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High energy inelastic electron-hadron scattering in peripheral kinematics: Sum rules for hadron form factors

E. A. Kuraev,[1] M. Sečanský,[1,*] and E. Tomasi-Gustafsson[1]

[1]*JINR-BLTP, 141980 Dubna, Moscow region, Russian Federation*

[2]*DAPNIA/SPhN, CEA/Saclay, 91191 Gif-sur-Yvette Cedex, France*

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Relations between differential cross section for inelastic scattering of electrons on hadrons and hadron form factors (sum rules) are derived on the basis of analytical properties of heavy photon forward Compton scattering on hadrons. Sum rules relating the slope of form factors at zero momentum transfer and anomalous magnetic moments of hadrons with some integrals on photoproduction on a hadron are obtained as well. The convergency of these integrals is provided by the difference of individual sum rules for different hadrons. The universal interaction of the Pomeron with nucleons is assumed. We derive the explicit formulas for the processes of electroproduction on proton and light isobar nuclei. Sudakov's parametrization of momenta for peripheral kinematics, relevant here, is used. The light-cone form of differential cross sections is also discussed. The accuracy of sum rules estimated in frames of pointlike hadrons and it is shown to be at the level of precision achievable by experiments. Suggestions and predictions for future experiments are also given.

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I. INTRODUCTION

The idea of construction of sum rules, relating the form factors of electrons with the cross sections of electroproduction processes at e^+e_- high energy collisions, was born in 1974. In a series of papers the cross sections of processes such as

$$e_+e_- \to (2e_-e_+)e_+; (e_-\gamma)e_+; (e_-2\gamma)e_+; (e_-\mu_+\mu_-)e_+$$

were calculated in the so-called peripheral kinematics, when the jets (consisting of particles noted in parentheses) are moving closely to the initial electron direction in the center of mass reference frame. These cross sections do not decrease as a function of the center of mass (CMS) total beam energy $\sqrt{s}$. Moreover, they are enhanced by a logarithmical factor $\ln(s/m^2)$ (where m is the electron mass), which is characteristic for the Weizsacker-Williams approximation. It was obtained for contribution to the process of muon pair production cross section from the so-called bremsstrahlung mechanism (corresponding to the virtual photon conversion into muon–antimuon pairs) [1,2]:

$$\sigma^{e^+e^-\to e^-\mu^+\mu^-e^+} = \frac{\alpha^4}{\pi\mu^2}\left[\left(\ln\frac{s^2}{m^2\mu^2}\right)\left(\frac{77}{18}\xi_2 - \frac{1099}{162}\right) + \text{const.}\right] \quad (1)$$

where $\xi_2 = \frac{\pi^2}{6}$.

In Ref. [3] Barbieri, Mignaco, and Remiddi calculated the slope of the Dirac form factor of the electron for $q^2 \to 0$:

$$F_1'(0) = \frac{\alpha^2}{8\pi^2\mu^2}\left(\frac{77}{18}\xi_2 - \frac{1099}{162}\right). \quad (2)$$

The fact that the same coefficients appear in Eqs. (1) and (2) suggests that a relation exists between the inelastic cross section and elastic form factors. This relation was later on derived in Ref. [2].

Here we suggest an extension of these studies to strong interaction particles, considering QED interactions in the lowest order of perturbation theory. We suggest sum rules which relate the nucleon and light nuclei form factors with the differential cross sections of electron scattering on the corresponding hadrons in peripheral kinematics. This kinematics corresponds to the region of very small values of Bjorken parameter x_B in deep inelastic scattering experiments.

This paper is organized as follows. After describing the relevant processes in peripheral kinematics, in terms of Sudakov variables (Sec. II), we recall the analytical properties of the advanced and the retarded parts of the virtual Compton scattering amplitudes (Sec. II). Then, briefly we discuss how to restore gauge invariance and how to formulate the modified optical theorem.

Following QED analysis in [2], we introduce light-cone projection of the Compton scattering amplitude integrated on a contour in the s_2 plane, where s_2 is the invariant mass squared of the hadronic jet.

Sum rules (Sec. III) arise when the Feynman contour in the s_2 plane is closed to the left singularities of the Compton amplitude and to the right ones on the real axes. Sum rules obtained in such a way contain the left hand cut contribution, which is difficult to be interpreted in terms of cross sections. Moreover, ultraviolet divergencies of contour integral arising from Pomeron Regge pole con-

*On leave of absence from Institute of Physics SAS, Bratislava tribution are present. Therefore, the final sum rules consist of differences, constructed in such a way to compensate the Pomeron contributions and the left hand cuts, as well.

The applications to different kinds of targets, as proton and neutron, deuteron and light nuclei, are explicitly given. Appendix A is devoted to an estimation of the left cut contribution which is proportional to the cross section of proton photoproduction of a $p\bar{p}$ pair taking into account the effect of identity of protons in final state in the framework of a simple model. In Appendix B the details of kinematics of recoil target particle momentum is investigated in terms of Sudakov's approach.

A. Sudakov parametrization

Let us consider the process presented in Fig. 1, where the inelastic electron—hadron interaction occurs through a virtual photon of momentum q. The particle momenta are indicated in the equation: $p^2 = M^2$, $p_1^2 = p_1'^2 = m^2$. The total energy is $s = (p + p_1)^2$ and the momentum transfer from the initial to the final electron is $t = (p_1 - p_1')^2$.

Let us introduce some useful notations, in order to calculate the differential cross section for the process of Fig. 1, where the hadron is a proton in peripheral kinematics, i.e., $s \gg -t \simeq M^2$. Therefore $s = (p_1 + p)^2 = M^2 + m^2 + 2pp_1 \simeq 2pp_1 = 2ME \gg M^2 \gg m^2$.

The differential cross section can be written as

$$d\sigma = \frac{1}{2 \cdot 2 \cdot 2s} \sum |\mathcal{M}|^2 d\Gamma. \qquad (3)$$

Let us define two lightlike vectors:

$$\tilde{p} = p - p_1 \frac{M^2}{s}, \qquad \tilde{p}_1 = p_1 - p\frac{m^2}{s}$$

and a transversal vector, $q_\perp$, such that $\tilde{p}q_\perp = p_1 q_\perp = 0$.

Therefore, $\tilde{p}^2 = \mathcal{O}(\frac{m^2M^4}{s^2})$ and similarly $\tilde{p}^2 = \mathcal{O}(\frac{m^4M^2}{s^2})$. Terms of order $\mathcal{O}(\frac{M^2}{s}, \frac{m^2}{M^2})$ compared to ones of order 1 we will neglect systematically. In the laboratory system, with an appropriate choice of the axis, the four vectors are written, in explicit form, as $\tilde{p}_1 \approx p_1 = E(1, 1, 0, 0)$, $\tilde{p} = \frac{M}{2}(1, -1, 0, 0)$, and $q_\perp = (0, 0, q_x, q_y)$, $q_\perp^2 = -\vec{q}^2 < 0$, which is essentially a two-dimensional vector. Let us express the four momentum of the exchanged photon in the Sudakov parametrization, in an infinite momentum frame, as a function of two (small) parameters α and β (Sudakov parameters):

$$q = \alpha\tilde{p} + \beta\tilde{p}_1 + q_\perp. \qquad (4)$$

The on-mass shell condition for the scattered electron can be written as

$$p_1'^2 - m^2 = (p_1 - q)^2 - m^2 = -\vec{q}^2 + s\alpha\beta - \alpha s - \beta m^2 = 0 \qquad (5)$$

where we use the relation

$$2p_1\tilde{p}_1 = 2p_1\left(p_1 - p\frac{m^2}{s}\right) = m^2. \qquad (6)$$

Similarly, one can find $2p\tilde{p} = M^2$.

From the definition, Eq. (4), and using Eq. (5), the momentum squared of the virtual photon is

$$q^2 = s\alpha\beta - \vec{q}^2 = -\frac{\vec{q}^2 + m^2\beta^2}{1 - \beta} < 0.$$

The variable β is related to the invariant mass of the proton jet, the set of particles moving close to the direction of the initial proton:

$$s_2 = (q + p)^2 - M^2 + \vec{q}^2 \simeq s\beta$$

neglecting small terms as $s\alpha\beta$ and αM^2 (Weizsacker-Williams approximation [4]). We discuss later the consequences of such approximation on the s-dependence of the cross section. In these notations the phase space of the final particle

$$d\Gamma = (2\pi)^4 \frac{d^3p_1'}{2\epsilon_1'(2\pi)^3} \Pi_1^n \frac{d^3q_i}{2\epsilon_i(2\pi)^3} \times \delta^4\left(p_1 + p - p_1' - \sum_i^n q_i\right), \qquad (7)$$

introducing an auxiliary integration on the photon transferred momentum $\int d^4q\delta^4(p_1 - q - p_1') = 1$, can be written as

$$d\Gamma = (2\pi)^{-3}\delta^4((p_1 - q)^2 - m^2)d^4q d\Gamma_H, \qquad (8)$$

where $d\Gamma_H$ is the hadron phase space:

$$d\Gamma_H = (2\pi)^4\delta^4\left(p + q - \sum_i^n q_i\right)\Pi_1^n \frac{d^3q_i}{2\epsilon_i(2\pi)^3}. \qquad (9)$$

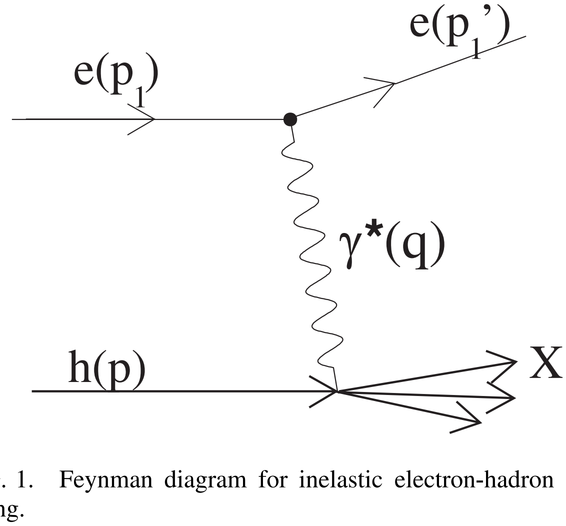

FIG. 1. Feynman diagram for inelastic electron-hadron scattering.

In the Sudakov parametrization:

$$d^4q = \frac{s}{2} d\alpha d\beta d^2q_\perp \simeq \frac{ds_2}{2s} d(s\alpha) d^2\vec{q}, \tag{10}$$

we obtain

$$d\Gamma = \frac{ds_2}{2s} d^2\vec{q}(2\pi)^{-3} d\Gamma_H. \tag{11}$$

Let us use the matrix elements to be expressed by the Sudakov parameters. Then it can be rewritten into the form

$$\mathcal{M} = \frac{4\pi\alpha}{q^2} \bar{u}(p_1')\gamma^\mu u(p_1)\mathcal{J}_H^\nu g_{\mu\nu}. \tag{12}$$

It is convenient to use here the Gribov representation of the numerator of the (exact) Green function in the Feynman gauge for the exchanged photon:

$$g_{\mu\nu} = (g_\perp)_{\mu\nu} + \frac{2}{s}(\tilde{p}_\mu \tilde{p}_{1\nu} + \tilde{p}_\nu \tilde{p}_{1\mu}). \tag{13}$$

