Hidden problems of parton analysis

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1. The double counting of the low $k_T < Q_0$ contrib^n is included in the NLO splitting and coefficient functions, but also hidden in the PDF input at $Q = Q_0$. Formally this is a power Q_0^2/μ^2 correction but it is non-negligible at moderate scales μ.

2. $\overline{MS}$ scheme keeps the ϵ/ϵ contribution generated by the infrared (IR) divergence after the dimensional regularization. Since the IR divergence is cut off by confinement (or the quark mass) these terms must be deleted.

3. The role of the smooth transition through the heavy quark thresholds and the need to work in the Physical scheme where at NLO (and higher orders) there is no admixture of the quarks to gluon PDF (and gluons to quark PDF) which occur in the MS-bar scheme.

Logic of parton analysis is:

We do not know QCD at large distances but we know the evolution (DGLAP) of PDFs at large scale μ.

$$\frac{\partial a(x,\mu)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu)}{2\pi} \sum_b \int_x^1 \frac{dz}{z} P_{ab}(z) b(\frac{x}{z},\mu) \qquad (a,b=q,g) \quad (1)$$

We measure/fit PDFs at $\mu = Q_0$ and start the evolution from the input PDF(x, Q_0)

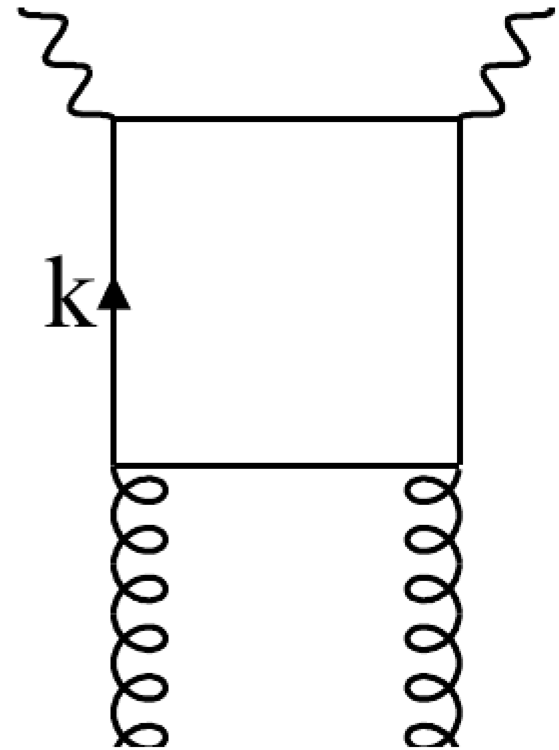
All contributions from $\mu < Q_0$ are included in the PDF(x, Q_0)

At present global parton analysis account for NNLO and up to N^3LO terms

1. ALL is OK at LO

The Problem – at NLO we have some finite contribution from $k^2 = \mu^2 < Q_0^2$.

This is a power Q_0^2/μ^2 correction but it becomes important when the scale is driven by the heavy (charm, beauty) quark mass and $Q_0 = 1 - 2$ GeV.



To avoid double counting we have to subtract the $k^2 < Q_0^2$ contrib. from C^{NLO} and P^{NLO}

$$C_{Lg}(z) = 4T_R z(1-z)\cdot(1 - zQ_0^2/Q^2)$$

$$C_{Lq}(z) = C_F 2z\cdot(1-(zQ_0^2/Q^2)^2)$$

$$C_{2g}(z) = T_R\left[(z^2+(1-z)^2)\ln\frac{1}{z} + [6z(1-z)-1]\cdot(1-zQ_0^2/Q^2)\right]$$

$$\begin{aligned} C_{2q} = C_F\Bigg\{ & \left(\frac{1+z^2}{1-z}\right)\ln\frac{1}{z} + 3z\cdot(1-(zQ_0^2/Q^2)^2) - \\ & -\delta(1-z)\left[\frac{5}{2} - \frac{\pi^2}{3} - 3\frac{Q_0^2}{Q^2} - \frac{3}{4}\frac{Q_0^2}{Q^2}\right] + \\ & + \left[2 - 2\left(\frac{1}{1-z}\right)_+\right]\cdot(1-zQ_0^2/Q^2) + \left(\frac{1/2}{1-z}\right)_+ \Bigg\} \end{aligned}$$

Problem 2. Dimensional $(d = 4 + 2\epsilon)$ regularization touches not only the UV but the IR region as well.

