Separation of the T-violating amplitude in the p-resonance of a compound nucleus on the phase shift of polarizing ability of a target

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The phase of the azimuthal component of a neutron spin interaction with a target is the sum of two angles. The tangent of the first angle is equal to the ratio of the *T*-violating amplitude *D* to the weak interaction amplitude C; the tangent of the second angle depends on the spin rotation in the residual pseudomagnetic field. The second angle has different signs in measurements with polarized and unpolarized neutrons and two measurements allow it to be compensated for. In the case of an unpolarized target, it is possible to find the phase of the azimuthal component relative to the phase of the weak interaction field.. The phase spectra of neutrons measured with polarized and unpolarized neutrons at a p-wave resonance on a polarized target allow the separation of the ratio *D/C*.
The algorithm described for separating the ratio *D/C* takes into account the influence of fringing fields of the Ramsey coil magnet and the target magnet.

1. Introduction

The presence of an electric dipole moment (EDM) of elementary particles or atoms testifies to a simultaneous violation of parity conservation and time-reversal invariance (P, T violation). The fundamentality of these symmetries has stimulated almost half a century of neutron EDM measurements, ever since the first Ramsey experiment was carried out in 1957[1]. The current upper limit on the EDM of the neutron is $d_n \leq 2.9 \bullet 10^{-26}$ *e cm* [2, 3]. The interaction resulting in such an EDM creates an energy splitting of the two spin states of 10^{-21} eV.

Search for a time-reversal symmetry violation in nuclear physics also has a fundamental significance. The experiment most widely discussed in the literature of the last 25 years is one on the passage of neutrons through a polarized target near the p-wave resonance in ^{139}La, where the strong dynamic enhancement of the weak interaction results in a 10% asymmetry in the counting rate of neutrons [4-6]. Theory predicts the same order of enhancement for a time-reversal violating interaction [7].

Just as for a neutron EDM this interaction of a neutron with a nucleus is *P*-odd and *T*-violating. The expected energy splitting between the two spin states of a neutron in this case is 10^{-15} eV. Excess of a *T*- violating effect of six orders of magnitude in energy in comparison with a neutron EDM shows that an experiment with the passage of neutrons through a sample of polarized ^{139}La should be less problematic than measurement of an EDM.

The detection of an asymmetry due to time reversal violation in ^{139}La is made difficult by virtue of the fact that the T-violating interaction takes place under a background of strong (energy scale 10^{-8} eV) and weak (10^{-12} eV) interactions.

In a review of the problem [8], the authors conclude that it is unlikely that the T-violating amplitude in ^{139}La can be measured in a classical polarization experiment.

Therefore, new approaches are necessary for designing an experiment. In this connection, we discuss the possibility of measuring the T-violating amplitude near the point of maximal interference of the strong interaction with the T-violating field. The purpose of the work reported here is to provide a description of the method for separating the ratio of the T-violating amplitude to the P-odd amplitude from the phase spectra of neutrons. The basic idea of a registration of the phase spectra is presented in Ref. [9].

2. Spin dependent neutron interaction with target.

From a neutron spin, its wave vector and the spin of a nucleus it is possible to build the following correlations:

1	2	3	4
$(\boldsymbol{\sigma}\cdot \boldsymbol{I})$,	$(\boldsymbol{\sigma}\cdot \boldsymbol{k})$,	$(\boldsymbol{k}\cdot\boldsymbol{I})$	$(\boldsymbol{\sigma}\cdot [\boldsymbol{k}\times\boldsymbol{I}])$
B'	C	C'	D

The first correlation corresponds to the strong spin-spin interaction B'; the second correlation is P-odd and describes a weak interaction with amplitude C.

The third correlation is also P-odd and manifests in nature as a weak interaction of the neutron with the polarized nucleus. An example of this is the measurement of the asymmetry in the counting rate of neutrons moving along and against the direction of nuclear polarization of ^{139}La. This asymmetry is three times more than the asymmetry arising under action of a field C [10].

