PHENOMENOLOGY OF COHERENT ELECTROPRODUCTION OF VECTOR MESONS ON SPINLESS TARGET

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Contents

Introduction

- Vector-meson electroproduction provides information on the reaction mechanism and target structure

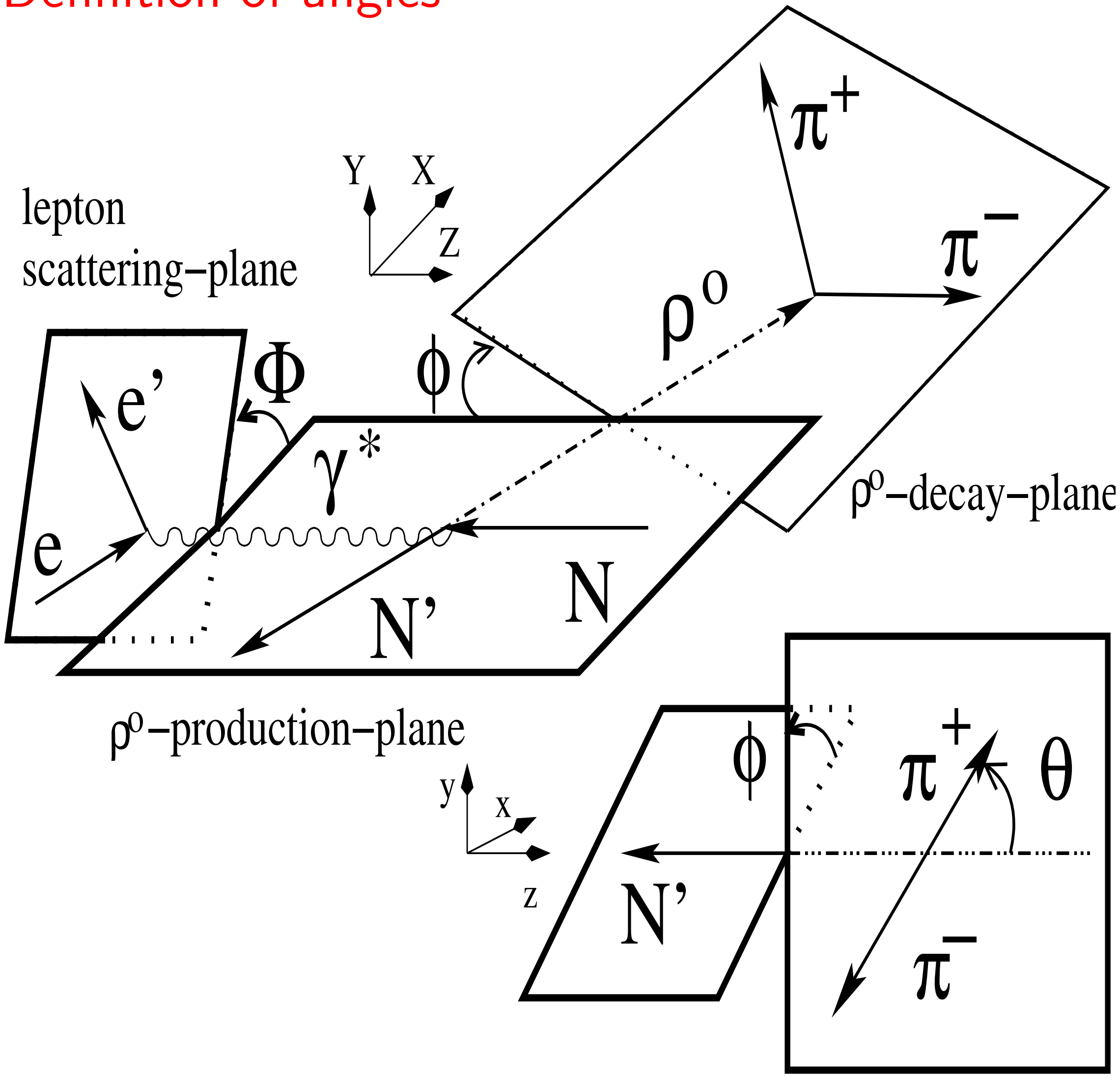
- Electroproduction of Vector mesons is one of two basic processes for extraction of Generalized Parton Distributions (GPD)
  Radyushkin, Ji