All three terms in the right-hand side of the previous equation give contributions to the matrix element proportional to

$$1 : \frac{s}{M^2} : \frac{M}{s}.$$

So, the main contribution (with power accuracy) is

$$\mathcal{M} = \frac{4\pi\alpha}{q^2} \frac{2}{s} \bar{u}(p_1')\hat{p}u(p_1)\mathcal{J}_H^\nu p_{1\nu} = \frac{8\pi\alpha s}{q^2} N \frac{\mathcal{J}_H^\nu p_{1\nu}}{s}, \tag{14}$$

with $N = \frac{1}{s}\bar{u}(p_1')\hat{p}u(p_1)$. One can see explicitly the proportionality of the matrix element of peripheral processes to s, in the high energy limit, $s \gg -t$. It follows from the relation $\sum_{pol}|N|^2 = \frac{1}{s^2}\,\mathrm{Tr}\hat{p}_1'\hat{p}\hat{p}_1\hat{p} = 2$ and from the fact, that the quantity $\frac{1}{s}\mathcal{J}_H^\nu p_{1\nu}$ is finite in this limit. Such term can be further transformed using the conservation of the hadron current $\mathcal{J}_H^\nu q_\nu \simeq (\beta p_1 + q_\perp)_\nu \mathcal{J}_H^\nu = 0$. The latter leads to

$$\frac{1}{s}\mathcal{J}_H^\nu p_{1\nu} = \frac{1}{s\beta}\vec{q}\cdot\vec{\mathcal{J}}_H = \frac{|\vec{q}|}{s_2}(\vec{e}\cdot\vec{\mathcal{J}}_H), \tag{15}$$

where $\vec{e} = \vec{q}/|\vec{q}|$ is the polarization vector of the virtual photon. As a result, one finds:

$$\sum|\mathcal{M}|^2 = \frac{(8\pi\alpha s)^2|\vec{q}|^2}{[(\vec{q})^2 + m^2(\frac{s_2}{s})^2]^2} \frac{2}{s_2^2}(\vec{\mathcal{J}}_H\cdot\vec{e})^2. \tag{16}$$

With the help of Eqs. (7), (14), and (16) one finds

$$d\sigma^{(e+p\to e+\mathrm{jet})} = \frac{\alpha^2 d^2q\vec{q}^2 ds_2}{\pi[(\vec{q})^2 + m^2(\frac{s_2}{s})^2]^2 s_2^2}(\vec{e}\cdot\vec{\mathcal{J}}_H)^2 d\Gamma_H. \tag{17}$$

Let us note that the differential cross section at $\vec{q}^2 \neq 0$ does not depend on the CMS energy $\sqrt{s}$. In the logarithmic (Weizsacker-Williams) approximation, the integral over the transverse momentum $\vec{q}$, at small $\vec{q}^2$, gives rise to large logarithm:

$$\sigma_{\mathrm{tot}}^\ell = \frac{\alpha}{\pi}\ln\left(\frac{s^2Q^2}{M^4m^2}\right)\int_{s_{\mathrm{th}}}^\infty \frac{ds_2}{s_2}\sigma_{\mathrm{tot}}^{(\gamma+p\to X)}(s_2), \tag{18}$$

where $s_{\mathrm{th}} = (M + m_\pi)^2 - M^2$, Q^2 is the characteristic momentum transfer squared, $Q^2 \simeq M^2$, and we introduced the total cross section for real polarized photons interacting with protons:

$$\sigma_{\mathrm{tot}}^{(\gamma^* p\to X)}(s_2, q^2 = 0) = \frac{\alpha\pi}{s_2}\int(\vec{\mathcal{J}}_H\cdot\vec{e})^2 d\Gamma_H. \tag{19}$$

The differential cross section (18) is closely related (due to the optical theorem) with the s-channel discontinuity of the forward amplitude for electron-proton scattering with the same intermediate state: a single electron and a jet, moving in opposite directions [see Figs. 2(a) and 2(b)] where, by Cutkovsky rule, the denominators of the "cutted" lines in the Feynman graph of Fig. 2(b) must be replaced by:

$$\frac{1}{q^2 - M^2 + i0} \to -2\pi i\delta(q^2 - M^2). \tag{20}$$

For the spin-averaged forward-scattering amplitude we have

$$\Delta_s A(s) = \frac{4s\alpha}{\pi^2}\int\frac{d^2q_\perp\vec{q}^2}{(q^2)^2}\int\frac{ds_2}{s_2}\sigma^{(\gamma^* p\to X)}(s_2, q) \tag{21}$$

with

$$\sigma^{(\gamma^* p\to X)}(s_2, q) = \int\frac{4\pi\alpha}{2\cdot 2\cdot 2s_2}(\vec{\mathcal{J}}_H\cdot\vec{e})^2 d\Gamma_H. \tag{22}$$

From the formulas given above we obtain

$$\frac{d\Delta_s\bar{A}^{eY\to eY}}{d^2q} = 2s\frac{d\sigma^{eY\to eY}}{d^2q}, \tag{23}$$

where $\bar{A}^{eY\to eY}$ is the averaged on spin states forward-scattering amplitude. This relation is the statement of the optical theorem in differential form.

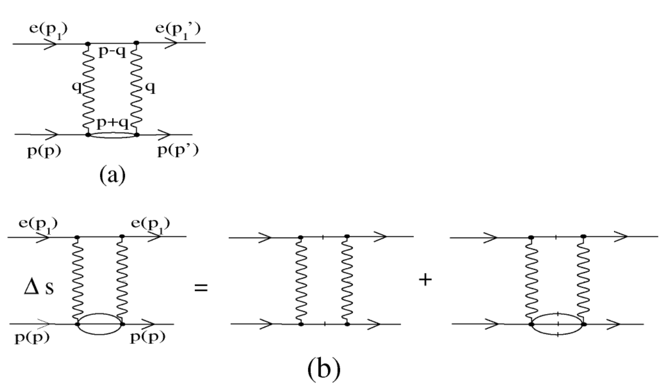

FIG. 2. Feynman diagram for e^+e^- scattering at the order α^3.

Let us now consider the discontinuity of the forward-scattering amplitude with the electron and proton intermediate state, we call it a "pole contribution." For the case of elastic electron-proton scattering we have

$$\frac{d\Delta_s \bar{A}^{ep\to ep}}{d^2q} = \frac{(4\pi\alpha)^2}{(q^2)^2 s(2\pi)^2} Sp, \tag{24}$$

with

$$Sp = Sp(\hat{P} + M)\Gamma(q)(\hat{P}' + M)\Gamma(-q)^*,$$

and

$$\Gamma(q) = F_1\hat{p}_1 - \frac{1}{2M}F_2\hat{q}\hat{p}_1.$$

A simple calculation gives $Sp = 2s^2[(F_1)^2 + \tau(F_2)^2]$, with $\tau = \vec{q}^2/(4M_p^2)$.

For the case of electron-deuteron scattering we use the electromagnetic vertex of deuteron in the form [5]

$$\langle\xi^{\lambda'}(P')|J_\mu^{\mathrm{EM}}(q)|\xi^\lambda(P)\rangle = d_\mu\left[F_1(\xi^{\lambda'*}\xi^\lambda) - \frac{F_3}{2M_d^2}(\xi^{\lambda'*}q)(\xi^\lambda q)\right] + F_2[\xi_\mu^\lambda(q\xi^{\lambda'*}) - \xi_\mu^{\lambda'*}(\xi^\lambda q)], \tag{25}$$

where $P^2 = (P')^2 = M_d^2$ (M_d) is the deuteron mass),$d_\mu = (P' + P)_\mu$, $q_\mu = (P' - P)_\mu$ and $\xi^\lambda(P)$ is the polarization vector of deuteron in chiral state λ. It has the properties:

$$\xi^2 = -1, \qquad (\xi(P)P) = 0;$$
$$\sum_\lambda \xi^\lambda(P)_\mu \xi^{\lambda*}(P)_\nu = g_{\mu\nu} - \frac{P_\mu P_\nu}{M_d^2}. \tag{26}$$

For the spin averaged forward-scattering amplitude one has

$$\frac{d\Delta_s \bar{A}^{ed\to ed}}{d^2q} = \frac{2s(4\pi\alpha)^2}{3(q^2)^2(2\pi)^2}\mathrm{Tr}, \tag{27}$$

with

$$Tr = 2(F_1)^2 + (F_1 + 2\tau_d(1+\tau_d)F_3)^2 + 2\tau_d(F_2)^2,$$
$$\tau_d = \frac{\vec{q}^2}{4M_d^2}. \tag{28}$$

We note that the amplitude corresponding to crossed box-type Feynman diagram has a zero s-channel discontinuity.

II. VIRTUAL COMPTON SCATTERING ON PROTON

Let us examine the different contributions to the total amplitude for virtual photon Compton scattering on a proton (hadron). Keeping in mind the baryon number conservation law, we can separate all possible Feynman diagram into four classes. In one, which will be named as a class of retarded diagram (the corresponding amplitude is denoted as A_1), the initial state photon is first absorbed by a nucleon line and then emitted by the scattered proton. Another class (advanced, A_2) corresponds to the diagrams in which the scattered photon is first emitted along the nucleon line and the point of absorption is located later on. The third class corresponds to the case when both photons do not interact with the initial nucleon line. The corresponding amplitude is denoted as A_P. The fourth class contains diagrams in which only one of external photons interacts with the nucleon line. The corresponding notation is A_{odd} (see Fig. 3):

$$A^{\mu\nu}(s,q) = A_1^{\mu\nu}(s,q) + A_2^{\mu\nu}(s,q) + A_P^{\mu\nu}(s,q) + A_{\mathrm{odd}}. \tag{29}$$

The amplitude $A_P^{\mu\nu}(s,q)$ corresponds to the Pomeron-type Feynman diagram [Fig. 4(e)] and gives the nonvanishing contribution to the total cross sections in the limit of a large invariant mass squared of initial particles $s_2 \to \infty$. The fourth class amplitude can be relevant in experiments measuring charge-odd effects and it will not be considered here. One can show explicitly that each of the 4 classes amplitudes are gauge invariant. The arguments in favor of it are essentially the same as was used in the QED case [6].

Let us discuss now the analytical properties of the retarded part of the forward Compton scattering of a virtual photon on a proton, $A_1(s_2, q)$ (see Fig. 4) at the s_2-plane. Because of general principles, the singularities—poles and branch points— are situated on the real axis.

These singularities are illustrated in Fig. 5. On the right side, the pole at $s_2 = 0$ corresponds to one nucleon exchange in the s_2 channel [Fig. 5(a)]; the right hand cut starts at the pion-nucleon threshold, $s_2 = (M + m_\pi)^2 - M^2$. The left cut, related with the u-channel 3-nucleon state of the Feynman amplitude, is illustrated in Fig. 4(f). It is situated rather far from the origin at $s_2 = -8M^2$. It can be shown that it is the nearest singularity of the right hand cut. Really the u-channel cut corresponding to the $2\pi N$ state

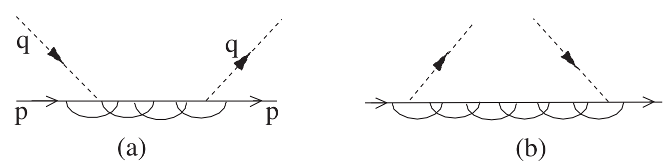

FIG. 3. Illustration of retarded (a) and advanced (b) virtual photon emission and absorption diagrams. The diagram containing Pomeron is not considered here.

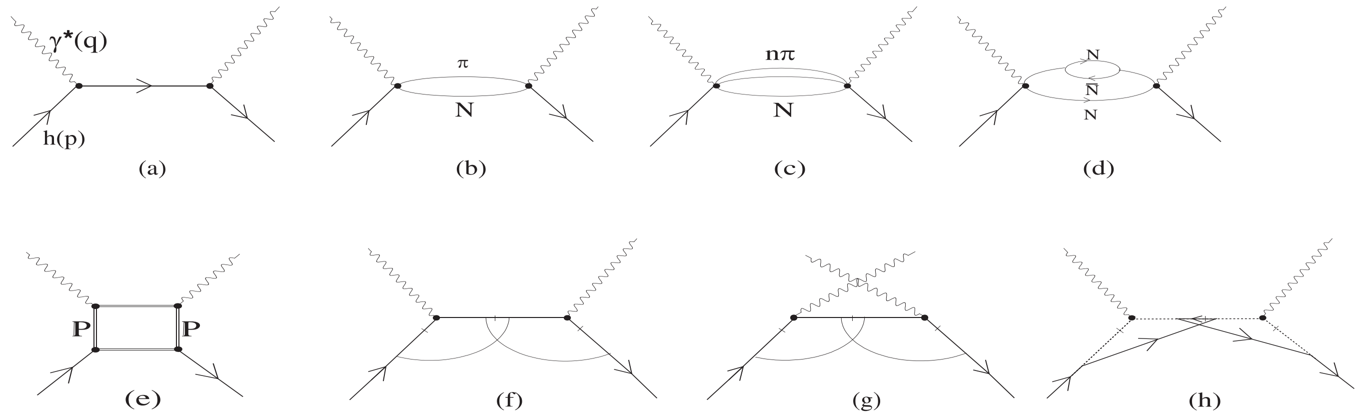

FIG. 4. Feynman diagrams for forward virtual Compton scattering on the proton, contributing to the retarded part of the amplitude: intermediate state in the s_2 channel for a single proton (a), $N\pi$ (b), $Nn\pi$ (c), $\sum N\bar{N}$ (d), two jets s_2 channel state, with two Pomeron t channel state (d), $3N$ intermediate state in u-channel (e), $3N$ intermediate state in u_2 channel (f). The Feynman diagram of the A_2 set which has s-channel discontinuity is illustrated in (g) and an example of the exotic $u$channel state in (h).