The action of field D leads to the fourth P, T-violating correlation that is the subject of the present study.

Let us choose a system of coordinates in which the nuclear polarization $\boldsymbol{I}$ is directed along the $\boldsymbol{z}$ axis, the wave vector $\boldsymbol{k}$ along the $\mathbf{y}$ axis and the T-violating field D along the $\boldsymbol{x}$ axis.

According to [9], it is possible to find the value of D from the phase spectra of neutrons measured in expanded setups modification of experiment that allows excluding the masking effects accompanying field D. In addition the calibration of a phase scale on weak interaction and a new way of separating the T-violating field by a measurement of the ratio ImD/ImC is considered.

In the first setup the neutrons polarized along the $\boldsymbol{z}$ axis pass through the radio-frequency Ramsey coil placed in an external magnetic field. At resonance, that is when the frequency of spin precession in an external field coincides with the radio-frequency ω, the neutron spin rotates by $\pi/2$. The distribution of spins in the $\boldsymbol{xy}$ plane depends on the phase of the field δ at the moment of entry of a neutron into the coil.

The transformation of the spin components by the coil is described by the matrix (1.3) shown in appendix 1.

Spins of neutrons scan all directions in this plane, and when a neutron is detected the phase of the rotating field of the coil is also recorded. The fixed phase differs from the initial one by the product of the frequency and the neutron time-of-flight from the coil to the detector. As a result, the counting rate of neutrons is a function of this phase and is distributed in an interval from 0 to 2π. The target in this case plays the role of the analyzer. This experimental configuration we shall designate as PCTD (polarizer, coil, target, detector).

For the second measurement, the target is a polarizer and the neutrons impinging on it are not polarized. After passage through the target, the beam of neutrons gets a polarization. Neutrons then pass through the Ramsey coil, the analyzer and hit the detector. This is the configuration TCAD (target, coil, analyzer, detector).

For each configuration calibration spectra are measured with the target polarization switched off. In this case the direction of the weak interaction field in the target is precisely known (or the phase angle of a field is precisely known), wqhich enables the phase scale to be calibrated and the phase shift of the field with a polarized target to be found.

3. Neutron spectra

The following expression describes a spectrum with initially polarized neutrons (configuration PCTD):

$$N_{pct}(\delta,t)=\frac{1}{2}Tr(U_t U_{ct} U_c \rho_p U_c^+ U_{ct}^+ U_t^{\ +}), \tag{1}$$

where the subscripts on the evolution operators correspond to the coil (c), the coil-target distance (ct) and the interaction with the target (t), and $\rho_p = (1 + \boldsymbol{\sigma} \cdot \boldsymbol{p_p})/2$ is the spin density matrix of the polarizer, normalized to unit neutron flux.

The corresponding expressions for the configuration TCAD are:

$$N_{tca}(\delta,t)=\frac{1}{2}Tr(\rho_a U_c U_{tc} U_t U_t^+ U_{tc}^+ U_c^{\ +}). \tag{2}$$

Here ρ_a is the density matrix of the analyzer, and the symbols a, c, t refer as before to the analyzer, coil and target.

The expressions for these spectra have the following form:

$$N_{pct(tca)}(\delta,t)=\frac{1}{2}\exp(-At)[N_0 + p_{p(a)} p'_{a(p)} \cos(\alpha_{pct(tca}))] \tag{3}$$

In (3) the exponent describes the spin-independent absorption of neutrons in the target, where N_0 takes into account the change in the normalization of the density matrix due to the absorption of neutrons; t is the time-of-flight through the target.

The quantity $p'_{a(p)}=\sqrt{{p'_{xa(p)}}^{2}+p'^{2}_{ya(p)}}$ is the azimuthal component of the analyzing (polarizing) power of the target, which has the angular orientation $\beta_{a(p)} = arctg(p'_{ya(p)} / p'_{xa(p)})$.

The angle α in (3) is the angle between vectors $\boldsymbol{p}$, $\boldsymbol{p'}$. This angle has a different meaning in the two cases.