- Amplitudes of of vector-meson production on nucleons at high Q^2 and small x_B in leading-logarithm approximation are proportional to gluon distributions $G(x_B, Q^2)$, $\Delta G(x_B, Q^2)$.

- Usual method of data treatment is Spin-Density-Matrix-Element (SDME) method

- The alternative and more economical method is amplitude method of data processing

- Definition of angles



Φ is angle between production plane $(\vec{q}, \vec{v})$ and lepton-scattering plane $(\vec{k}_e, \vec{k}'_e)$.
θ and ϕ are polar and azimuthal angles of π^+-momentum in ρ^0-meson rest frame

Formalism

- Reactions $e \to e' + \gamma^*$, $\gamma^*(\lambda_\gamma) + S \to V(\lambda_V) + S$, $V = \rho^0 \to \pi^+ + \pi^-$.
  S is spinless nucleus.
  γ^* is virtual photon with helicity λ_γ in center-of-mass (CM) $\gamma^* S$ system.
  V denotes produced vector meson with helicity λ_V in CM system.
  Amplitudes $F_{\lambda_V \lambda_\gamma}$ in CM system obey relations $F_{-\lambda_V -\lambda_\gamma} = (-1)^{\lambda_V - \lambda_\gamma} F_{\lambda_V \lambda_\gamma}$ due to parity conservation. Independent amplitudes are: F_{11}, F_{10}, F_{1-1}, F_{01}, F_{00}.

- The von Neumann Formula
  $\mathcal{N} \rho_{\lambda_V \tilde{\lambda}_V} = \sum_{\lambda_\gamma, \tilde{\lambda}_\gamma} F_{\lambda_V \lambda_\gamma} F^*_{\tilde{\lambda}_V \tilde{\lambda}_\gamma} \varrho_{\lambda_\gamma \tilde{\lambda}_\gamma}$
  $\varrho_{\lambda_\gamma \tilde{\lambda}_\gamma} = \varrho^U_{\lambda_\gamma \tilde{\lambda}_\gamma} + P_b \varrho^L_{\lambda_\gamma \tilde{\lambda}_\gamma}$ is spin-density matrix of virtual photon $(\mathrm{tr}\{\varrho\} = 1)$

known from QED.
$\rho_{\lambda_V \tilde{\lambda}_V}$ is spin-density matrix of vector meson $(\mathrm{tr}\{\rho\} = 1)$.
$\mathcal{N} = \mathcal{N}_T + \epsilon \mathcal{N}_L$ is normalized factor.
$\mathcal{N}_T = |F_{11}|^2 + |F_{01}|^2 + |F_{-11}|^2$,
$\mathcal{N}_L = |F_{10}|^2 + |F_{00}|^2 + |F_{-10}|^2$,
ϵ is flux ratio of longitudinally polarized $(\lambda_\gamma = 0)$ virtual photons to transversely polarized $(\lambda_\gamma = \pm 1)$ photons produced by lepton beam.

