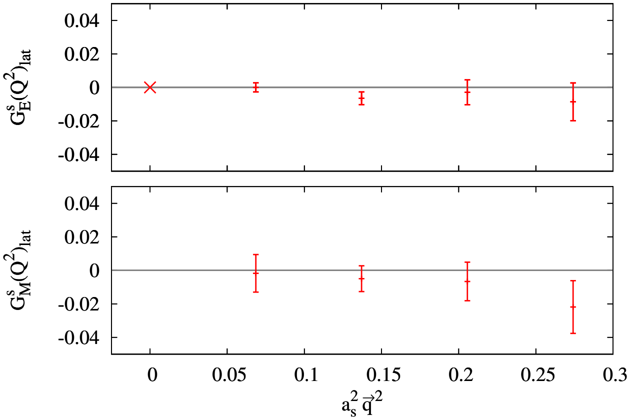
comment disconnected ?



$g_T = 1.028 \pm 0.011$

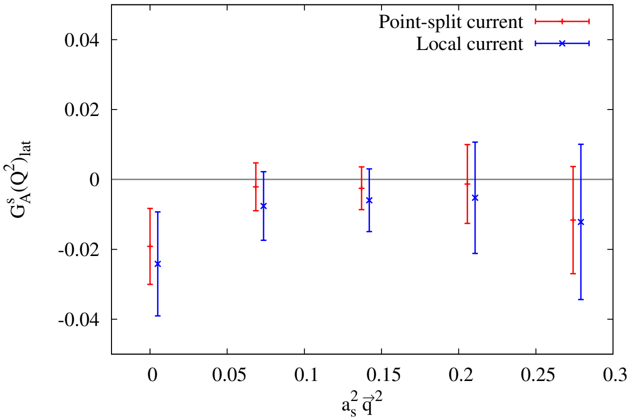
Negele et al. [arxiv:1206.4527]

Vector and axial form factor



[1012.0562]

Vector and axial form factor



[1012.0562]

Summary

Results

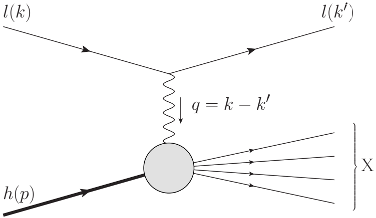
- Pseudoscalar form factor can also be investigated
- Scalar and tensor charge to constraint BSM physic
- Investigation of the "singlet" strange form factor : very beginning

Other observables

- σ-terms , strangeness of the nucleon y_N (scalar matrix element)
- $g_A^{(8)}$ and $\langle x \rangle^{(8)}_{q,\mu^2}$: easier because disconnected contributions vanish in the $SU(3)$ limit : expected to be a small correction
- ...

Parton Distribution Functions (PDF)

- Deep inelastic scattering (DIS) : $l(k)N(p) \longrightarrow l(k')X$



$$Q^2 = -q^2$$
$$x = Q^2/2p\cdot q$$

- factorization :
  - ⋆ $\frac{d\sigma^{(lN)}}{d^3k'}(p,q) \sim \int_0^1 d\xi \sum_q \frac{d\sigma^{(lq)}}{d^3k'}(\xi p, q)\, q(\xi)$
  - ⋆ lepton-parton cross section is perturbative for large Q^2
  - ⋆ $q(\xi)$ encodes non perturbative dynamics

Parton Distribution Functions (PDF)

- Definition (unpolarized PDFs)

$$q(x,\mu) = \int \frac{d\lambda}{2\pi} e^{ixp\cdot\lambda n} \langle p,s|\bar{q}\left(-\frac{\lambda}{2}n\right) \not{n} W_n(-\frac{\lambda}{2}n, \frac{\lambda}{2}n) q\left(\frac{\lambda}{2}n\right)|p,s\rangle\Big|_{\mu^2}$$

with

$$W_n(-\frac{\lambda}{2}n, \frac{\lambda}{2}n) = \mathcal{P}\exp\left(\mathrm{ig}\int_{-\lambda/2}^{\lambda/2} \mathrm{d}\alpha\, \mathrm{A}(\alpha \mathrm{n}) \cdot \mathrm{n}\right).$$

- PDFs involve quark and gluon fields seperated along the light-cone

  $\rightsquigarrow$ difficult to construct explicitly in Euclidean space

Moments of PDF

- Definition :

$$\langle x^n \rangle_{q,\mu^2} = \int_{-1}^{1} dx\, x^n q(x,\mu^2) = \int_{0}^{1} dx\, x^n \left\{ q(x,\mu^2) - (-1)^n \bar{q}(x,\mu^2) \right\}$$

- Forward matrix elements of twist-two operators :

$$\langle p,s| \bar{q}(0) \gamma^{\{\mu_1} iD^{\mu_2} \cdots iD^{\mu_n\}} q(0) |p,s\rangle \Big|_{\mu^2} = 2 \langle x^n \rangle_{q,\mu^2} p^{\{\mu_1} \cdots p^{\mu_n\}}$$