$\alpha_{pct} = \delta + \alpha_{ct} - \beta_a$, where α_{ct} is the spin rotation in the coil-target region including the coil and δ is the phase of the rotating field at the moment of the entry of a neutron into the coil.

For the TCA mode this angle $\alpha_{tca} = \beta_p + \alpha_{tc} - \delta$. Here α_{tc} is the spin rotation in the field between the target and the coil.

Let us assume that the gyromagnetic ratio for the neutron is equal to one. In this case, the intensity of the magnetic field, as well as the fields in the target, and the precession frequency have the same dimension.

We show further expressions for the analyzing and polarizing characteristics of the target, using the notation of [8]:

$$p'_{xa(p)} = 2\,\mathrm{Im}(b'^*_x \cos qt) \pm 2\,\mathrm{Im}(b'_y b'^*_z)\,, \tag{4}$$

$$p'_{ya(p)} = 2\,\mathrm{Im}(b'^*_y \cos qt) \pm 2\,\mathrm{Im}(b'_z b'^*_x)\,, \tag{5}$$

$$p'_z = 2\,\mathrm{Im}(b'^*_z \cos qt)\,. \tag{6}$$

The second terms in (4) and (5) represent an interference of the fields *C* and *D* with the residual pseudo-magnetic field. Expression (6) would also contain such a term but, because of the resonant behavior of *C* and *D*, this term is equal to zero. The terms in (4)-(6) are shown in more detail in appendix 2.

In relations (4), (5) the upper signs correspond to the target playing the role of analyzer. The lower signs correspond to the case where non-polarized neutrons fall on the target and the target is a polarizer.

The value of $\boldsymbol{b}' = \boldsymbol{b}\,(\sin(qt))/q$, $q = (\boldsymbol{b}\cdot\boldsymbol{b})^{1/2}$.

The vector $\boldsymbol{b}$ describes the resulting field of the target and has the following components: $b_x = p_t D/2$, $b_y = C/2$ and $b_z = B/2$.

The value N_0 in (3) is :

$$N_0 = \frac{|b|^2}{|q|^2}|\sin(qt)|^2 + |\cos qt|^2\,. \tag{7}$$

As the target is in an external magnetic field the residual field is equal to $\mathrm{Re}\,2b_z = \mathrm{Re}\,B = p_t\,\mathrm{Re}\,B' - H$, where p_t is the target polarization, and $\mathrm{Re}\,B'$ is the nuclear constant.

Now we can find the angular orientation of the analyzing (polarizing) power of the target, supposing that the interaction b_x has the same resonant behavior as b_y.

$$tg\beta_{a(p)} = tg(\beta_1 \pm \beta_2), \tag{8}$$

where $tg\beta_1 = \dfrac{\mathrm{Im}(b'_z b'^*_x)}{\mathrm{Im}(b'_y b'^*_z)} = -\dfrac{\mathrm{Im}\,b_x}{\mathrm{Im}\,b_y}$ and $tg\beta_2 = \dfrac{\mathrm{Im}(b'^*_y \cos qt)}{\mathrm{Im}(b'_y b'^*_z)}$.

Here we use the equality $\dfrac{\mathrm{Im}(b'^*_x \cos qt)}{\mathrm{Im}(b'^*_y \cos qt)} = \dfrac{\mathrm{Im}\,b_x}{\mathrm{Im}\,b_y}$, which follows from $\dfrac{\mathrm{Re}\,b_x}{\mathrm{Im}\,b_x} = \dfrac{\mathrm{Re}\,b_y}{\mathrm{Im}\,b_y}$ and the expressions (2.1-2.3) of appendix 2.

The hierarchy of thespin-dependent interactions is *B*>> *C*>> *D* and each subsequent interaction is between three and four orders of magnitude less than the previous one.