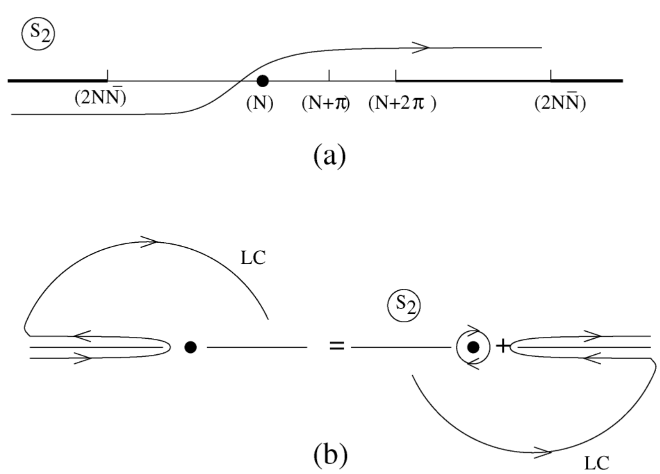

FIG. 5. Illustration of singularities along the s_2 real axis with the open contour C (a), and with the contour C closed (b), corresponding to Fig. 4. LC stands for large circle contribution.

cannot be realized without exotic quantum number states [see Fig. 4(h)].

III. SUM RULES

Following [6] let us introduce the quantity

$$\int_C ds_2 \frac{p_1^\mu p_1^\nu A_{1\mu\nu}^{\gamma^* p \to \gamma^* p}}{s^2(\vec{q}^2)^2} = \frac{dI}{d\vec{q}^2}, \tag{30}$$

with the Feynman contour C in the s_2 plane as it is shown in Fig. 5(a).

Sum rules appear when one considers the equality of the path integrals along the contours obtained by deforming C such that it is closed to the left and to the right side (Fig. 5). As a result one finds

$$\frac{d\sigma_{\rm left}}{d\vec{q}^2} - \frac{d\sigma_{\rm el} - d\sigma_{\rm el}^B}{d\vec{q}^2} = \frac{d\sigma_{\rm inel}}{d\vec{q}^2}, \tag{31}$$

where $\frac{d\sigma_{\rm left}}{d\vec{q}^2}$ indicates the contribution of the left cut;[1]

$$\frac{d\sigma_{\rm el}^B}{d\vec{q}^2} = \frac{4\pi\alpha^2 Z^2}{(\vec{q}^2)^2}.$$

The latter is generally the Born cross section of the scattering of an electron on any hadron with charge Z when the strong interaction is switched off, and $\frac{d\sigma_{\rm el}}{d\vec{q}^2}$ is the elastic electron-hadron cross section when the strong interaction is switched on, in the lowest order of QED coupling constant. This quantity can be expressed in terms of electromagnetic form factors of corresponding hadrons.

Using the notation for the generalized square of form factors as Φ^2, we have the following expression for the process of electron scattering on a hadron Y with charge Z:

$$Z^2 - \Phi^2(-\vec{q}^2) = \frac{2(\vec{q}^2)^2}{\pi\alpha^2}\frac{d\sigma^{eY\to eX}}{d\vec{q}^2}. \tag{32}$$

For the case of the spin-zero target, the quantity Φ^2 coincides with its squared charge form factor.

For the cases of electron scattering on a spin one-half (proton, ^{3}He,^{3}H), which are described by two form factors (Dirac's one F_1 and Pauli one F_2) we have

$$Z_i^2 - F_{1i}^2(-\vec{q}^2) - \tau_i F_{2i}^2(-\vec{q}^2) = \frac{2(\vec{q}^2)^2}{\pi\alpha^2}\frac{d\sigma^{eY_i\to eX_i}}{d\vec{q}^2}, \tag{33}$$

[1]The coincidence of numbers in (1) and (2) is derived from the absence of the left cut contribution, which is known for planar Feynman diagram amplitudes.

with

$$\tau_i = \frac{\vec{q}^2}{4M_i^2}; \qquad Z_p = Z_{^3\mathrm{H}} = 1; \qquad Z_{^3\mathrm{He}} = 2.$$

For scattering of electrons on deuteron we have

$$1 - \frac{1}{3}[2F_1^2(-\vec{q}^2) + [F_1(-\vec{q}^2) + 2\tau_d(1+\tau_d)F_3(-\vec{q}^2)]^2 + 2\tau_d F_2^2(-\vec{q}^2)] = \frac{2(\vec{q}^2)^2}{\pi\alpha^2}\frac{d\sigma^{ed\to eX}}{d\vec{q}^2}. \tag{34}$$

These equations can be tested in experiments with electron-hadron colliders.

Let us consider the differential form of these sum rules applying the operator : $\frac{d}{d\vec{q}^2}(\vec{q}^2)^2 F(\vec{q}^2)|_{\vec{q}^2=0}$ to both sides of Eq. (34), which can be expressed in terms of charge radii, anomalous magnetic moments, etc. and of the total photoproduction cross section.

Considering formally this derivative at $\vec{q}^2 = 0$, we obtain

$$\frac{d}{d\vec{q}^2}\Phi_Y^2|_{\vec{q}^2=0} = \frac{2}{\pi^2\alpha}\int_{s_{\mathrm{th}}}^{\infty}\frac{ds_2}{s_2}\sigma_{\mathrm{tot}}^{\gamma Y\to X}(s_2). \tag{35}$$

Unfortunately, the sum rule in this form cannot be used for experimental verification due to the divergence of the integral in the right-hand side of this equation. Its origin follows from the known fact of increasing of photoproduction cross sections at large values of initial center of mass energies squares s_2. It is commonly known that this fact is the consequence of Pomeron-Regge pole contribution. The universal character of Pomeron interaction with nucleons can be confirmed by the Particle Data Group-2004 (PDG) result [7]

$$[\sigma^{\gamma p}(s_2) - \sigma^{\gamma n}(s_2)]|_{s_2\to\infty} = [2\sigma^{\gamma p}(s_2) - \sigma^{\gamma d}(s_2)]|_{s_2\to\infty} = 0. \tag{36}$$

In Ref. [8] a sum rule which contains the difference between proton and neutron sum photoproduction cross sections was derived

$$\frac{1}{3}\langle r_p^2\rangle + \frac{1}{4M^2}[\kappa_n^2 - \kappa_p^2] = \frac{2}{\pi^2\alpha}\int_{\omega_n}^{\infty}\frac{d\omega}{\omega} \times [\sigma^{\gamma p\to X}(\omega) - \sigma^{\gamma n\to X}(\omega)]. \tag{37}$$

We use here the known relations

$$F_1(-\vec{q}^2) = 1 - \frac{1}{6}\vec{q}^2\langle r^2\rangle + O((\vec{q}^2)^2); \qquad F_2(0) = \kappa,$$

with $\langle r^2\rangle$, κ, are the charge radius squared and the anomalous magnetic moment of nucleon (in units h/Mc).

It was verified in this paper that this sum rule is fulfilled within the experimental errors: both sides of the equation equal 1.925 mb. Here the Pomeron contribution is compensated in the difference of proton and neutron total cross photoproduction cross sections.

In Ref. [9], the similar combination of cross sections was considered for $A = 3$ nuclei:

$$\frac{2}{3}\langle r_{^3\mathrm{He}}^2\rangle - \frac{1}{3}\langle r_{^3\mathrm{H}}^2\rangle - \frac{1}{4M^2}[\kappa_{^3\mathrm{He}}^2 - \kappa_{^3\mathrm{H}}^2] = \frac{2}{\pi^2\alpha}\int_{\omega_{\mathrm{th}}}^{\infty}\frac{d\omega}{\omega}[\sigma^{\gamma^3\mathrm{He}\to X}(\omega) - \sigma^{\gamma^{33}\mathrm{H}\to X}(\omega)]. \tag{38}$$

In a similar way, the combination of cross sections of electron scattering on proton and deuteron leads to the relation

$$\frac{1}{3}\langle r_d^2\rangle - \frac{F_3(0)}{3M_d^2} - \frac{1}{6M_d^2}F_2(0)^2 - 2\left[\frac{1}{3}\langle r_p^2\rangle - \frac{1}{4M_p^2}\kappa_p^2\right] = \frac{2}{\pi^2\alpha}\int_{\omega_{th}}^{\infty}\frac{d\omega}{\omega}[\sigma_{\mathrm{tot}}^{\gamma d\to X}(\omega) - 2\sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)], \tag{39}$$

with ω_{th} for the deuteron and the proton being different: $(\omega_{\mathrm{th}})_d = 2,2$ MeV, $\qquad (\omega_{\mathrm{th}})_p = m_\pi + \frac{m_\pi^2}{2M_p} \approx 140$ MeV.

We use here the similar expansion for F_1 deuteron form factor $F_1(0) = 1$ and introduce its square charge radius. Other quantities can be found in Ref. [10]:

$$F_2(0) = -\frac{M_d}{M_p}\mu_d; \qquad \mu_d = 0.857;$$

$$2F_3(0) = 1 + F_2(0) - M_d^2 Q_d; \qquad Q_d = 0.2859\ \mathrm{fm}^2.$$

We find for the right side of Eq. (39) by using [7,11]

$$\frac{2}{\pi^2\alpha}\left\{\int_{0.020}^{0.260}\frac{d\omega}{\omega}\sigma_{\mathrm{tot}}^{\gamma d\to X}(\omega) + \int_{0.260}^{16}\frac{d\omega}{\omega}[\sigma_{\mathrm{tot}}^{\gamma d\to X}(\omega) - 2\sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)]\right\} = 0.8583\ \mathrm{fm}^2 = 8.583\ \mathrm{mb} \tag{40}$$

Using the data for γp and γd cross sections and the parametrization from Ref. [11] we find from Eqs. (39) and (40)

$$\langle r_d\rangle \approx 1.94\ \mathrm{fm}.$$

This quantity is in a satisfactory agreement with prediction of models based dispersion relations [12] where is $\langle r_d\rangle \sim 2$ fm.

The reason for a discrepancy (which does not exceed the errors of 3% inherent to our approach) can be attributed to the lack of data for γp and γd cross sections near the thresholds.

IV. CONCLUSIONS

The left cut contribution has no direct interpretation in terms of cross section. In analogy with the QED case it can be associated with the contribution to the cross section of the process of proton-antiproton pair electroproduction on proton $ep \to e2p\bar{p}$, arising from taking into account the identity of final state protons.

Fortunately, the threshold of this process is located quite far. Using this fact we can estimate its contribution in the framework of QED-like model with nucleons and pions (ρ-mesons), omitting the form-factor effects (so we put them equal to coupling constants of nucleons with pions and vector mesons).

In this paper we applied an optical theorem, which connects the s-channel discontinuity of the forward-scattering amplitude with the total cross section. It is valid for the complete scattering amplitude, whereas we consider only part of it, A_1. We can explicitly point out on the Feynman diagram [see Fig. 4(g)], contributing to A_2, which has 3 nucleon s-channel states. The relevant contribution can be interpreted as the identity effect of proton photoproduction of a $p\bar{p}$ pair.