After the singular $1/\epsilon$ terms are crossed out, the $\overline{MS}$ scheme still keeps the finite ϵ/ϵ contributions.

$1/\epsilon$ comes from the IR logarithm $\int_0 dk^{2+2\epsilon}/k^2$
while in the numerator ϵ comes from
the number of gluon transverse polarizations
and phase space $(k^2)^\epsilon$ factor.

These extra ϵ/ϵ contributions are not visible in $\overline{MS}$ due to the **PDF redefinition**

$$a^{\overline{\mathrm{MS}}}(x) \;=\; a^{\mathrm{phys}}(x) - \frac{\alpha_s}{2\pi}\int \frac{dz}{z}\sum_b \delta P_{ab}(z) b^{\mathrm{phys}}(x/z)$$

$$P_{ab}^{\overline{\mathrm{MS}}}(z) \;=\; P_{ab}^{\mathrm{LO}}(z) + \epsilon\delta P_{ab}(z)$$

Formally this is just different scheme but this redefinition means that in this $\overline{MS}$ scheme at NLO we deal with the **mixture of gluon and quark PDFs**

At LO the split. funct. are

$$P_{qq}^{real}(z) = C_F\left[\frac{1+z^2}{1-z}(1+\epsilon\ln(1-z))+\epsilon(1-z)\right]$$

$$P_{qg} = T_R\left[(z^2+(1-z)^2)(1+\epsilon\ln(1-z))+\epsilon 2z(1-z)\right]$$

$$P_{gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}(1+\epsilon\ln(1-z))+\epsilon z\right]$$

$$P_{gg}^{real}(z) = 2C_A\left[\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right)(1+\epsilon\ln(1-z))\right]$$

This extra ϵ/ϵ contribution is unphysical.
It comes from very large distances $r >> 1/\Lambda_{QCD}$ and is actually killed by confinement and/or quark masses.

Next, the large r (i.e. small k^2) contr. are included in the input PDF(Q_0) and should be subtracted from the coefficient and splitting functions.

Problem 3. Conventionally all the quarks in DGLAP evolution are **massless** (this is the price to keep the renorm. group.)

At each heavy quark threshold the number of "light" quarks, $n_f \to n_f + 1$ increases by 1.

To see the difference in comparison of including actual quark masses we consider the behavior of $\alpha_s(Q^2)$ at NLO

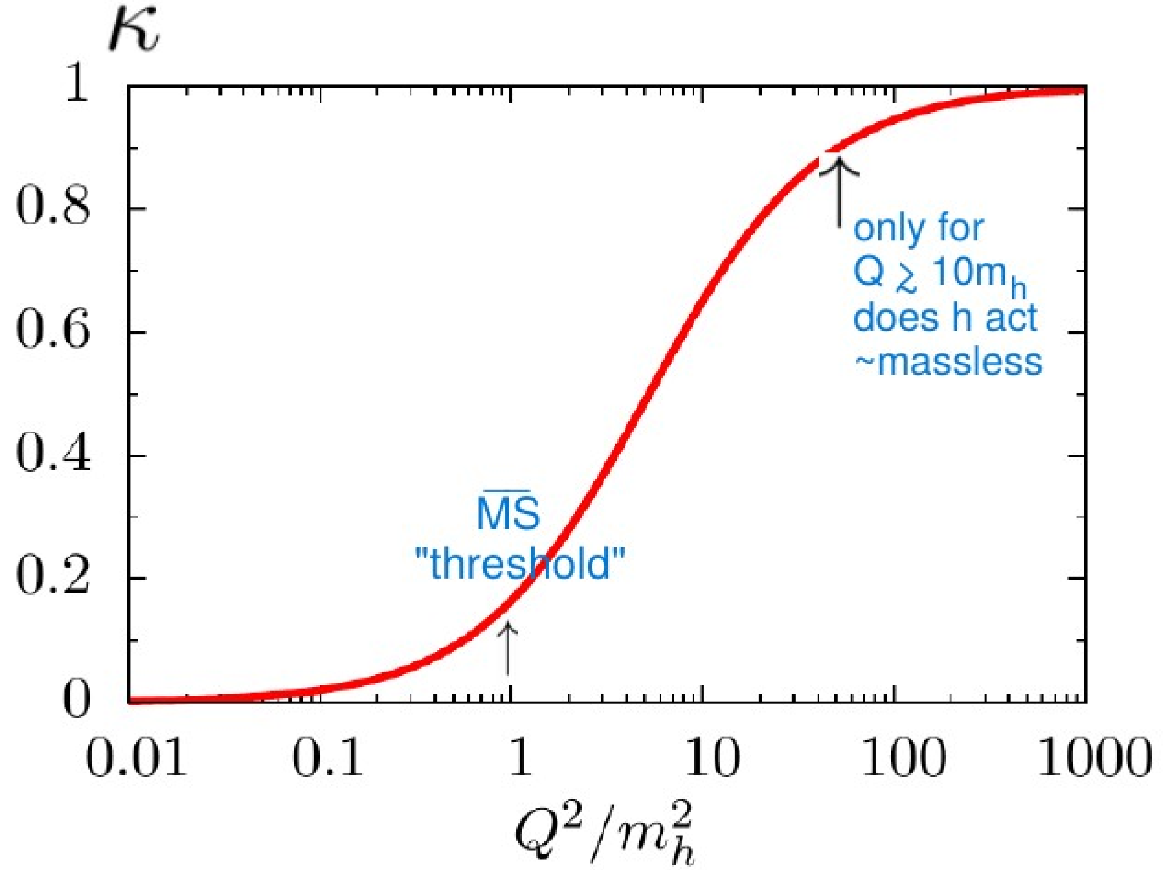
$$\frac{d}{d \ln Q^2}\left(\frac{\alpha_s}{4\pi}\right) = -\beta_0 \left(\frac{\alpha_s}{4\pi}\right)^2 - \beta_1 \left(\frac{\alpha_s}{4\pi}\right)^3$$