Let us consider the compensation of the pseudomagnetic field when the following condition is satisfied:

$$ReB \approx Re2q >> Im2q,\ C,\ D. \tag{9}$$

Then expressions (4), (5) become:

$$p'_{xa(p)} \approx -ap_t \operatorname{Im} Dt \pm b \operatorname{Im} Ct\,, \tag{4a}$$

$$p'_{ya(p)} \approx -a \operatorname{Im} Ct \mp bp_t \operatorname{Im} Dt\,, \tag{5a}$$

$$p'_z \approx -ap_t \operatorname{Im} Bt\,. \tag{6a}$$

Here the factor $a = \sin(ReBt)\,/ReBt$ specifies the reduction of fields C and D depending on the precession of the spin in the residual pseudomagnetic field. The factor $b = \dfrac{2\sin^2(\operatorname{Re} Bt/2)}{\operatorname{Re} Bt}$ shows how the spin precession changes the interference term.

Due to the hierarchy of forces, there are two limiting cases: $qt << 1$ and $qt \geq 1$. In the former case, when interference contributions are minimized, $a \approx 1$, $b \approx \operatorname{Re} Bt/2$ and measurement of the field D is made in the direction of the $\boldsymbol{x}$ axis. In most discussions of the separation of the T-violating field, this limiting case is studied under condition (9). We note that in the case of full compensation of the pseudomagnetic field the interference contribution does not disappear entirely.

The actions of fields C, D are minimal when $qt \geq 1$, but in this case interference contributions become enhanced. For an experiment, this case seems preferable due to the enhancement interference of the action of the field D.

The dependences of a, b on the angle of spin rotation in a residual field are shown in figure 2. The equality $\operatorname{Re} Bt = tg(\operatorname{Re} Bt/2)$ corresponds to the positions of the interference maxima.

At angle $ReBt = \pi$, when the factor a=0, the action of the basic fields on the spin of the neutron are minimal and the spins interact with the interference fields. Field D is now along the $\boldsymbol{y}$ axis, and field C is along the $\boldsymbol{x}$ axis.

The value $p' = \sqrt{p'^2_x + p'^2_y} = c \operatorname{Im} Ct \sqrt{1 + (p_t \operatorname{Im} Dt / \operatorname{Im} Ct)^2} \approx c \operatorname{Im} Ct$ enters into the amplitude of spectra (3). The dependence of the factor c on the angle of spin rotation is also shown in figure 2.

We have the approximate value for the angle β_2.

$tg\beta_2 = ctg(\operatorname{Re} Bt/2) = tg(\pi/2 - \operatorname{Re} Bt/2)$, so that $\beta_{a(p)} = -p_t \dfrac{\operatorname{Im} D}{\operatorname{Im} C} \pm (\pi/2 - Bt/2)$.

The counting rate in expression (3) is maximal if the angular position of the neutron spin coincides with the phase β. However, the angular position of the neutron spin is not observed, so to find the field phase β it is necessary to calibrate the phase scale in each case. This can be done in following way. The polarization of the target and the target magnetic field are switched off. The fields B and D are proportional to the target polarization, thus they disappear and in the target there remains only the weak interaction field C which is along the $\boldsymbol{y}$ axis and has, according to figure 1, a phase $\boldsymbol{\beta} = \pi/2$. Concerning the calibration spectrum, the phase in the first expression (3) becomes $\Delta_1 = \alpha_{tl} - \beta_a$ as shown in figure 3. For the configuration TCAD this phase shift will be $\Delta_2 = \beta_p - \alpha_{tr}$. For minimization of the phase β_2, the angle of spin rotation in a residual pseudomagnetic field must be close to π. The sum of phase shifts Δ_1 and Δ_2 is $\Delta = 2p_t \frac{\operatorname{Im} D}{\operatorname{Im} C} + \alpha_{tl} - \alpha_{tr}$.