$T^{\{\mu_1 \cdots \mu_n\}}$: symmetrization and substraction of the traces

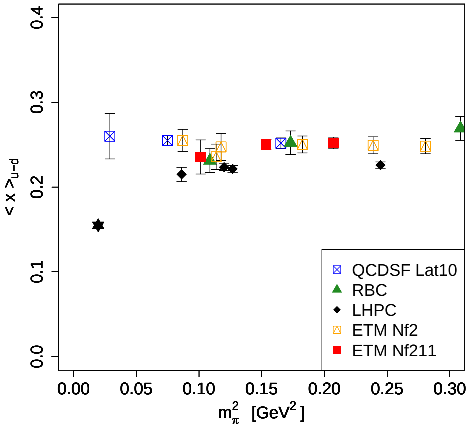
D^{μ} : covariant derivative

- Moments are related to local operators that can be calculated in Eucliean space.
- Benchmark quantity :

$$\langle p,s| \bar{\psi} \gamma^{\{\mu} iD^{\nu\}} \tau^3 \psi |p,s\rangle \Big|_{\mu^2} = 2 \langle x \rangle_{u-d,\mu^2} p^{\{\mu} p^{\nu\}}, \quad \text{with} \quad \psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

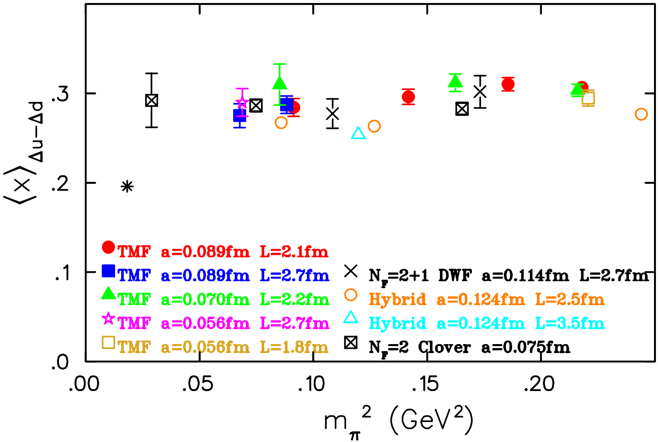
A long-standing puzzle

Up-to-date results for $\langle x \rangle_{u-d}$ ($\overline{\rm MS}$-scheme $\mu = 2$ GeV)



- Discrepancy of 40%
- The same discrepancy is obtained for many other nucleon matrix elements (*e.g* : g_A, the axial coupling of the nucleon)

$\langle x \rangle_{\Delta q, \mu^2}$



[ETM, 1104.1600]

GPDs

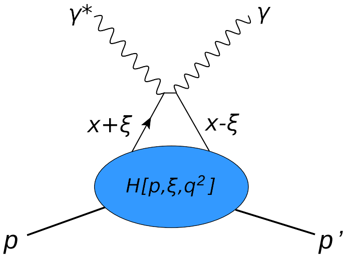


Figure: "Handbag" diagram.

$$F_{\not n}(x,\xi,q^2) = \frac{1}{2}\bar{u}_N(p')\left[\not n H(x,\xi,q^2) + i\frac{n_\mu q_\nu \sigma^{\mu\nu}}{2m_N}E(x,\xi,q^2)\right]u_N(p) \quad (11)$$

$$F_{\not n\gamma_5}(x,\xi,q^2) = \frac{1}{2}\bar{u}_N(p')\left[\not n\gamma_5 \tilde{H}(x,\xi,q^2) + \frac{n\cdot q\gamma_5}{2m_N}\tilde{E}(x,\xi,q^2)\right]u_N(p). \quad (12)$$

where u_N is a nucleon spinor and H, E, $\tilde{H}$, $\tilde{E}$ are the twist-2 chirality even GPDs.

Generalized form factors

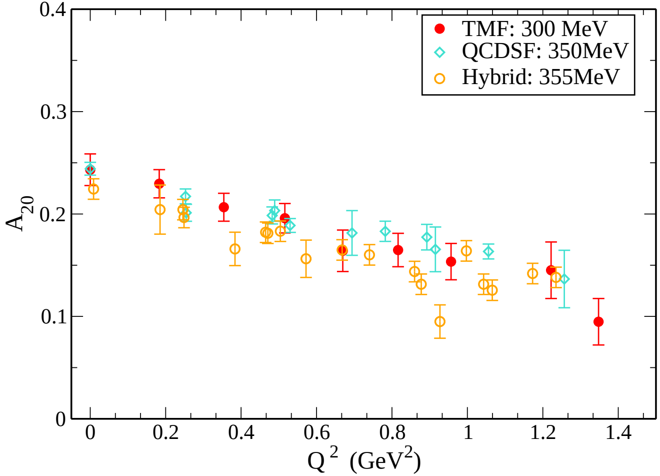
$$\mathcal{O}_V^{\mu_1\cdots\mu_n} = \bar{\psi}\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \ldots i \overleftrightarrow{D}^{\mu_n\}} \psi \tag{13}$$