The explicit calculation in the framework of our approach is given by Appendix A. The corresponding contributions to the derivative on $\vec{q}^2$ at $\vec{q}^2 = 0$ of scattering amplitudes entering the sum rules have an order of magnitude

$$I = \frac{g^4 M^2}{\pi^3 s_{2\,\mathrm{min}}^2}.$$

In order to estimate the strong coupling constant, we use the PDG value for the total cross section of scattering of the pion on the proton $\sigma_{\mathrm{tot}}^{\pi p} = 20$ mb. Keeping in mind the ρ-meson t-channel contribution $\sigma^{\pi p} \sim g^4/(4\pi m_\rho^2)$ and the minimal value of three nucleon invariant mass-squared $s_{2\,\mathrm{min}} = 8M_p^2$, we have $I \approx (1/15)$ mb. Comparing this value with typical values of right- and left-hand sides of sum rules of order of $2mb$, we estimate the error arising by omitting the left cut contribution as well as replacing our incomplete cross sections by the measurable ones on the level of 3%. Gottfried sum rules [15], which are also related to the present question, suffer from ultraviolet divergency due to Pomeron exchange contribution.

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APPENDIX A: EFFECT OF IDENTITY OF PROTONS TO THE CROSS SECTION OF $2p\bar{p}$ PHOTOPRODUCTION CROSS SECTION

The contribution to the s-channel discontinuity of the part of the scattering amplitude $A_2^{\gamma^* p \to \gamma^* p}$, arising from the interference of the amplitudes for the creation of a proton-antiproton pair of bremsstrahlung type, due to the identity of protons in the final state has the form:

$$\Delta_s A_2(s, q) = -\frac{16g^4}{s^2} \int ds_2 d\Gamma_3 \frac{S}{(q_1^2 - m_\pi^2)(q_2^2 - m_\pi^2)}, \tag{A1}$$

where $q_1 = P_1 - P$, $q_2 = P_2 - P$ and

$$S = \tfrac{1}{4}\mathrm{Sp}(\hat{P} + M)\gamma_5(\hat{P}_1 + M)V_1(\hat{P}_3 - M)V_2(\hat{P}_2 + M)\gamma_5,$$

and $V_1 = \gamma_5 \frac{\hat{q} - \hat{P}_3 + M}{d_3}\hat{p}_1 + \hat{p}_1 \frac{\hat{P}_1 - \hat{q} + M}{d_1}\gamma_5$;
$V_2 = \gamma_5 \frac{-\hat{q} + \hat{P}_2 + M}{d_2}\hat{p}_1 + \hat{p}_1 \frac{-\hat{P}_3 + \hat{q} + M}{d_3}\gamma_5$;
$d_{1,2,3} = (q - P_{1,2,3})^2 - M^2$, $s_2 = (P + q)^2 - M - q^2$.

The elements of phase volume can be written as

$$d\Gamma_3 = (2\pi)^4 \delta^4(P + q - P_1 - P_2 - P_3)\Pi_1^3 \frac{d^3 P_i}{2E_i(2\pi)^3}.$$

Here we consider pions to be interacting with nucleons with a coupling constant g. The similar expression can be written for the case when one or both pions are replaced by the ρ-meson. It can be shown that the corresponding contributions are approximately one order of magnitude smaller than those from pions.

We use Sudakov's parametrization of momenta:

$$q = \alpha\tilde{P} + \beta p_1 + q_\perp; \qquad P = \tilde{P} + \frac{M^2}{s} p_1; \qquad P_i = \alpha_i \tilde{P} + \beta_i p_1 + P_{i\perp}. \tag{A2}$$

Using the formulas given above, all the relevant quantities can be written as

$$\int ds_2 d\Gamma_3 = \frac{1}{4(2\pi)^3} \frac{d^2 P_1 d^2 P_2 d\alpha_1 d\alpha_2}{\alpha_1 \alpha_2 \alpha_3}; \qquad \alpha_1 + \alpha_2 + \alpha_3 = 1; \qquad \vec{P}_1 + \vec{P}_2 + \vec{P}_3 = \vec{q} \tag{A3}$$

and

$$s_2 = s\beta = -M^2 + \sum_1^3 \frac{\vec{P}_i^2 + M^2}{\alpha_i}; \qquad q_i^2 = -\frac{\vec{P}_i^2 + (1 - \alpha_i)^2}{\alpha_i}, \quad i = 1, 2; \qquad d_i = -s_2\alpha_i + 2\vec{q}\vec{P}_i - \vec{q}^2, \quad i = 1, 2, 3. \tag{A4}$$

Note that the quantities V_i can be written as

$$V_1 = s\gamma_5 A_{13} + \frac{\gamma_5 \hat{q}\hat{p}_1}{d_3} - \frac{\hat{p}_1\hat{q}\gamma_5}{d_1};$$
$$V_2 = s\gamma_5 A_{23} - \frac{\gamma_5 \hat{q}\hat{p}_1}{d_2} + \frac{\hat{p}_1\hat{q}\gamma_5}{d_3}, \tag{A5}$$

with

$$A_{13} = \frac{\alpha_1}{d_1} - \frac{\alpha_3}{d_3}; \qquad A_{23} = \frac{\alpha_2}{d_2} - \frac{\alpha_3}{d_3}.$$

In this form the gauge invariance of the contribution to the forward-scattering amplitude is explicitly seen; namely, this quantity turns out to zero at $\vec{q} \to 0$, (we see that replacements $\hat{q}\hat{p}_1 = \hat{q}_\perp \hat{p}_1$, $\hat{p}_1\hat{q} = \hat{p}_1\hat{q}_\perp$ in $V_{1,2}$ can be done).

The calculation of the trace leads to the result:

$$\frac{S}{s^2} = A_{13}A_{23}S_1 + A_{13}\left[\frac{1}{d_3} + \frac{1}{d_2}\right]S_2 + A_{23}\left[\frac{1}{d_3} + \frac{1}{d_1}\right]S_3 + \left[\frac{1}{d_3} + \frac{1}{d_1}\right]\left[\frac{1}{d_3} + \frac{1}{d_2}\right]S_4, \tag{A6}$$

with

$$S_1 = M^4 + \tfrac{1}{2}M^2\vec{q}^2 + (PP_1)(P_2P_3) + (P_3P_1)(P_2P) - (PP_3)(P_2P_1);$$
$$S_4 = -\frac{\alpha_3\vec{q}^2}{2}[M^2\alpha_3 + \alpha_1(PP_2) + \alpha_2(PP_1) - (P_1P_2)];$$
$$S_2 = \frac{M^2}{2}[\alpha_2(\vec{q}\vec{P}_2) - \alpha_3(\vec{q}\vec{P}_3) - (\alpha_1 + 2\alpha_3)(\vec{q}\vec{P}_1)] + \frac{1}{2}[(\vec{q}\vec{P}_3)[(P_1P_2) - \alpha_2(PP_1) - \alpha_1(PP_2)] + (\vec{q}\vec{P}_2)[(P_1P_3) + \alpha_3(PP_1) - \alpha_1(PP_3)] - (\vec{q}\vec{P}_1)[(P_3P_2) - \alpha_2(PP_3) - \alpha_3(PP_2)]];$$
$$S_3 = \frac{M^2}{2}[\alpha_3(\vec{q}\vec{P}_3) - \alpha_1(\vec{q}\vec{P}_1) + (\alpha_2 + 2\alpha_3)(\vec{q}\vec{P}_2)] + \frac{1}{2}[(\vec{q}\vec{P}_1)[-(P_3P_2) - \alpha_3(PP_2) + \alpha_2(PP_3)] + (\vec{q}\vec{P}_2)[(P_1P_3) - \alpha_3(PP_1) - \alpha_1(PP_3)] - (\vec{q}\vec{P}_3)[(P_1P_2) - \alpha_1(PP_2) - \alpha_2(PP_1)]]. \tag{A7}$$

The invariants entering this expression have a form

$$2PP_i = \frac{\vec{P}_i^2 + M^2(1 + \alpha_i^2)}{\alpha_i};$$
$$2P_iP_j = \frac{(\alpha_i\vec{P}_j - \alpha_j\vec{P}_i)^2 + M^2(\alpha_i^2 + \alpha_j^2)}{\alpha_i\alpha_j}.$$

Numerical integration of this expression confirms the estimate given above within 10%.

APPENDIX B: CORRELATION BETWEEN MOMENTUM AND THE SCATTERING ANGLE OF RECOIL PARTICLE IN LAB FRAME

The idea of expanding four vectors of some relativistic problem using two of them as a basis (Sudakov's parametrization) becomes useful in many regions of quantum field theory. It was crucial in studying the double logarithmical asymptotic of amplitudes of processes with large transversal momenta. Being applied to processes with peripheral kinematics, it essentially coincides with the infinite momentum frame approach.

Here we demonstrate its application to the study of the kinematics of the peripheral process of jet formation on a resting target particle. One of the experimental approaches to studying them is to measure the recoil particle momentum distribution. For instance, this method is used in the process of electron-positron pair production by linearly polarized photon on electrons in a solid target (atomic electrons). Here the correlation between the recoil momentum-initial photon plane and the plane of photon polarization is used to determine the degree of photon polarization [14].

Sudakov's parametrization allows us to give a transparent explanation of correlation between the angle of emission of the recoil target particle of mass M with the recoil momentum value in the laboratory reference frame [14]:

$$\frac{|\vec{P}'|}{M} = \frac{2\cos\theta_p}{\sin^2\theta_p}; \qquad \frac{E'}{M} = \frac{1 + \cos^2\theta_p}{\sin^2\theta_p}, \tag{B1}$$

where $\vec{P}'$, E' are the 3-momentum and energy of recoil particle, $(E')^2 - (P')^2 = M^2$; θ_p is the angle between the beam axes $\vec{k}$ in the rest frame of the target particle.

$$\gamma(k) + P(P) \to \text{jet} + P(P'), \qquad s = 2kP = 2M\omega,$$
$$P - P' = q, \qquad P^2 = (P')^2 = M^2.$$

The kinematics considered here corresponds to the main contribution to the cross section for a jet moving close to projectile direction. Using the Sudakov representation for transfer momentum $q = \alpha\tilde{P} + \beta k + q_\perp$, and the recoil particle on-mass shell condition $(P - q)^2 - M^2 \approx -s\beta - \vec{q}^2 = 0$, we obtain for the ratio of squares of transversal and longitudinal components of the 3-momentum of the recoil particle:

$$\tan^2\theta_p = \frac{\vec{q}^2}{(\beta\omega)^2} = \frac{4M^2}{\vec{q}^2}, \qquad \vec{q}^2 = (\vec{P}')^2\sin^2\theta_p. \tag{B2}$$

The relation noted in the beginning of this section follows immediately.

This correlation was first mentioned in Ref. [13], where the production on electrons from the matter was investigated. It was proven in Ref. [14].

This relation can be applied in experiments with collisions of high energy protons scattered on protons in the matter.

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Charge asymmetry for electron (positron)-proton elastic scattering at large angles

E. A. Kuraev, V. V. Bytev, and S. Bakmaev
JINR-BLTP, 141980 Dubna, Moscow Region, Russian Federation

E. Tomasi-Gustafsson
DAPNIA/SPhN, CEA/Saclay, F-91191 Gif-sur-Yvette Cedex, France
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Charge asymmetry in electron (positron) scattering arises from the interference of the Born amplitude and the box-type amplitude corresponding to two virtual photons exchange. It can be extracted from electron-proton and positron-proton scattering experiments, in the same kinematical conditions. Considering the virtual photon Compton scattering tensor, which contributes to the box-type amplitude, we separate proton and inelastic contributions in the intermediate state and parametrize the proton form factors as the sum of long-distance and short-distance interaction terms. Arguments based on analyticity are given in favor of cancellation of contributions from proton strong-interaction form factors and of inelastic intermediate states in the box-type amplitudes. In the framework of this model, with a realistic expression for nucleon form factors, numerical estimations are given for moderately high energies.

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I. INTRODUCTION

The nucleon structure is traditionally investigated by using electromagnetic probes and assuming that the interaction occurs through the exchange of a virtual photon, which carries the momentum transfer, q, from the incident to the scattered lepton. Recently, considerable attention was devoted to the 2γ exchange amplitude both in scattering and annihilation channels [1–4], in connection with experimental data on electromagnetic proton form factors (FFs) [5].