The difference of phases, due to the precession of the spin in the fringing fields of the target magnet can be the source of a systematic error and to eliminate it, it is necessary to carry out several rounds of measurements. In these the polarization of the target is changed in such a manner that the angle of spin precession in a residual field ranges from -π to π. By virtue of the linear dependence of Δ on p_t, we have:

$$\frac{\partial \Delta}{\partial p_t} = 2\frac{\operatorname{Im} D}{\operatorname{Im} C}.$$

Finally, we estimate the sensitivity of the described algorithm defining the phase shift β_1. For this purpose we note that expression (7) has another approximate form $N_0 = \frac{|b|^2}{|q|^2}|\sin qt|^2 + |\cos qt|^2 \approx \cosh(\operatorname{Im} qt) \approx 1$ due to the weak absorption of neutrons in the spin-dependent fields of the target. Thus the experimental spectra are described by the following expressions:

$N_1 = \frac{n}{2\pi}(1 + A\cos\alpha)$, $N_2 = \frac{n}{2\pi}(1 + A\cos(\alpha + \chi))$ for the non-polarized and polarized targets respectively. In these expressions n is the number of detected neutrons (identical in both spectra), and the asymmetry arising from the action of the weak interaction field is $A \approx 0.1$. Taking the difference of the two spectra and integrating from 0 to π gives $\chi = \frac{\pi}{2An}(n_1 - n_2)$. Then the root-mean-square deviation of the phase shift is (in view of the second half of the spectrum) $\sigma(\chi) = \frac{\pi\sqrt{2}}{2A\sqrt{n}}$. For χ we use the value $\chi \le 5 \cdot 10^{-4}$ from [13]. Then the statistics necessary for a measurement of the phase shift to 30% is $n \approx 2 \cdot 10^{10}$. Such statistics can be collected in one week at a neutron flux of 10^5 in the region of 1 eV.

5. **Conclusions**

The results obtained above show a preference for a separation of the *T*-violating field in the region of the first interference maximum, where the angle of the spin rotation in a residual pseudomagnetic field is close to $\pm\pi$.

The ratio $\operatorname{Im} D / \operatorname{Im} C$, representing the part of a phase azimuthal components of an effective field of a target, is separated from spectra measured with polarized and unpolarized neutrons. For each of these asymmetries the phase scale is calibrated with respect to the weak interaction.

Since the problem is reduced to measurements of phase shifts the strict equality of polarizer and analyzer efficiencies and high accuracies of their orientations are not required. It is shown that the fringing fields of the Ramsey coil magnets are taken into account by calibrations of the phase scale and are not a source of systematic errors. The algorithm described also allows the influence of the fringing fields of the target magnet on the result of the measurement of the ratio $\frac{\operatorname{Im} D}{\operatorname{Im} C}$ to be eliminated.

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Appendix 1

The evolution operator for a Ramsey coil at resonance, that is when the frequency of spin precession in the external field ω_0 coincides with the frequency of a rotating field ω, is:

$$U_c = \exp(-i\omega t\sigma_z)\exp(-i\omega_1 t(\boldsymbol{\sigma}\cdot\boldsymbol{n})), \tag{1.1}$$

where ω_1 is the frequency of spin precession in a field equal to the amplitude of the rotating field, $\boldsymbol{\sigma}$ is the Pauli matrix and $\boldsymbol{n}$ is the unit vector of the field direction at the moment of entry of the neutron into the coil, determined by the phase of the field δ:

$$n_x = \cos\delta,\ n_y = \sin\delta \ \text{and} \ n_z = 0.$$

This evolution operator transforms the components of a vector of polarization $\boldsymbol{p_0}$ as follows:

$$(\boldsymbol{\sigma}\cdot\boldsymbol{p_1}) = U_c\,(\boldsymbol{\sigma}\cdot\boldsymbol{p_0})\,U_c^{\dagger}. \tag{1.2}$$