Formalism

- Conservation of angular momentum in decay $\rho^0 \to \pi^+ + \pi^-$

  $|\rho^0; J = 1, J_z = \lambda_V\rangle \to |\pi^+\pi^-; L = 1, L_z = \lambda_V\rangle \to Y_{1\lambda_V}(\theta, \phi)$

- Angular distribution of decay pions

$$\mathcal{N}\mathcal{W}(\Phi, \theta, \phi) = \sum_{\lambda_V, \tilde{\lambda}_V} Y_{1\lambda_V}(\theta, \phi) Y^*_{1\tilde{\lambda}_V}(\theta, \phi) \mathcal{N}\rho_{\lambda_V \tilde{\lambda}_V}(\Phi, \epsilon)$$

- Angular distribution of decay pions for unpolarized beam

$$\mathcal{N}\mathcal{W}^U(\Phi, \vartheta, \varphi) = \frac{3\sin^2\theta}{8\pi}\{d_1 + 2\eta d_2 \cos\Phi - 2\epsilon d_3 \cos(2\Phi) - 2d_4 \cos(2\varphi) +$$
$$+2\eta\big(d_5 \cos(2\varphi - \Phi) - d_6 \cos(2\varphi + \Phi)\big) + \epsilon d_7 \cos(2\varphi - 2\Phi) + \epsilon d_8 \cos(2\varphi + 2\Phi)\} +$$
$$+\frac{3\cos^2\theta}{4\pi}\{d_9 + 2\eta d_{10} \cos\Phi + \epsilon d_{11} \cos(2\Phi)\} - \frac{3\sqrt{2}\sin(2\theta)}{8\pi}\{d_{12}\cos\varphi +$$
$$+\eta\big(d_{13}\cos(\varphi - \Phi) + d_{14}\cos(\varphi + \Phi)\big) + \epsilon\big(d_{15}\cos(\varphi - 2\Phi) - d_{16}\cos(\varphi + 2\Phi)\big)\},$$

where $\eta = \sqrt{\epsilon(1 + \epsilon)}$. All d_j $(1 \leq j \leq 16)$ can be extracted from data.

Formalism

- Angular distribution of decay pions for polarized beam

$$P_b\,\mathcal{N}\mathcal{W}^L(\Phi,\theta,\phi) = P_b\,\frac{3\sin^2\theta}{4\pi}\{\zeta d_{17}\sin\Phi - \xi d_{18}\sin(2\phi) - \zeta d_{19}\sin(2\phi-\Phi) + -\zeta d_{20}\sin(2\phi+\Phi)\} + P_b\,\frac{3\cos^2\theta}{2\pi}\zeta d_{21}\sin\Phi + +P_b\,\frac{3\sqrt{2}\sin(2\theta)}{8\pi}\{\xi d_{22}\sin\phi + \zeta\big(d_{23}\sin(\phi-\Phi) + d_{24}\sin(\phi+\Phi)\big)\},$$

where $\zeta = \sqrt{\epsilon(1-\epsilon)}$ and $\xi = (1-\epsilon^2)^{\frac{1}{2}}$.
All d_j $(17 \le j \le 24)$ can be extracted from data.

- Unnormalized spin-density-matrix elements (SDME) for unpolarized beam

$$d_1 = |F_{11}|^2 + |F_{1-1}|^2 + 2\epsilon|F_{10}|^2, \quad (1)$$

$$d_2 = \mathrm{Re}\{(\mathrm{F}_{11} - \mathrm{F}_{1-1})\mathrm{F}^*_{10}\}, \quad (2)$$

$$d_3 = \mathrm{Re}\{\mathrm{F}_{1-1}\mathrm{F}^*_{11}\}, \quad (3)$$

$$d_4 = \mathrm{Re}\{\mathrm{F}_{1-1}\mathrm{F}^*_{11}\} - \epsilon|\mathrm{F}_{10}|^2, \quad (4)$$

$$d_5 = \mathrm{Re}\{\mathrm{F}_{10}\mathrm{F}^*_{11}\}, \quad (5)$$

$$d_6 = \mathrm{Re}\{\mathrm{F}_{1-1}\mathrm{F}^*_{10}\}, \quad (6)$$

$$d_7 = |F_{11}|^2, \quad (7)$$

$$d_8 = |F_{1-1}|^2, \quad (8)$$

$$d_9 = |F_{01}|^2 + \epsilon|F_{00}|^2, \quad (9)$$

$$d_{10} = \mathrm{Re}\{\mathrm{F}_{00}\mathrm{F}^*_{01}\}, \quad (10)$$

$$d_{11} = |F_{01}|^2, \quad (11)$$

$$d_{12} = \mathrm{Re}\{(\mathrm{F}_{11} - \mathrm{F}_{1-1})\mathrm{F}^*_{01} + 2\epsilon\mathrm{F}_{10}\mathrm{F}^*_{00}\}, \quad (12)$$