$$\beta_0(n_f) = 11 - \frac{2}{3} n_f, \qquad \beta_1(n_f) = 102 - \frac{38}{3} n_f.$$

$$\beta_0 = 11 - \frac{2}{3} n_{eff}$$

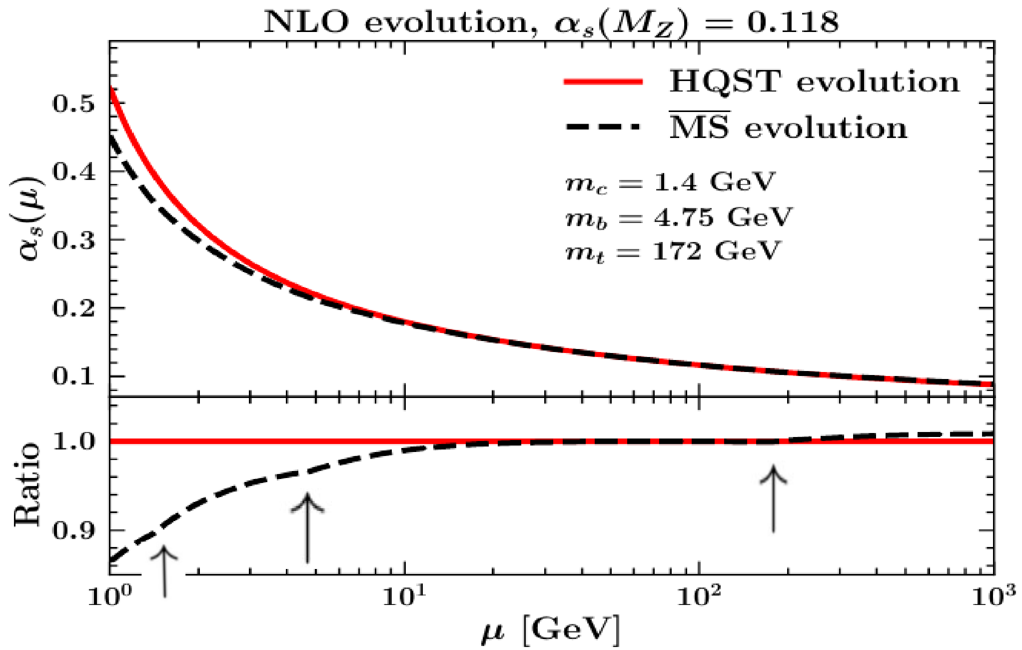
$$n_f \to n_{eff} = \sum_{i=1}^{n_f} \kappa(\xi_i)$$

$$\xi_i = m_i^2 / Q^2$$



For each heavy quark, h, we must include in n_f a factor κ

$$\kappa(\xi) = 1 - 6\xi + 12 \frac{\xi^2}{\sqrt{1+4\xi}} \ln \frac{\sqrt{1+4\xi}+1}{\sqrt{1+4\xi}-1}$$



α_s running at NLO in the physical HQST (red) and $\overline{MS}$ (black) schemes.