The equality $(\boldsymbol{p_1}) = M\,(\boldsymbol{p_0})$ defines a three-dimensional matrix of polarization vector transformation. This matrix is:

$$M_c = \begin{pmatrix} \begin{matrix} cos(\omega t+\delta)\cos\delta + \\ +\cos\omega_1 t\sin\delta\sin(\omega t+\delta) \end{matrix} & \begin{matrix} cos(\omega t+\delta)\sin\delta - \\ -\cos\omega_1 t\cos\delta\sin(\omega t+\delta) \end{matrix} & \sin\omega_1 t\sin(\omega t+\delta) \\ \begin{matrix} \sin(\omega t+\delta)\cos\delta - \\ -\cos\omega_1 t\sin\delta\cos(\omega t+\delta) \end{matrix} & \begin{matrix} \sin(\omega t+\delta)\sin\delta + \\ +\cos\omega_1 t\cos\delta\cos(\omega t+\delta) \end{matrix} & -\sin\omega_1 t\cos(\omega t+\delta) \\ -\sin\omega_1 t\sin\delta & \sin\omega_1 t\cos\delta & \cos\omega_1 t \end{pmatrix}$$

For a neutron with energy E_0 (a maximum p-resonance) an angle of spin rotation is $\omega_1 t_0 = \pi/2$.

Appendix 2

Here we show in detail the expressions for the quantities in relations (5)-(7):

$$2\operatorname{Im}(b_i'^{*}\cos qt) = \frac{\sin(2\operatorname{Re} qt)}{|q|^2}\left(\operatorname{Re} b_i \operatorname{Im} q - \operatorname{Re} q \operatorname{Im} b_i\right) - \frac{\sinh(2\operatorname{Im} qt)}{|q|^2}\left(\operatorname{Re} b_i \operatorname{Re} q + \operatorname{Im} b_i \operatorname{Im} q\right), \tag{2.1}$$

where i = x, y, z.

$$2\operatorname{Im}(b_y' b_z'^{*}) = \frac{2\left[\sin^2(\operatorname{Re} qt) + \sinh^2(\operatorname{Im} qt)\right]}{|q|^2}\left(\operatorname{Re} b_z \operatorname{Im} b_y - \operatorname{Re} b_y \operatorname{Im} b_z\right). \tag{2.2}$$

$$2\,\mathrm{Im}(b_z' b_x'^{*}) = \frac{2\left[\sin^2(\mathrm{Re}\,qt) + \sinh^2(\mathrm{Im}\,qt)\right]}{|\,q\,|^2}\left(\mathrm{Re}\,b_x\,\mathrm{Im}\,b_z - \mathrm{Re}\,b_z\,\mathrm{Im}\,b_x\right). \tag{2.3}$$

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Figure captions

Fig. 1. The phase diagrams of azimuthal component of neutron spin interaction with a target.

Fig. 2. Dependences of the quantities *a, b* and *c* on the angle of spin rotation in a residual pseudomagnetic field.

Fig. 3. Phase shift in integrated asymmetry concerning the calibration when the field of the target magnet and the polarization of the target are switched off.