$$\mathcal{O}_A^{\mu_1\cdots\mu_n} = \bar{\psi}\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \ldots i \overleftrightarrow{D}^{\mu_n\}} \gamma_5 \psi \,. \tag{14}$$

$$\langle N(p',s')|\mathcal{O}^{\mu\nu}_{\not{n}}|N(p,s)\rangle = \bar{u}_N(p',s')\Big[A_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\,\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}(q^2)\,\frac{1}{m}q^{\{\mu}q^{\nu\}}\Big]u_N(p,$$

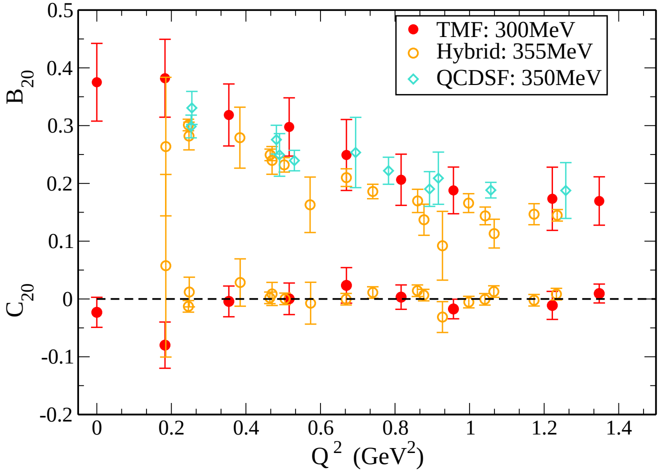
$$\langle N(p',s')|\mathcal{O}^{\mu\nu}_{\not{n}\gamma_5}|N(p,s)\rangle = \bar{u}_N(p',s')\Big[\tilde{A}_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}}\gamma^5 + \tilde{B}_{20}(q^2)\frac{q^{\{\mu}P^{\nu\}}}{2m}\gamma^5\Big]u_N(p,s)$$

A_{20}, B_{20} and C_{20}



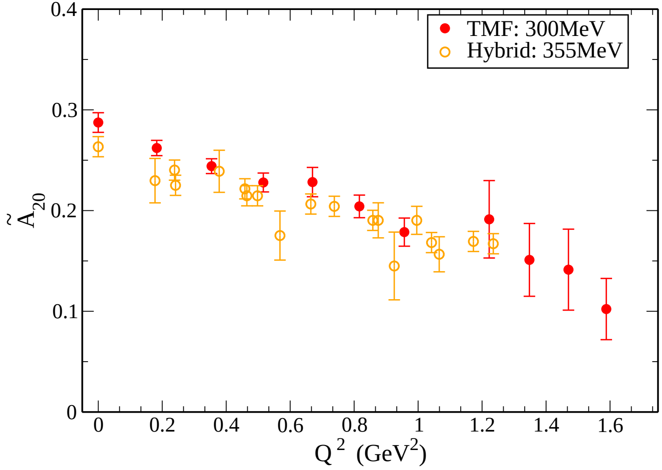
[ETM, 1104.1600]

A_{20}, B_{20} and C_{20}



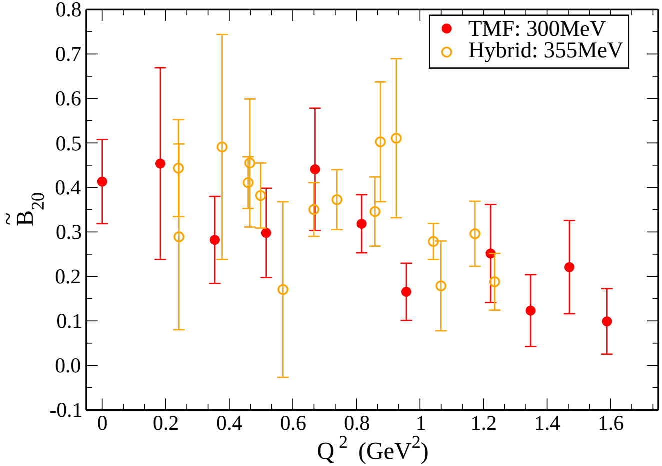
[ETM, 1104.1600]

$\widetilde{A}_{20}$ and $\widetilde{B}_{20}$



[ETM, 1104.1600]

$\widetilde{A}_{20}$ and $\widetilde{B}_{20}$



[ETM, 1104.1600]