Lower panel: ratio of the two evolutions;

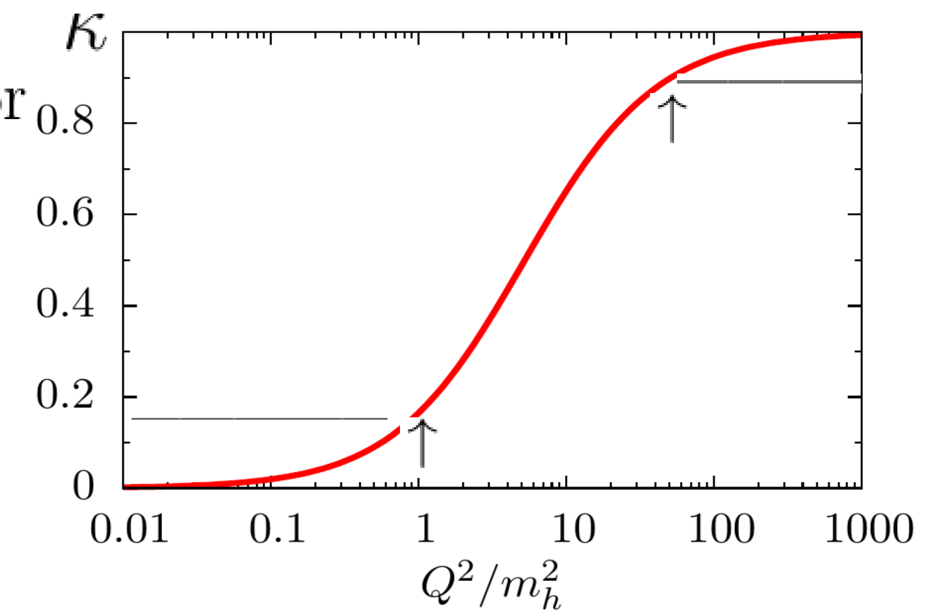
the cusps displayed in the $\overline{MS}$ curve are apparent at the c, b, t quark thresholds.

Heavy quark, h, acts with the factor

$$\kappa(\xi) = 1 - 6\xi + 12\frac{\xi^2}{\sqrt{1+4\xi}} \ln \frac{\sqrt{1+4\xi}+1}{\sqrt{1+4\xi}-1},$$

$$\xi = m_h^2/Q^2$$

$$\kappa(1) = 0.165, \quad \kappa(0.02) = 0.9$$



$$P_{hg}(\xi,x) = 2\,T_R\,\eta\left(x^2+(1-x)^2+(1-\eta)2x(1-x)\right)\theta(\eta-4x+3\eta x)\,,$$

$$P_{hh}^{\rm real}(\xi,x) = 2\,C_F\,\left(\frac{1+x^2}{1-x}\frac{1}{1+\xi(1-x)}+x(x-3)\frac{\xi}{(1+\xi(1-x))^2}\right),$$

$$P_{gh}(\xi,x) = 2\,C_F\,\left(\frac{1+(1-x)^2}{x}\frac{1}{1+\xi x}+(x-1)(x+2)\frac{\xi}{(1+\xi x)^2}\right),$$

$$P_{gg}(\xi,x) = P_{gg}^{(n_l)}(x)-\delta(1-x)2\,T_R\sum_h\frac{\eta_h^3}{(4-3\eta_h)^2}\left(1-\frac{4}{3}\frac{\eta_h^2}{(4-3\eta_h)}+\frac{\eta_h^3}{(4-3\eta_h)^2}\right)$$

$\eta = 1/(1+\xi)$ and $P_{gg}^{(n_l)}$ is the $\overline{\rm MS}$ result with n_l active flavours

Note the presence of $\xi = m_h^2/Q^2$

$$C_a^{(1),{\rm HQST}}(x,Q,m_h) = C_a^{(1),{\rm FFNS}}(x,Q,m_h)\;-$$

$$-\frac{\alpha_s(Q)}{4\pi}\sum_{b=g,q,h}C_b^{(0)}(x,Q,m_h)\otimes\int_0^{Q^2}\frac{d\mu^2}{\mu^2}\left(P_{ba}^{(0),{\rm HQST}}(\xi,x)-P_{ba}^{(0),\overline{\rm MS}}(x)\right)$$

We plan to examine the effect by doing a NLO fit to F_2 data and comparing the results using the $\overline{MS}$ and physical schemes.