The extraction of the box-type [two-photon exchange amplitude (TPE)] contribution to the elastic electron-proton scattering amplitude is one of long-standing problems of experimental physics. It can be obtained from electron-proton and positron-proton scattering at the same kinematical conditions. Similar information about the TPE amplitude in the annihilation channel can be obtained from the measurement of the forward-backward asymmetry in proton-antiproton production in electron-positron annihilation (and from the time-reversal process).

The theoretical description of the TPE amplitude is strongly model dependent. Two reasons should be mentioned: The experimental knowledge of nucleon FFs is restricted to a small kinematical region, and the contribution of the intermediate hadronic states can only be calculated with large uncertainty, the precision of the data being insufficient to constrain the models.

A general approximation for proton electromagnetic form factors follows the dipole approximation,

$$G_E(q^2) = \frac{G_M(q^2)}{\mu} = G_D(Q^2) = (1 + Q^2/0.71\ \mathrm{GeV}^2)^{-2},\\ Q^2 = -q^2 = -t, \tag{1}$$

where μ is the magnetic moment of proton. However, recent experiments [5] showed a deviation of the proton electric FF from this prescription, when measured following the recoil polarization method [6], which is more precise than the traditional Rosenbluth separation [7]. Such deviation was tentatively explained, where the presence of the two-photon exchange contribution was advocated.

The motivation of this paper is to perform the calculation of the charge-odd correlation

$$A^{\mathrm{odd}} = \frac{d\sigma^{e^-p} - d\sigma^{e^+p}}{2d\sigma_B^{ep}}, \tag{2}$$

in the process of electron-proton scattering in the framework of an analytical model (AM), free from uncertainties connected with the inelastic hadronic state in the intermediate state of the TPE amplitude. In the framework of this model it is possible to show that there is a compensation between the effects from strong-interaction FFs and those from the inelastic intermediate states, within an accuracy discussed in the following.

Our paper is organized as follows. In Sec. II, we introduce a new decomposition of proton FFs, separating the long-distance and short-distance contributions. In Sec. III we formulate the analytical model and calculate the contribution of the long-distance part of the FFs. In Sec. IV the resulting expression for the asymmetry is obtained and we present the results of numerical integration for asymmetries. Conclusions are given in Sec. V, where we estimate the accuracy of the obtained results. The Appendix contains some details of the calculation.

II. NEW FORM OF THE PROTON FORM FACTORS

The cross section of elastic ep scattering,

$$e(p_1) + p(p) \to e(p_1') + p(p'), \tag{3}$$

in the Born approximation in the laboratory (Lab) frame [$p = (M, 0, 0, 0)$, $p_1 = E(1, 1, 0, 0)$] has the form

$$\frac{d\sigma_B}{d\Omega} = \frac{\sigma_M \sigma_{\rm red}}{\varepsilon(1+\tau)}, \quad \sigma_M = \frac{\alpha^2 \cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}}\frac{1}{\rho},$$
$$\rho = 1 + \frac{2E}{M}\sin^2\frac{\theta}{2}, \quad \tau = \frac{Q^2}{4M^2}, \quad t = \frac{s(1-\rho)}{\rho},$$
$$Q^2 = -q^2 = -t = 2p_1p_1', \quad s = 2ME, \tag{4}$$
$$u = -\frac{s}{\rho} = -2pp_1', \quad s + t + u = 0,$$
$$\varepsilon^{-1} = 1 + 2(1+\tau)\tan^2\frac{\theta}{2},$$

with

$$\sigma_{\rm red} = \tau G_M^2 + \varepsilon G_E^2, \quad G_M = F_1 + F_2, \quad G_E = F_1 - \tau F_2. \tag{5}$$

Here θ is the laboratory electron scattering angle.

In the analysis of the TPE amplitude we consider the electromagnetic interactions in the lowest order of perturbation theory. Hadron electromagnetic FFs in the spacelike region, which parametrize the interaction with the external electromagnetic vertex as

$$\bar{u}(p')\left[\Gamma_1(q^2)\gamma_\mu + \frac{1}{2M}\hat{q}\gamma_\mu\Gamma_2(q^2)\right]u(p),$$
$$q = p' - p, \quad p^2 = p'^2 = M^2, \tag{6}$$

are functions of one kinematical variable, q^2. We divide the form factors into two parts:

$$F_1(q^2) = F_{1l}(q^2) + F_{1s}(q^2), \quad F_2(q^2) = F_{2s}(q^2) \tag{7}$$

with

$$F_{1s}(0) = 0 \text{ and } F_{2s}(0) = \mu - 1.$$

The first part of the FFs, F_{1l}, describes the contribution of the long-distance interaction (small value of Q^2, or exchange of small virtuality gluons). At $Q^2 = 0$, $F_{1l} = 1$ (normalization of FFs). At larger Q^2 we suppose that this contribution will vanish owing to Sudakov suppression; the main contribution will be given by

$$F_{1l}(q^2) = \exp\left(-\frac{\alpha_s C_F}{4\pi}\ln^2\frac{Q^2}{m_q^2}\right), \tag{8}$$

where $m_c \sim 200$ MeV is the constituent quark mass and $C_F = (N_f^2 - 1)/(2N_f) = 4/3$, with $N_f = 3$ the effective number of flavors.

So we can chose $F_{1l} = \Theta(\Lambda_{\rm qcd}^2 - Q^2)$. The second contribution, $F_{2s}(Q^2)$, corresponding to the contribution of the short-distance interaction (large Q^2, or large virtualities of exchanged gluons), equals zero at small Q^2 and is proportional to $(1/Q^2)^2$ at $Q^2 > \Lambda_{\rm qcd}^2$ because of the quark counting rule. Here we suppose $\Lambda_{\rm qcd}^2 \approx m_\pi^2$. The dependence on $\Lambda_{\rm qcd}$ or m_q turns out to be very weak.

The contribution of the large distances to $F_{2l}(Q^2)$ is neglected because in all physical values the quantity F_2 is accompanied by the factor Q^2, which makes the contribution of F_{2l} negligible.

In most of the kinematical domain, one can safely neglect the interference between F_{1l} and F_{1s}, as they act in different regions of transferred momenta: $F_1^2 = F_{1l}^2 + F_{1s}^2$. However, the derivative at $Q^2 \to 0$, which is connected to the charge radius r_p is sensitive to the interference term:

$$\frac{d}{dQ^2}F_1^2(Q^2)\big|_{Q^2=0} = 2F_{1s}'(0).$$

Parametrizations of spacelike form factors in terms of fractional polynomials have already been suggested in the literature (see, e.g., Ref. [8]). The present choice of the strong-interaction FF is consistent with the following parametrization:

$$F_{1s}(Q^2) = \frac{Q^2 r_p^2\left[1 + \sum_1^n c_k\left(\frac{Q^2}{Q_0^2}\right)^k\right]}{6\left[1 + \sum_1^{n+3} d_k\left(\frac{Q^2}{Q_0^2}\right)^k\right]},$$
$$F_{2s}(Q^2) = \frac{(\mu-1)\left[1 + \sum_1^n e_k\left(\frac{Q^2}{Q_0^2}\right)^k\right]}{1 + \sum_1^{n+3} f_k\left(\frac{Q^2}{Q_0^2}\right)^k}, \tag{9}$$

where c, d, e, and f can be considered as fitting parameters. The last coefficients of the series c_n, d_{n+3}, e_n, and f_{n+3} are constrained by the high-Q^2 asymptotic limit. The low-Q^2 properties—$F_{1s}(0) = 0$, $F_{1s}'(0) = \frac{1}{6}r_p^2$, and $F_{2s}(0) = \mu - 1$—are explicitly taken into account.

III. FORMULATION OF THE ANALYTICAL MODEL

Let us now discuss the arguments in favor of mutual cancellation of the terms of the order of F_s^2 with the contribution of the inelastic hadronic intermediate states in the TPE amplitude.

The TPE amplitude contains the virtual photon Compton scattering tensor. It can be split into two terms, when only strong-interaction contributions to Compton amplitude are taken into account. One term (the elastic term) is the generalization of the Born term with the strong-interaction FFs at the vertexes of the interaction of the virtual photons with the hadron. We suppose that the hadron before and after the interaction with the photons remains unchanged. The second (inelastic) term corresponds to inelastic channels formed by pions and nucleons or the excited states of the nucleon such as the the πN and $\pi N\bar{N}$ states.

Denoting the loop momenta in the TPE amplitude as k, we can write the loop momenta phase volume as

$$d^4k = \frac{1}{2s}d^2k_\perp ds_e ds_p,$$

where $s_e = (p_- - k)^2$ and $s_p = (p+k)^2$ are the invariant mass squared of the electron and proton blocks, and $k_\perp$ is a two-dimensional Euclidean vector orthogonal to the momenta of the initial particles: $k_\perp p_- = k_\perp p = 0$. By considering both Feynman diagrams, the box and the crossed diagram (Fig. 1), the integration over $s_e(-\infty < s_e < \infty)$ can be done by calculating the residue by s_e in the electron Green function (omitting higher QED corrections). The contour for the s_p integration

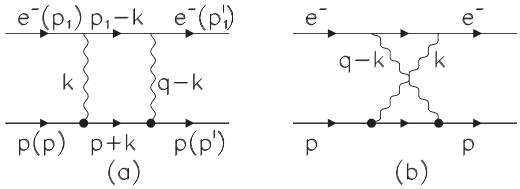

FIG. 1. Feynman diagrams for two-photon exchange in elastic *ep* scattering: (a) box diagram and (b) crossed box diagram.

has a Feynman contour form: $-\infty - i0 < s_p < \infty + i0$ (see Fig. 2).

Let us now consider the analytic properties of the Compton scattering amplitude with both photons off-mass shell as a function of the complex variable s_p. The singularities on the physical sheet are a pole, located at $s_p = M^2$, which corresponds to a proton in the intermediate state, and a series of cuts corresponding to inelastic states of a nucleon accompanied by pions, and nucleon-antinucleon pairs. The first cut lies at $s_{\pi p} = (M + m_\pi)^2$. The left cut lies at $s_2 < -(3M_N)^2$ and corresponds to a $p\bar{p}p$ state in the u channel. Its contribution, for the case $Q^2 \sim s \gg M^2$, is suppressed by powers of $(M^2/Q^2)^n$, compared with contributions corresponding to singularities of the right cut, since the operator of higher twist become relevant. Neglecting the contribution of the left cut we can close the s_p integration contour to the pole and the $s_p > s_{\pi p}$ cuts, ensuring their mutual cancellation. Similar considerations were developed in the QED framework [9]. The application of this result to forward elastic *ep* scattering amplitude in the high-energy limit allows us to derive sum rules for the strong-interaction contribution to the proton FF. The contribution of the left cut, in this case, has to be precisely calculated. This will be published elsewhere.

Based on this hypothesis (which was proved rigorously in the QED framework and is here taken as an assumption) we consider only the long-distance interaction part of the amplitude corresponding to the one-nucleon (nonexcited) state in the hadronic block and omit the pure strong-interaction contributions.

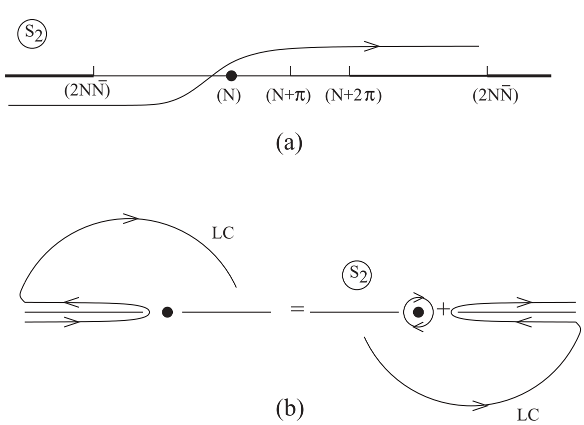

FIG. 2. Illustration of singularities along the s_2 real axis with the open contour C (a) and with the contour C closed (b). LC stays for the large circle contribution.