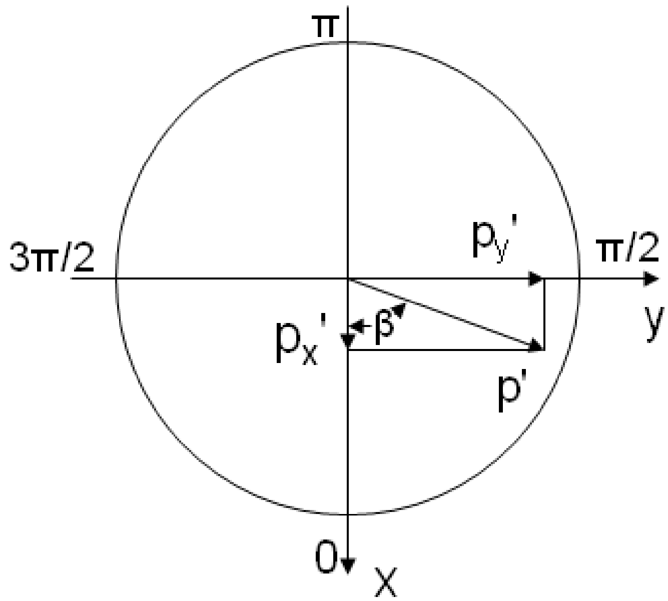
π
3π/2
π/2
p_y'
y
p_x'
β
p'
0
X

Fig. 1

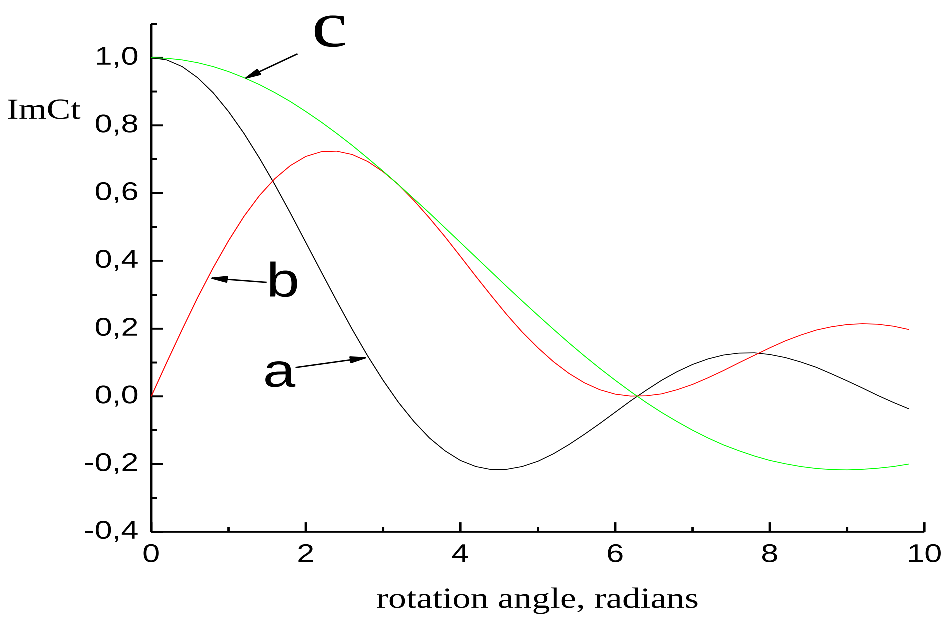
c
b
a
ImCt
1,0
0,8
0,6
0,4
0,2
0,0
-0,2
-0,4
0
2
4
6
8
10
rotation angle, radians

Fig. 2

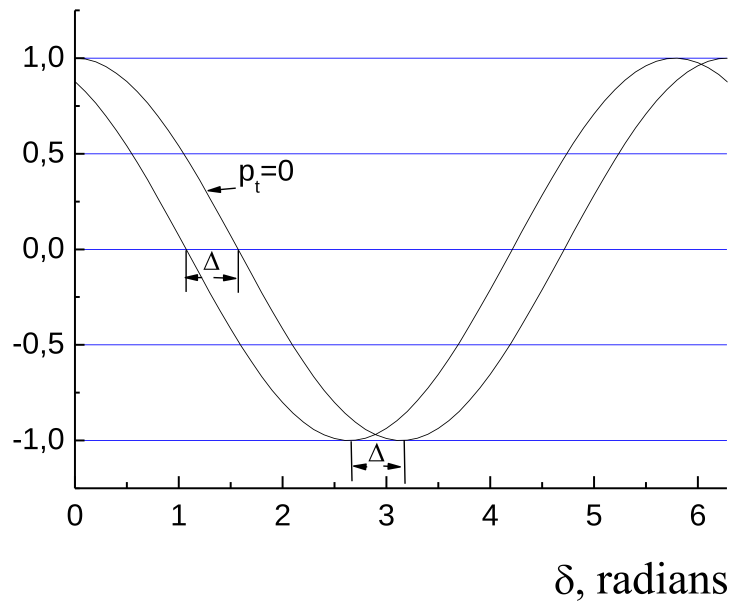

Fig. 3