- Unnormalized spin-density-matrix elements for unpolarized beam (continuation)

$$d_{13} = \mathrm{Re}\{\mathrm{F}_{11}\mathrm{F}^*_{00} + \mathrm{F}_{10}\mathrm{F}^*_{01}\}, \quad (13)$$

$$d_{14} = \mathrm{Re}\{\mathrm{F}_{10}\mathrm{F}^*_{01} - \mathrm{F}_{1-1}\mathrm{F}^*_{00}\}, \quad (14)$$

$$d_{15} = \mathrm{Re}\{\mathrm{F}_{11}\mathrm{F}^*_{01}\}, \quad (15)$$

$$d_{16} = \mathrm{Re}\{\mathrm{F}_{1-1}\mathrm{F}^*_{01}\}. \quad (16)$$

- Unnormalized spin-density-matrix elements for longitudinally polarized beam

$$d_{17} = \mathrm{Im}\{(\mathrm{F}_{11} - \mathrm{F}_{1-1})\mathrm{F}^*_{10}\}, \quad (17)$$

$$d_{18} = \mathrm{Im}\{\mathrm{F}_{1-1}\mathrm{F}^*_{11}\}, \quad (18)$$

$$d_{19} = \mathrm{Im}\{\mathrm{F}_{11}\mathrm{F}^*_{10}\}, \quad (19)$$

$$d_{20} = \mathrm{Im}\{\mathrm{F}_{1-1}\mathrm{F}^*_{10}\}, \quad (20)$$

$$d_{21} = \mathrm{Im}\{\mathrm{F}_{01}\mathrm{F}^*_{00}\}, \quad (21)$$

$$d_{22} = \mathrm{Im}\{(\mathrm{F}_{11} + \mathrm{F}_{1-1})\mathrm{F}^*_{01}\}, \quad (22)$$

$$d_{23} = \mathrm{Im}\{\mathrm{F}_{11}\mathrm{F}^*_{00} - \mathrm{F}_{10}\mathrm{F}^*_{01}\}, \quad (23)$$

$$d_{24} = \mathrm{Im}\{\mathrm{F}_{10}\mathrm{F}^*_{01} + \mathrm{F}_{1-1}\mathrm{F}^*_{00}\}. \quad (24)$$

Solution of Basic Equations

- Moduli of helicity amplitudes

$$|F_{11}|^2 = d_7,$$
$$|F_{00}|^2 = (d_9 - d_{11})/\epsilon,$$
$$|F_{01}|^2 = d_{11},$$
$$|F_{10}|^2 = (d_3 - d_4)/\epsilon,$$
$$|F_{1-1}|^2 = d_8.$$

- Ratios of amplitudes to F_{00}. Formulas can be obtained for polarized beam only.

$$\frac{F_{11}}{F_{00}} = \frac{\epsilon d_7\{d_{13} - d_{14} + i(d_{23} + d_{24})\}}{(d_9 - d_{11})(d_7 + d_3 + id_{18})}.$$
$$\frac{F_{01}}{F_{00}} = \frac{\epsilon(d_{10} + id_{21})}{(d_9 - d_{11})}.$$
$$\frac{F_{10}}{F_{00}} = \frac{2d_5 - d_2 - i(d_{17} + 2d_{20})}{d_{13} - d_{14} - i(d_{23} + d_{24})}.$$
$$\frac{F_{1-1}}{F_{00}} = \frac{\epsilon(d_3 + id_{18})\{d_{13} - d_{14} + i(d_{23} + d_{24})\}}{(d_9 - d_{11})(d_7 + d_3 + id_{18})}.$$