Proton FFs enter in the box amplitude in a form that can be schematically written as

$$\int \frac{d^4k}{i\pi^2(e)(p)} \frac{F_{1l}(k^2)+F_s(k^2)}{(k)} \frac{F_{1l}(\bar{k}^2)+F_s(\bar{k}^2)}{(\bar{k})},$$
$$(k) = k^2-\lambda^2, \quad (\bar{k}) = (k-q)^2-\lambda^2,$$
$$(e) = (k-p_1)^2-m_e^2, \quad (p) = (k+p)^2-M^2, \tag{10}$$

where we extract the long-distance interaction part and do not distinguish Dirac and Pauli form factors. This integrand expression can be rearranged as

$$\frac{F_{1l}(k^2)F_{1l}(\bar{k}^2)}{(k)(\bar{k})} + \frac{F_s(k^2)F_s(\bar{k}^2)}{(k)(\bar{k})} + \frac{F_{1l}(k^2)F_s(\bar{k}^2)}{(k)(\bar{k})} + \frac{F_s(k^2)F_{1l}(\bar{k}^2)}{(k)(\bar{k})}. \tag{11}$$

The first term turns out to be zero because of the assumption about the "long-distance" contribution behavior of the Dirac form factor (7) [i.e., nonzero contributions of $F_{1l}(k^2)F_{1l}(\bar{k}^2)$ are in different kinematical regions]. The second one can be omitted because of the hypothesis of cancellation of the interaction at small distance with the contribution from the inelastic intermediate state. The last two terms coincide and can be written as

$$2\frac{F_s(q^2)}{q^2} \int \frac{d^4k}{i\pi^2(e)(p)} \frac{1}{(k)} \theta\left(m_0^2 - |k^2|\right). \tag{12}$$

The relevant integral is calculated in the Appendix.

The virtual photon emission contribution to the cross section has a form [according to Eq. (11)]

$$\frac{d\sigma_v}{d\Omega} = \frac{\alpha^3}{2\pi t^2 M^2 \rho^2} a, \tag{13}$$

with

$$a = \int \frac{d^4k}{i\pi^2} \frac{1}{(k)} \frac{S_T}{(p)} \left[\frac{S_e}{(e)} + \frac{S_{\bar{e}}}{(\bar{e})}\right] \theta\left(m_0^2 - |k^2|\right) \tag{14}$$

and

$$\begin{aligned}
(\bar{e}) &= (k+p_1')^2 - m_e^2,\\
S_e &= \frac{1}{4}\mathrm{Tr}\, \hat{p}_1' \gamma_\mu (\hat{p}_1 - \hat{k})\gamma_\nu \hat{p}_1 \gamma_\eta,\\
S_{\bar{e}} &= \frac{1}{4}\mathrm{Tr}\, \hat{p}_1' \gamma_\nu (\hat{p}_1' + \hat{k})\gamma_\mu \hat{p}_1 \gamma_\eta,\\
S_T &= \frac{1}{4}\mathrm{Tr}(\hat{p}+M)\Gamma_\eta(-q)(\hat{p}'+M)\Gamma_\nu(k)\\
&\quad \times (\hat{p}+\hat{k}+M)\Gamma_\mu(q-k),\\
\Gamma_\mu(q) &= \left[F_1(q^2) + \frac{\hat{q}}{2M}F_2(q^2)\right]\gamma_\mu.
\end{aligned} \tag{15}$$

Using the formulas given in the Appendix, we can write the virtual photon contribution to the differential cross section with the two-photon exchange, $d\sigma_v$, as

$$d\sigma_v^{\mathrm{odd}} = \frac{2\alpha}{\pi} \ln\rho \left(\ln\frac{2EM}{\lambda^2} - \frac{1}{2}\ln\rho\right) d\sigma_{\mathrm{Born}}. \tag{16}$$

The IR divergence from the virtual photon emission contribution is, as usual, canceled when summing the contribution from the emission of soft real photons:

$$\frac{d\sigma^{\rm soft}}{d\Omega} = \left[\frac{d\sigma_{Bt}}{d\Omega} + \frac{d\sigma_{B\rm box}}{d\Omega}\right]\delta^{\rm odd}_{\rm soft} = \frac{d\sigma^{\rm soft}_{Bt}}{d\Omega} + \frac{d\sigma^{\rm soft}_{B\rm box}}{d\Omega}. \quad (17)$$

The quantity $\delta^{\rm odd}_{B\rm soft}$ was considered in Refs. [2,11]:

$$\begin{aligned}\delta^{\rm odd}_{B\rm soft} &= -2\frac{4\pi\alpha}{16\pi^3} \\ &\quad\times \int \frac{d^3k}{\omega}\left(\frac{p_1'}{p_1'k} - \frac{p_1}{p_1k}\right)\left(\frac{p'}{p'k} - \frac{p}{pk}\right)\Bigg|_{S_0,\omega\leqslant\Delta E} \\ &= \frac{2\alpha}{\pi}\left[\ln\frac{1}{\rho}\ln\frac{2\rho\Delta E}{\lambda} + \ln x\ln\rho + \mathrm{Li}_2\left(1-\frac{1}{\rho x}\right)\right. \\ &\quad\left. - \mathrm{Li}_2\left(1-\frac{\rho}{x}\right)\right], \\ x &= \frac{\sqrt{1+\tau}+\sqrt{\tau}}{\sqrt{1+\tau}-\sqrt{\tau}}, \qquad (18)\end{aligned}$$

with λ and ΔE, respectively, the mass and the maximal energy of the soft photon.

IV. RESULTS AND DISCUSSION

The final result for the asymmetry $A^{\rm odd}$ (2) is

$$\begin{aligned}A^{\rm odd} &= \frac{2\alpha}{\pi}\left[\ln\frac{1}{\rho}\ln\frac{(2\Delta E)^2}{ME} - \frac{5}{2}\ln^2\rho + \ln x\ln\rho\right. \\ &\quad\left. + \mathrm{Li}_2\left(1-\frac{1}{\rho x}\right) - \mathrm{Li}_2\left(1-\frac{\rho}{x}\right)\right], \\ \rho &= \left(1-\frac{Q^2}{s}\right)^{-1}. \qquad (19)\end{aligned}$$

The finite part of the asymmetry is calculated for different values of Q^2 and θ. The results are plotted in Fig. 3, as a function of θ. The asymmetry vanishes for $\theta = 0$ and reaches the largest values for $\theta = \pi$. It is measurable, of the order of 5%. The largest contribution to the asymmetry is given by the first term in Eq. (19), which depends on the soft photon energy. The calculation is done for $\Delta E = 0.01E$.

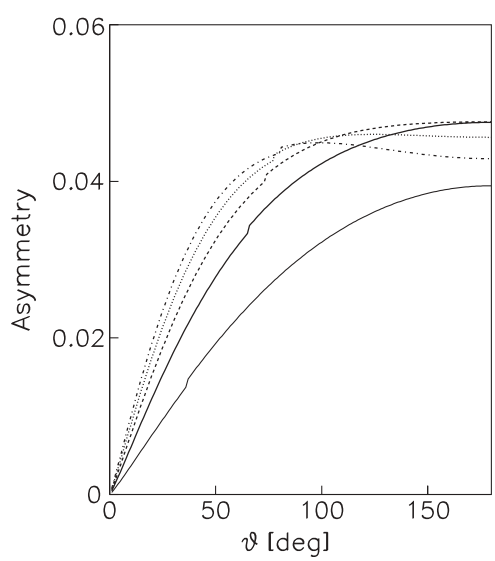

FIG. 3. Charge-odd correlation in electron-positron scattering as a function of the scattering angle θ, for $Q^2 = 1$ GeV^2 (thin solid line), 3 GeV^2 (thick solid line), 5 GeV^2 (dashed line), 7 GeV^2 (dotted line), and 9 GeV^2 (dash-dotted line). The calculation corresponds to $\Delta E = 0.01E$ [see Eq. (19)].

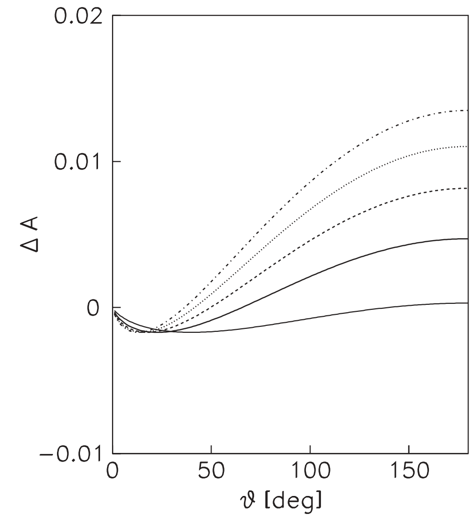

FIG. 4. Difference between the present asymmetry and the calculation [11], ΔA [Eq. (20)], for $Q^2 = 1$ GeV^2 (thin solid line), 3 GeV^2 (thick solid line), 5 GeV^2 (dashed line), 7 GeV^2 (dotted line), and 9 GeV^2 (dash-dotted line).

In Ref. [11], the approach used to calculate the TPE has to be considered as a model: One of the exchanged photons is quasireal. The contribution of inelastic intermediate states is also ignored. The comparison with the results from Ref. [11], $A^{\rm odd}_{\rm MT}$, is shown in Fig. 4, where the absolute difference

$$\Delta A = A^{\rm odd}_{\rm MT} - A^{\rm odd} = \frac{2\alpha}{\pi}\left(\ln\frac{1}{\rho}\ln\frac{s}{Q^2} + \frac{1}{2}\ln^2\rho\right) \quad (20)$$

between the two calculation is shown. The present results are in general smaller, except at small angles, where they are comparable. The main difference is due to the term related to $\ln(Q^2/s)$, which gives a different ΔE-dependent contribution.

V. CONCLUSIONS

Charge asymmetry in electron-proton elastic scattering contains essential information on the contribution of the real part of the 2γ exchange to the reaction amplitude. This amplitude can shed light on Compton scattering of virtual photons on the proton. It contains a part corresponding to a proton intermediate state, which carries the information on proton FFs. Another term corresponds to excited nucleon states and inelastic states such as $N\pi$, $N2\pi$, and $N\bar{N}N$. Their theoretical investigation is strongly model dependent. Taking into account the FFs in the TPE amplitude only is inconsistent, because the amplitude becomes non-gauge-invariant. Only the consideration of FFs in both inelastic intermediate states and the TPE amplitude is consistent.

Similar charge and angular asymmetry effects can also be due to Z-boson exchange, but such a contribution is small for moderate- to high-energy colliders. The ratio of the corresponding contributions can be evaluated as $\sim (\pi g_V g_A s)/(\alpha M_Z^2) < 5 \times 10^{-3}$ for $s < 10\ \mathrm{GeV}^2$ [where g_V (g_A) is the vector (axial) coupling constant of the Z boson to the fermion].

The analytical calculation of the 2γ amplitude with FFs encounters mathematical difficulties. In Ref. [12] the results for the box amplitude with arbitrary FFs was investigated. Other works use different approaches to include FFs, and the results are quantitatively different [3,11].

The analytical model presented in this paper is based on two main assumptions:

(i) the separation of the nucleon FFs into two terms, one of which corresponds to the long-distance contribution, which contributes explicitly to a narrow region of transferred momenta, close to zero, and
(ii) the mutual compensation of the short-distance interaction contributions to the TPE amplitude, arising from nucleon form factors and from inelastic states.

The last assumption, which has been proved in QED and which holds for zero scattering angle amplitude, has to be considered as an approximation when applied to the high-energy limit of large-angle scattering. The tendency of the elastic proton state and the Δ resonance in the real part of the TPE amplitude to cancel was previously noted in the literature [13], so it can be considered as indirect confirmation of our model.

The last two terms in Eq. (11) are similar to the kinematical conditions that were considered in Refs. [11,14] (one photon is almost on-mass shell and the second carries the whole momentum), but in the framework of our model we include all the phase volume and also the contributions from the inelastic intermediate states.

The numerical results obtained here are in agreement with our previous calculation [2,4].

Other works [13] devote attention to the excited intermediate states such as Δ and N^* resonances, introducing additional uncertainties. In our approach, excited states should not be included, as they correspond to poles in the second physical sheet. Only contributions from $n\pi N$ states, with any number of pions, are included.