Conclusion

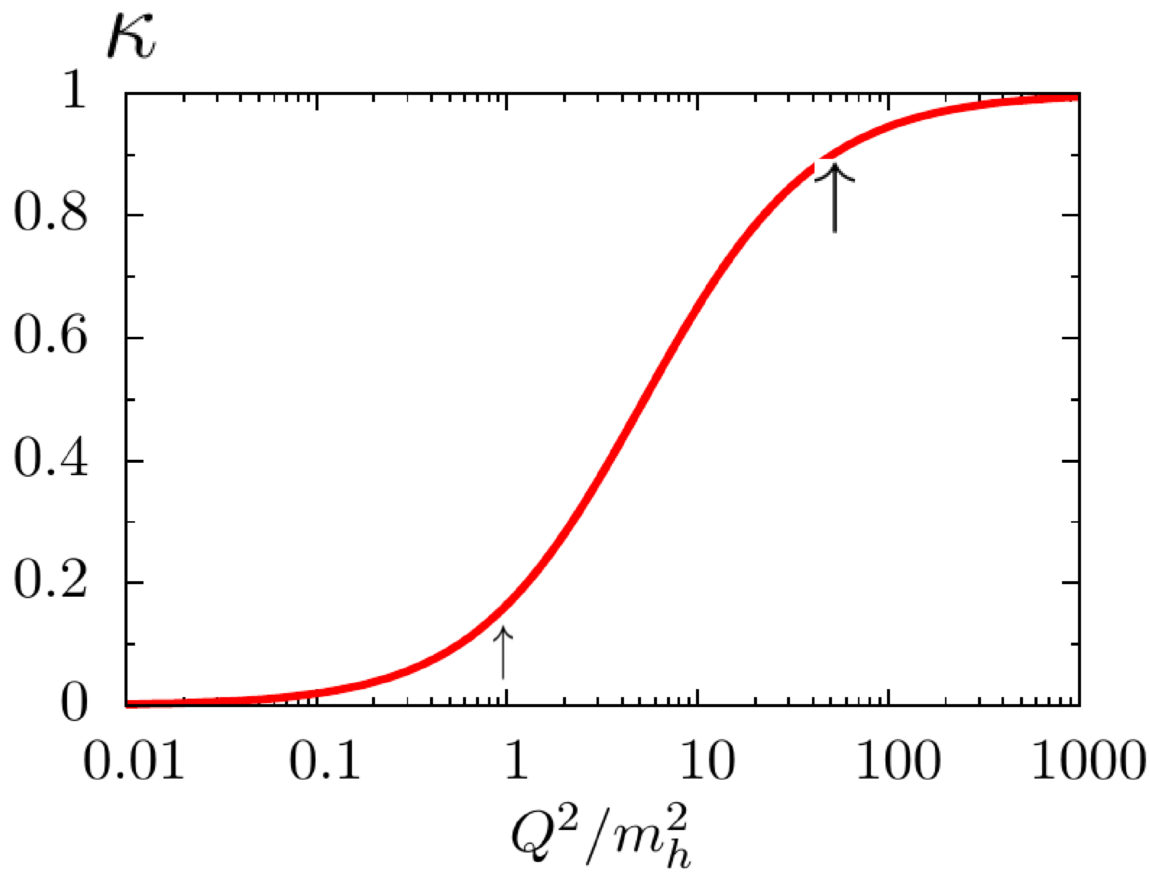
To provide the high ($\sim$ %) precision we have
to work in Physical scheme
implementing the Q_0 subtraction
and accounting for the heavy quark mass explicitly

THANK YOU

$$\beta_0 = 11 - \frac{2}{3} n_{eff}$$

$$n_f \to n_{eff} = \sum_{i=1}^{n_f} \kappa(\xi_i)$$

$$\xi_i = m_i^2 / Q^2$$



For each heavy quark, h, we must include in n_f a factor κ

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