Solution of Basic Equations

- Ratios of helicity amplitudes to F_{11} (limit $Q^2 \to 0$)

$$\frac{F_{00}}{F_{11}} = \frac{(d_9 - d_{11})(d_7 + d_3 + id_{18})}{\epsilon d_7\{d_{13} - d_{14} + i(d_{23} + d_{24})\}}.$$

$$\frac{F_{01}}{F_{11}} = \frac{d_{15} + d_{16} - id_{22}}{d_7 + d_3 - id_{18}}.$$

$$\frac{F_{10}}{F_{11}} = \frac{d_5 - i(d_{17} + d_{20})}{d_7}.$$

$$\frac{F_{1-1}}{F_{11}} = \frac{d_3 + id_{18}}{d_7}.$$

- Angular distribution normalized to unity

$\int_0^{2\pi} \frac{d\Phi}{2\pi} \int_0^{\pi} \sin\vartheta d\vartheta \int_0^{2\pi} d\varphi \mathcal{W}(\Phi, \vartheta, \varphi) = 1.$

$\mathcal{N}\mathcal{W} \to \mathcal{W}$, $d_j \to \widetilde{d}_j = d_j/\mathcal{N}$, $\quad \mathcal{N} = d_1 + d_9$. $\widetilde{d}_j$ are called normalized SDMEs.
Formulas for amplitude ratios retain true if $d_j \to \widetilde{d}_j$.

Comparison of SDME and Amplitude Methods

- SDME method. Properties of normalized SDMEs ($\widetilde{d}_j$, Schilling-Wolf SDMEs $r^{\alpha}_{\lambda_V \tilde{\lambda}_V}$)

  All SDMEs $\widetilde{d}_j$ are considered as independent in a fit of experimental angular distribution. Total number of independent SDMEs $\widetilde{d}_j$ is 23 ($\widetilde{d}_1 + \widetilde{d}_9 = 1$).
  Since $\widetilde{d}_j$ can be expressed through 4 complex amplitude ratio, SDMEs $\widetilde{d}_j$ are mutually dependent. There are 15 equations of constraints.
  Examples: $\widetilde{d}_2 = \widetilde{d}_5 - \widetilde{d}_6$, $\quad (\widetilde{d}_3 \widetilde{d}_{11} - \widetilde{d}_{15} \widetilde{d}_{16})^2 = (\widetilde{d}_7 \widetilde{d}_{11} - \widetilde{d}^2_{15})(\widetilde{d}_8 \widetilde{d}_{11} - \widetilde{d}^2_{16})$.
  The region for SDME values which guaranties positive definiteness of angular distribution is unknown. If $\mathcal{W} \leq 0$ maximum likelihood method is inapplicable.
  Likelihood function is linear with respect to $\widetilde{d}_j$.

- Amplitude method
  The number of real fit functions is 8.
  Four complex helicity-amplitude ratios are independent.
  Angular distribution is positive for any non-zero helicity amplitudes.
  Likelihood function is nonlinear with respect to amplitude ratios.