Our main assumption about the compensation of pure strong-interaction-induced contributions to FFs and inelastic channels allows us to avoid additional uncertainty connected with inelastic channels. Experiments measuring charge-odd observables in ep scattering will be critical for the verification of the validity of our model.

The numerical results show that charge-odd correlations are of the order of a few percent, in the kinematical region considered here. Such a value is expected to be larger at larger Q^2 values and could be measured in very precise experiments at present facilities.

ACKNOWLEDGMENTS

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APPENDIX

The calculation of the integral

$$I = \int \frac{\mathrm{d}^4 k}{i\pi^2} \frac{1}{(k)(e)(p)} \theta\left(m_0^2 - |k^2|\right) \tag{A1}$$

is performed by using Sudakov's momentum parametrization. For this we build two (almost) lightlike vectors

$$\begin{aligned} &\tilde{p}_1 = p_1 - p\frac{m^2}{s}, \quad \tilde{p} = p - p_1\frac{M^2}{s}, \quad s = 2pp_1, \\ &\tilde{p}_1^2 = 0, \quad \tilde{p}^2 = 0, \\ &2\tilde{p}p = M^2, \quad 2\tilde{p}_1 p_1 = m^2. \end{aligned} \tag{A2}$$

The integration is done over the momentum variables:

$$k = \alpha\tilde{p} + \beta\tilde{p}_1 + k_\perp, \quad k_\perp p = k_\perp p_1 = 0, \quad k^2 = s\alpha\beta - \vec{k}^2,$$

and the phase volume is

$$d^4 k = \frac{s}{2} d\alpha d\beta d^2\vec{k}.$$

The analysis of the location of the poles of the denominators in α and β planes,

$$\begin{aligned} &(k) = k^2 - \lambda^2 = s\alpha\beta - \vec{k}^2 - \lambda^2 + i0, \\ &(e) = (k - p_1)^2 - m^2 + i0 = s\alpha\beta - \vec{k}^2 - s\alpha - m^2\beta + i0, \\ &(p) = (k + p)^2 - M^2 + i0 = s\alpha\beta - \vec{k}^2 + s\beta + M^2\alpha + i0, \end{aligned} \tag{A3}$$

leads to two regions of nonzero contribution, $0 < \alpha,\ \beta < 1$ and $-1 < \alpha,\ \beta < 0$, with equal contributions. We obtain

$$\begin{aligned} I = 2\int_0^1 d\alpha \int_0^1 d\beta \frac{d\vec{k}^2}{i\pi} \frac{1}{s\alpha\beta - \vec{k}^2 - \lambda^2 + i0} \frac{1}{-s\alpha - m^2\beta} \\ \times \frac{1}{s\beta + M^2\alpha} \theta\left(m_0^2 - |k^2|\right). \end{aligned} \tag{A4}$$

The real part of I (which is relevant for us) can be extracted by using the identity $1/(x + i0) = \mathcal{P}(1/x) - i\pi\delta(x)$ and performing the integration on $\vec{k}^2$:

$$\int_0^{m_0^2} dz\delta(z - a) = \theta(a), \quad a = s\alpha\beta - \lambda^2, \quad m_0^2 > a > 0.$$

The final answer is

$$\mathrm{Re}I = 2\left[\frac{1}{2}\ln^2\frac{s}{mM} + \ln\frac{s}{mM}\ln\frac{mM}{\lambda^2} - \frac{1}{2}\ln^2\frac{M}{m} - \frac{\pi^2}{2}\right]. \tag{A5}$$

It is interesting to compare this result with the same integral without the cut on $|k^2|$:

$$\begin{aligned} J &= \mathrm{Re}\int \frac{d^4 k}{i\pi^2} \frac{1}{(k)(e)(p)} \\ &= \ln^2\frac{s}{mM} - \ln^2\frac{M}{m} + 2\ln\frac{s}{mM}\ln\frac{mM}{\lambda^2} - \frac{4\pi^2}{3}. \end{aligned} \tag{A6}$$

By using similar expressions for another integral of this type, it is straightforward to recover Eq. (19).

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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

THE TWO-PHOTON EXCHANGE AMPLITUDE IN ep AND $e\mu$ ELASTIC SCATTERING: A COMPARISON

E. A. Kuraev

Joint Institute for Nuclear Research, Dubna

E. Tomasi-Gustafsson[1]

DSM, IRFU, SPhN, Saclay, Gif-sur-Yvette, France

In this note we give arguments in favor of the statement that the contribution of the box diagram calculated for electron–muon elastic scattering can be considered an upper limit to electron–proton scattering. As an exact QED calculation can be performed, this statement is useful for constraining model calculations involving the proton structure.

Показано, что вклад от диаграмм Фейнмана с двухфотонным обменом в амплитуду упругого электрон-мюонного рассеяния можно рассматривать как верхнюю границу для соответствующей амплитуды электрон-протонного рассеяния. Поскольку для первого случая расчет может быть выполнен явно в рамках квантовой электродинамики, это утверждение полезно для расчетов в рамках моделей, описывающих структуру протона во втором случае.

PACS: 25.30.Bf, 13.40.-f, 13.40.Gp

The problem of the two-photon exchange amplitude (TPE) contribution to elastic electron–proton scattering amplitude has been widely discussed in the past, and the main attention was devoted to single-particle polarization effects [1]. This amplitude has in principle a complex nature. Experimentally its real part, more exactly the real part of the interference between one- and two-photon exchange, can be obtained from electron–proton and positron–proton scattering in the same kinematical conditions. A similar information in the annihilation channel (electron–positron annihilation into proton–antiproton and in the reversal process) can be obtained from the measurement of the forward-backward asymmetry in the angular distribution of one of the emitted particles in the reaction center-of-mass (CMS) system.

Recently, a lot of attention was devoted to the two-photon exchange amplitude (TPE) in electron–proton elastic scattering as a possible solution to a discrepancy between polarized and unpolarized measurements devoted to the determination of the proton form factors [2]. Whereas no experimental evidence has been found on the presence of TPE effects (real part) in non-linearities of the Rosenbluth fit, for example [3], the imaginary part is responsible for beam transverse single-spin asymmetry, which although very small, of the order

[1]E-mail: etomasi@cea.fr

10^{-5}–10^{-6} (ppm), but has been experimentally measured by experimental collaborations, and firstly reported in [4].

These results triggered new theoretical work and the asymmetry at first order of perturbation theory was calculated by several groups, see, for instance, [5]. Inelastic intermediate states of the TPE amplitude give rise to contributions containing the square of large logarithm (logarithm of the ratio of the four momentum squared to the electron mass squared). At higher order of perturbation theory, such contributions were calculated in [6] where it was shown that they cannot be neglected.

The contribution of the interference of the Born amplitude with the imaginary part of TPE, which is responsible for one-spin asymmetry, is proportional to the electron mass. Therefore, its presence does not contradict the Kinoshita–Lee–Nauenberg theorem [7] about cancellation of mass singularities, since the corresponding cross sections are suppressed by the lepton mass.

The theoretical description of TPE amplitude is strongly model-dependent, as it involves modeling of the proton and of its excited states, but it is still possible to derive rigorous results and predict exact properties of the two-photon box: model-independent statements based on symmetry properties of the strong and electromagnetic interaction have been suggested in [8, 9]. It has been proved that, due to C-parity conservation, the amplitude for $e^+ + e^- \to p + \bar{p}$, taking into account the interference between one- and two-photon exchange, is an odd function of $\cos\theta$, where θ is the angle of the emitted proton in the CMS of the reaction.

For the scattering channel, the interference between one- and two-photon exchange amplitudes induces a kinematical term $\sqrt{(1+\epsilon)/(1-\epsilon)}$ in the «reduced cross section», $\epsilon^{-1} = 1 + 2(1+\tau)\tan^2(\theta_e/2)$, where θ_e is the electron scattering angle, in the lab. system. This term violates the commonly accepted linear behavior of the reduced cross section $\sigma_{\rm red} = a + b\epsilon$. This property must be satisfied by all model calculations.

A second possibility is to do an exact calculation of the box diagram, which is possible for electron–electron and electron–muon scattering, and in the crossed channel (i.e., replacing the proton with a lepton) [10], where the muon can be considered a structureless proton. Even if such a calculation is not rigorous when applied to the interaction on proton, the interest of a pure QED calculation is that the results should be considered as an upper limit for any calculation involving protons, as it will be discussed in this work.

The discussion of TPE box diagram in ep scattering cannot be restricted to one-proton intermediate state, but inelastic amplitudes should be consistently taken into account. Concerning the real part, the contributions to the amplitude from the proton on one side and from the inelastic intermediate states on the other side, are not gauge-invariant, if considered separately. Only their sum is gauge-invariant: the Ward–Takahashi identities relate the vertex function and the nucleon Green function [11]. On the contrary, for the imaginary part, these contributions are separately gauge-invariant, as the intermediate nucleon is on shell, as well as the external nucleons, therefore they must have comparable values.

Analyticity arguments lead to a (almost complete) compensation of elastic and inelastic contributions in the whole amplitude. This statement is rigorous in QED [12], and has been recently extended to electron–hadron scattering at small scattering angles. Moreover, based on such a statement, sum rules which relate peripheral cross section and elastic form factors have been derived in QCD and their validity verified on experimental data (see [13] and refs. therein). Therefore, one can state that elastic and inelastic contributions are of the same order of magnitude, which is sufficient for our aim here.

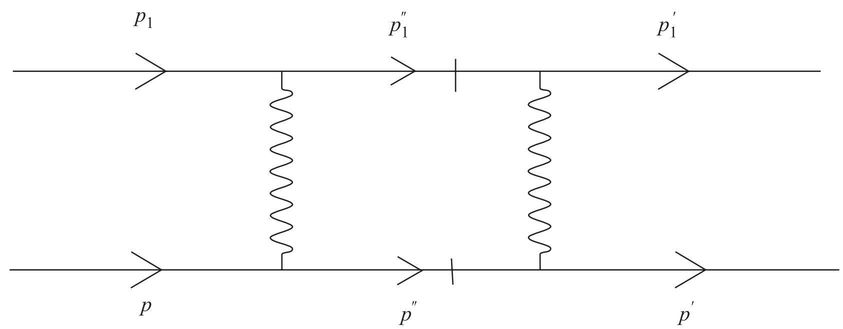

Fig. 1. s-channel discontinuity of the Feynman box amplitude for $e\mu$ scattering

The notion of «nucleon form factors (FF)» cannot be applied to the two-photon exchange amplitude (TPE) since one of the nucleons is off mass shell. Nevertheless, the s-channel imaginary part of TPE, which corresponds to a single on mass shell nucleon and on mass shell electron in the intermediate state, can be analyzed in terms of FFs. Moreover, it provides the gauge-invariant contribution to the imaginary part of the whole TPE. We build a simple model, calculating the $e\mu$ box Feynman diagram with one muon (nucleon) in the intermediate state. For the proton case, the muon mass is taken equal to the proton mass, and the proton structure is described by form factors.

We can neglect the spin dependence and we calculate scalar four-dimensional integrals with point-like particles (in case of $e\mu$ scattering), and including proton form factors (for ep scattering). A complete calculation was performed in [14] where similar scalar Feynman integrals with three and four denominators are involved.