- If Monte-Carlo codes describe detector good enough results of both methods should be in agreement

Virtual-photon Longitudinal-to-transverse Cross-section Ratio

- Exact formula for virtual-photon longitudinal-to-transverse differential-cross-section ratio for vector-meson productions on spinless targets

$$R \equiv \frac{d\sigma_L}{dt} \Big/ \frac{d\sigma_T}{dt} = \frac{|F_{10}|^2 + |F_{00}|^2 + |F_{-10}|^2}{|F_{11}|^2 + |F_{01}|^2 + |F_{-11}|^2},$$

$$R = \frac{d_9 - d_{11} + 2(d_3 - d_4)}{\epsilon(d_7 + d_8 + d_{11})},$$

$$R = \frac{r^{04}_{00} + r^{1}_{00} + 2(r^{1}_{11} - r^{04}_{1-1})}{\epsilon(2r^{1}_{1-1} - r^{1}_{00})}.$$

- New approximate formula for virtual-photon longitudinal-to-transverse differential-cross-section ratio for scattering from nucleons

$$R \approx \frac{r^{04}_{00} + r^{1}_{00} + 2(r^{1}_{11} - r^{04}_{1-1})}{\epsilon\{1 - r^{04}_{00} - r^{1}_{00} - 2(r^{1}_{11} - r^{04}_{1-1})\}}.$$

- Standard approximate formula for virtual-photon longitudinal-to-transverse differential-cross-section ratio for scattering from nucleons

$$R \approx R_{04} = \frac{r^{04}_{00}}{\epsilon\{1 - r^{04}_{00}\}}.$$

Summary

- Exact formulas for helicity-amplitude ratios in terms of spin-density-matrix elements (SDMEs) of vector mesons produced on spinless targets are obtained. It is shown that single-valued formulas for helicity-amplitude ratios can be obtained only if the beam is longitudinally polarized.

- Making use of the amplitude ratios instead of SDMEs as free fit parameters in maximum likelihood method reduces the number of free real parameters from 23 to 8.

- A comparison between SDMEs directly extracted from the experimental data and calculated from the helicity-amplitude ratios permits to estimate systematic uncertainties of description with Monte Carlo codes of the applied detector properties.

- Exact formula for virtual-photon longitudinal-to-transverse differential-cross-section ratio, R in terms of SDMEs for vector-meson production on spinless targets is established. There is no need in Rosenbluth decomposition.

- A new approximate formula for virtual-photon longitudinal-to-transverse differential-cross-section ratio, R in terms of SDMEs for vector-meson production on nucleons is proposed. It is more precise than R_{04} for high energies.

Back-up Slides. Spin-density Matrix of Virtual Photon

- General Formula for Spin-density Matrix of Virtual Photon

  $\varrho_{\lambda_\gamma \tilde{\lambda}_\gamma} = \varrho^U_{\lambda_\gamma \tilde{\lambda}_\gamma} + P_b \varrho^L_{\lambda_\gamma \tilde{\lambda}_\gamma}$

- Spin-density Matrix of Virtual Photon for Unpolarized Beam

$$\varrho^U_{\lambda_\gamma \tilde{\lambda}_\gamma}(\epsilon, \Phi) =$$

$$= \frac{1}{2} \begin{pmatrix} 1 & \sqrt{\epsilon(1+\epsilon)} e^{-i\Phi} & -\epsilon e^{-2i\Phi} \\ \sqrt{\epsilon(1+\epsilon)} e^{i\Phi} & 2\epsilon & -\sqrt{\epsilon(1+\epsilon)} e^{-i\Phi} \\ -\epsilon e^{2i\Phi} & -\sqrt{\epsilon(1+\epsilon)} e^{i\Phi} & 1 \end{pmatrix},$$

- Additive Term to Spin-density Matrix of Virtual Photon due to Longitudinal Polarization of Beam

$$\varrho^L_{\lambda_\gamma \tilde{\lambda}_\gamma}(\epsilon, \Phi) =$$

$$= \frac{\sqrt{1-\epsilon}}{2} \begin{pmatrix} \sqrt{1+\epsilon} & \sqrt{\epsilon} e^{-i\Phi} & 0 \\ \sqrt{\epsilon} e^{i\Phi} & 0 & \sqrt{\epsilon} e^{-i\Phi} \\ 0 & \sqrt{\epsilon} e^{i\Phi} & -\sqrt{1+\epsilon} \end{pmatrix}.$$