Our aim is not to do a complete calculation of the box diagram, but to find an upper limit of this term: in every step, one should compare the relevant integrals. The purpose of this note is to prove that modeling of the proton by Q^2 decreasing form factors leads to a smaller contribution of the box diagram, compared to the QED case. We will prove this statement for the imaginary part of the amplitude corresponding to the box diagram with one proton line connecting two γpp vertexes and the validity for the relevant part of the full amplitude, $\mathcal{A}$, can be inferred through dispersion relations:

$$\mathcal{A}(s,t) = \frac{1}{\pi}\int \frac{ds' \operatorname{Im}\mathcal{A}(s',t)}{(s'-s-i\epsilon)}. \tag{1}$$

Let us consider the cases where the target T is a proton or a muon (Fig. 1) with the following convention for the particle four momenta:

$$e(p_1) + T(p) \to e(p_1'') + T(p'') \to e(p_1') + T(p'). \tag{2}$$

The following kinematical relations hold in the center-of-mass frame:

$$p_1 + p = p_1' + p', q = p_1 - p_1', p_1' + p' = p_1'' + p'', q_1 = p_1 - p_1'', q_2 = p_1'' - p_1',$$

$$Q_1^2 = -q_1^2 = -(p_1 - p_1'')^2 = 2(\mathbf{p})^2(1-c_1),$$
$$Q_2^2 = -q_2^2 = -(p_1'' - p_1')^2 = 2(\mathbf{p})^2(1-c_2),$$
$$Q^2 = -q^2 = -(p_1 - p_1'')^2 = 2(\mathbf{p})^2(1-c),$$

where $c_1 = \cos\theta_1$, $c_2 = \cos\theta_2$, $c = \cos\theta$, and $\theta_1 = \widehat{\mathbf{p}_1\mathbf{p}_1''}$, $\theta_2 = \widehat{\mathbf{p}_1''\mathbf{p}_1'}$, and $\theta = \widehat{\mathbf{p}_1\mathbf{p}_1'}$. The momenta carried by the virtual photons are $q_1 = k$ and $q_2 = q - k$.

The contribution to the Feynman amplitude corresponding to the diagram of Fig. 1 can be written as

$$\mathcal{M} = \frac{1}{(2\pi)^2}\int\frac{\mathcal{N}d\Gamma}{(Q_1^2+\lambda^2)(Q_2^2+\lambda^2)}, \tag{3}$$

where $\mathcal{N} = F(Q_1^2)F(Q_2^2)(\mathcal{N} = 1)$ for $ep(e\mu)$ scattering, λ is a fictitious photon mass and $d\Gamma$ is the phase volume of the loop intermediate state. The notation $F(Q_1^2)F(Q_2^2)$ indicates the bilinear combination of the Dirac F_1 and Pauli F_2 nucleon form factors.

Taking into account the fact that the intermediate particles are on shell, one can write for the proton case:

$$d\Gamma = d^4p_1''\delta(p_1''^2 - m^2)\delta(p''^2 - M^2)d^4p''\delta^4(p_1 + p - p_1'' - p'') =$$
$$= \frac{d^3p_1''}{2\epsilon_1''}\frac{d^3p''}{2\epsilon''}\delta^4(p_1 + p - p_1'' - p'') = \frac{d^3p_1''}{4\epsilon_1''\epsilon''}\delta(\sqrt{s} - \epsilon_1'' - \epsilon''),$$
$$\epsilon_1'' = \frac{s - M^2}{2\sqrt{s}}, \quad \epsilon'' = \frac{s + M^2}{2\sqrt{s}}. \tag{4}$$

Finally, one can write

$$d\Gamma = \frac{s - M^2}{8s}dO_1'', \tag{5}$$

where dO_1'' is the solid angle of the electron in the intermediate state, which can be expressed as a function of the angles defined above as

$$dO_1'' = \frac{2dQ_1^2\,dQ_2^2}{\sqrt{\mathcal{D}_1 Q_0^2}}, \quad \mathcal{D}_1 = 2(Q_1^2+Q_2^2)Q^2Q_0^2 - 2Q^2Q_1^2Q_2^2 - (Q_1^2-Q_2^2)^2Q_0^2 - (Q^2)^2Q_0^2, \tag{6}$$

with the relation $Q_0^2 = 2\mathbf{p}^2 = (s - M^2)^2/(2s)$. The positivity of the function $\mathcal{D}$ defines the solid angle kinematically available for the reaction.

Therefore, one can write the contributions corresponding to the «QED» diagram in Fig. 1, in case of a muon target:

$$\mathcal{M}_\mu = \frac{1}{\sqrt{8s}}\int\frac{dQ_1^2\,dQ_2^2}{\sqrt{\mathcal{D}_1}(Q_1^2+\lambda^2)(Q_2^2+\lambda^2)}. \tag{7}$$

Introducing a generalized form factor for the proton, one finds for the «QCD» diagram of Fig. 1, in case of a proton target:

$$\mathcal{M}_p = \frac{1}{\sqrt{8s}}\int\frac{dQ_1^2\,dQ_2^2\,F(Q_1^2)\,F(Q_2^2)}{\sqrt{\mathcal{D}_1}(Q_1^2+\lambda^2)(Q_2^2+\lambda^2)}. \tag{8}$$

We imply that both amplitudes are similarly infrared regularized, which, for the purpose of our paper, is equivalent to consider λ as a finite quantity. Therefore, the condition $F(Q_1^2)\,F(Q_2^2) < 1$ is equivalent to the statement that the value of the electron–muon scattering amplitude can be considered an upper estimation of the amplitude for electron–proton scattering.

Nucleon form factors are functions which are rapidly decreasing with Q^2. The Pauli and Dirac form factors, F_1 and F_2, are related to the Sachs form factors by

$$F_1(Q^2) = \frac{\tau G_M(Q^2) + G_E(Q^2)}{\tau + 1}, \quad F_2(Q^2) = \frac{G_M(Q^2) - G_E(Q^2)}{\tau + 1}, \quad \tau = \frac{Q^2}{4M^2}, \tag{9}$$

with the following normalization: $F_1(0) = 1$, $F_2(0) = \mu_p - 1 = 1.79$, where μ_p is the magnetic moment of the proton in units of Born magneton.

Let us consider the dipole approximation as a good approximation at least for the magnetic proton form factor G_M, although it has been shown that the electric form factor G_E deviates from the dipole form [2]. In any case, any parametrization closer to the data will give even lower values as compared to the dipole form. In this approximation, we have

$$\begin{gathered} F_1^D(Q^2) = \frac{(\tau\mu_p + 1)G_D(Q^2)}{\tau + 1}, \quad F_2^D(Q^2) = \frac{(\mu_p - 1)G_D(Q^2)}{\tau + 1}, \\ G_D(Q^2) = [1 + Q^2(\text{GeV})^2/0.71]^{-2}. \end{gathered} \tag{10}$$

In Fig. 2 we show $F_1(Q^2)$ (solid line), $F_2(Q^2)$ (dashed line), which are smaller than unity practically overall the Q^2 range. The product $F_1(Q_1^2)F_1(Q_2^2)$ is shown in Fig. 3 as a bidimensional plot, and in Fig. 4, as a projection on the Q_1^2 axis for $Q_2^2 = 0.05$ GeV^2 (solid line), $Q_2^2 = 1.2$ GeV^2 (dashed line), $Q_2^2 = 2$ GeV^2 (dotted line).

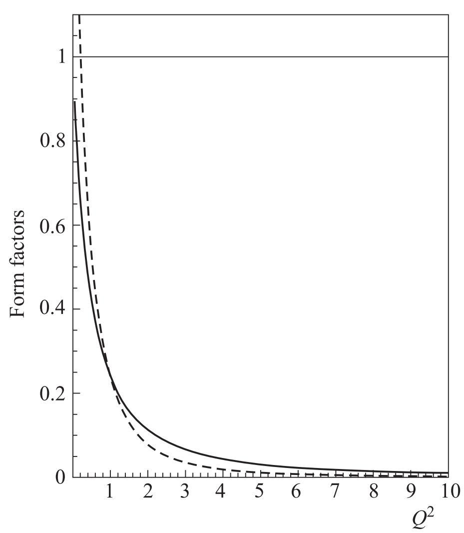

Fig. 2. Form factors as a function of Q^2: solid line — $F_1(Q^2)$, dashed line — $F_2(Q^2)$

One can see that the condition $F(Q_1^2)F(Q_2^2) < 1$ is satisfied, starting from very low values of Q^2. Let us stress that $F_1(Q^2)$ is normalized to 1 and decreases with Q^2, being therefore smaller than unity; in the expression of the hadronic current, $F_2(Q^2)$ is multiplied by q_μ, which lowers its contribution at small Q^2, whereas at larger Q^2 it does not compensate the steep Q^{-6} behavior of this form factor, as expected from quark counting rules [16]. This is the reason why we can replace the bilinear combination $F(Q_1^2)F(Q_2^2)$ by $F_1(Q_1^2)F_1(Q_2^2)$.

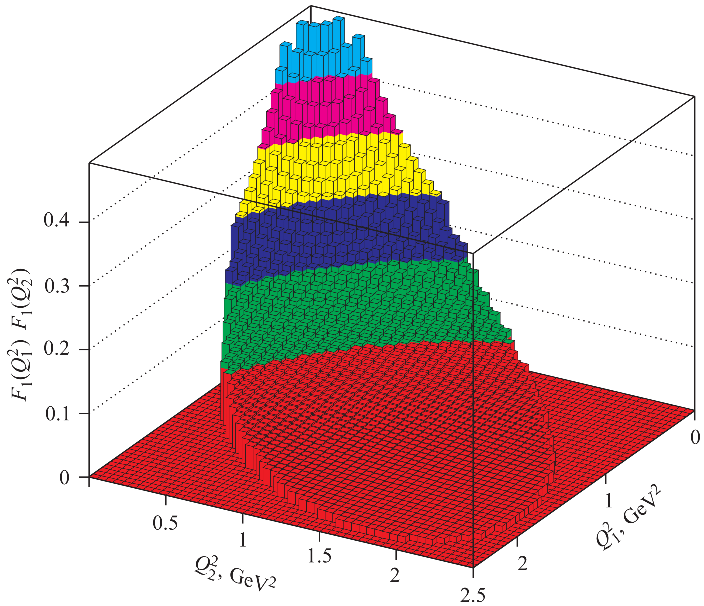

Fig. 3. Bidimensional plot of $F_1(Q_1^2)F_1(Q_2^2)$ as a function of Q_1^2 and Q_2^2

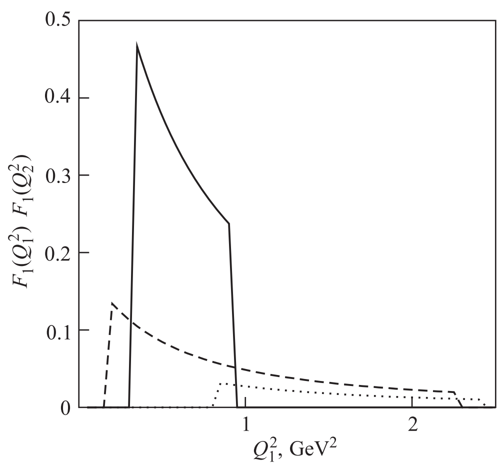

Fig. 4. Projection on $F_1(Q_1^2)F_1(Q_2^2)$ on the Q_1^2 axis for $Q_2^2 = 0.05$ GeV^2 (solid line), $Q_2^2 = 1.2$ GeV^2 (dashed line), $Q_2^2 = 2$ GeV^2 (dotted line)

Furthermore, we note that a destructive interference of the contributions of the single proton and its excited states takes place in (1), which results in additional suppression of ep amplitude compared to $e\mu$. A mutual compensation of the amplitudes for «elastic» proton intermediate state with the excited hadronic states exists, and the reason lies in the superconvergent character of the dispersion relation (1), where the total amplitude is implied. Indeed, considering the amplitude for virtual Compton scattering in the complex s plane, closing the integration contour to the right-hand singularities (which correspond to the proton intermediate state (pole) and to excited hadron states (cuts)) a compensation takes place, up to the small contribution of the left-hand cut. Details are given in [13]. Therefore, all model calculations for ep elastic scattering as [15] should result in smaller contribution of the two-photon amplitude, as compared to QED calculations [10].

In conclusion, let us note that any (quantitative) application of our considerations to polarization phenomena is outside the purpose of this paper. Our statements were done for the imaginary part of the scattering amplitude, and extended to the full amplitude with the help of dispersion relations. As far as the real amplitude of ep scattering is concerned, the whole TPE amplitude is smaller than the contribution of the one-proton intermediate state of the TPE amplitude, due to a compensation of elastic and inelastic states. Reasons in favor of this cancellation were given in the literature [13]. But, even if we neglect the compensation effects, the QED $e\mu$ amplitude dominates the relevant one-proton real ep TPE amplitude, due to the steep falling of form factors. Our statement, about the QED dominance, is relevant to the whole ep amplitude. This is the main conclusion of the present work.

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