

DGLAP and BFKL

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1. Logarithmic theory;

$\alpha \ll 1$ but $\alpha \ln(Q^2/Q_0^2) \sim 1$

2. Space-time picture;

(detector resolution; EFT)

3. Evolution ($t_i \gg t_{i+1}$)

A) DGLAP

i) evolution equation (splitting functions)

ii) conservation law(s)

iii) IR regularization (plus prescription)

iv) double Log's

v) anomalous dimension

vi) factorization scheme(s)

vii) unintegrated PDF(x, k_T)

B. BFKL

i) evolution equation

gluon reggeization (bootstrap)

ii) diffusion in $\ln k_T$

iii) $j = 1 + \omega_0$ intercept

iv) evolution 'trajectory'

v) Mueller-Navelet, Mueller-Tang

vi) correlations

vii) secondary Reggeons

If coupling α is dimensionless
next loop integral reads

$$\alpha \int \frac{d^4 k}{k^4} \implies \alpha \ln(\dots)$$

For $\alpha \ll 1$ we can sum up $\sum_n C_n (\alpha \ln(\dots))^n$

There is **No charge** without the field
around it $e \rightarrow e\gamma \rightarrow e(e^+e^-) \rightarrow \dots$

time-life of fluctuation $\Delta t \sim 1/\Delta E$

DGLAP - $\Delta t \sim 1/k_{i,T}$ (in Breit frame)

BFKL - $\Delta t \sim E_i/k_{i,T}^2$ (in target rest frame)

DGLAP evolution

$$\frac{da(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s}{2\pi} \sum_b \left[\int P_{ab}(z) b\left(\frac{x}{z}, Q^2\right) \frac{dz}{z} - a(x, Q^2) \int P_{ba}(z) dz \right]$$

$$a = q(x), g(x)$$

$$P_{qq} = C_F \frac{1+z^2}{1-z}$$

$$P_{gq} = C_F \frac{1+(1-z)^2}{z}$$

$$P_{qg} = T_R (z^2 + (1-z)^2)$$

$$P_{gg} = 2N_c \left(z(1-z) + \frac{1-z}{z} + \frac{z}{1-z} \right)$$

symmetry

$$z \rightarrow 1-z$$

$$\sum_a \int x a(x, Q^2) dx = \text{const} \quad \int q_f(x, Q^2) dx = \text{const}$$

'+' prescription

$$\frac{f(z)}{(1-z)_+} = \frac{f(z) - f(1)}{1-z}$$

Soft $(1-z) \rightarrow 0$ gluon does not change
white PDF

IR divergence is canceled

Double Log(s)

$$P_{gg}(z \ll 1) = 2N_c/z \implies \alpha_s \int (dq^2/q^2)(dz/z)$$

$$xg(x, Q^2) \sim \exp\left(\sqrt{\frac{4N_c\alpha_s}{\pi} \ln(1/x) \ln(Q^2/Q_0^2)}\right)$$

$$a(x, Q^2) \propto (Q^2/Q_0^2)^\gamma$$

At $x \ll 1$ anom. dimension.

$$\gamma = \sqrt{\alpha_s N_c / \pi} (\ln(1/x) / \ln(Q^2/Q_0^2))$$

increases with x decreases

Factorization

Two blocks are separated by the LOG^c cell $\int^{k_{i+1}} dk_i^2/k_i^2$ with $k_i \ll k_{i+1}$

Splitting and coefficient functions are calculated with $k_i^2 = 0$.

(power k_i^2/k_{i+1}^2 corrections are neglected)

Strong Δt_i and/or $\theta_i = k_{i,t}/E_i$ ordering
(both in DGLAP and in BFKL)

Scheme dependence starting from NLO

$$a^{\overline{\text{MS}}}(x) = a^{\text{phys}}(x) - \frac{\alpha_s}{2\pi} \int \frac{dz}{z} \sum_b \delta P_{ab}(z) b^{\text{phys}}(x/z)$$

due to ϵ/ϵ terms in $\overline{\text{MS}}$

Input PDF(x, Q_0)
+ boundary cond.ⁿ PDF($x = 1, Q$) = 0

$$a(x, Q^2) = \int dx_0 \sum_b b(x_0, Q_0^2) \cdot G_{ab}(x_0, Q_0; x, Q)$$

input evolution

(In BFKL \implies boundary at $Q = Q_0$,
input at $x = x_0$)

Unintegrated PDF(x, k_T, μ)

$$a(x, \mu) = \int^{\mu} d^2 k_T f_a(x, k_T, \mu)$$

$$f_a = \frac{T(k_T, \mu)}{k_T^2} \frac{\alpha_s}{2\pi} \sum_b \int P_{ab}(z) b\left(\frac{x}{z}, Q^2 = \frac{k_T^2}{1-z}\right) dz$$

BFKL evolution (gluon only)

$$\frac{df(x, q_T)}{d \ln(1/x)} = \frac{N_c \alpha_s}{\pi^2} \int K(q_T, q'_T) f(x, q'_T) d^2 q'_T$$

$$K(q, q') f(q') = \frac{1}{(q - q')^2} \left[f(q') - \frac{q^2 f(q)}{q'^2 + (q - q')^2} \right]$$

gluon reggeizatⁿ

No IR divergence at $q \rightarrow q'$

DGLAP evolution

$$\frac{da(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s}{2\pi} \sum_b \left[\int P_{ab}(z) b\left(\frac{x}{z}, Q^2\right) \frac{dz}{z} - a(x, Q^2) \int P_{ba}(z) dz \right]$$

$$a = q(x), g(x)$$

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symmetry

$$z \rightarrow 1-z$$

$$\sum_a \int x a(x, Q^2) dx = \text{const} \quad \int q_f(x, Q^2) dx = \text{const}$$

Gauge

DGLAP - axial $G_{\mu\nu} = G_{\mu\nu}^\perp$

$$G_{\mu\nu} = \left[g_{\mu\nu} - \frac{q_\mu n_\nu + n_\mu q_\nu}{(q \cdot n)} \right] \quad q^2 = n^2 = 0$$

BFKL - different n for different cells
need to sum up the diagrams

DGLAP $\implies f(x, q) \propto 1/q^2 \times (q^2/Q_0^2)^\gamma$
with anom. dim. $\gamma \sim \alpha_s \ll 1$

BFKL at $\alpha_s \ln(1/x) \gg 1 \implies \gamma \rightarrow 1/2$
 $f(x, q) \propto x^{-\omega_0} / \sqrt{q^2 Q_0^2} \quad \omega_0^{LO} = \frac{N_c \alpha_s}{\pi} 4 \ln 2$
(beam - target symmetry)

Diffusion:

i) in b_t $\Delta b_{ti} \sim 1/q_{Ti}$

ii) in $\ln q_{Ti}$ $\Delta \ln q_{Ti} \sim O(1)$

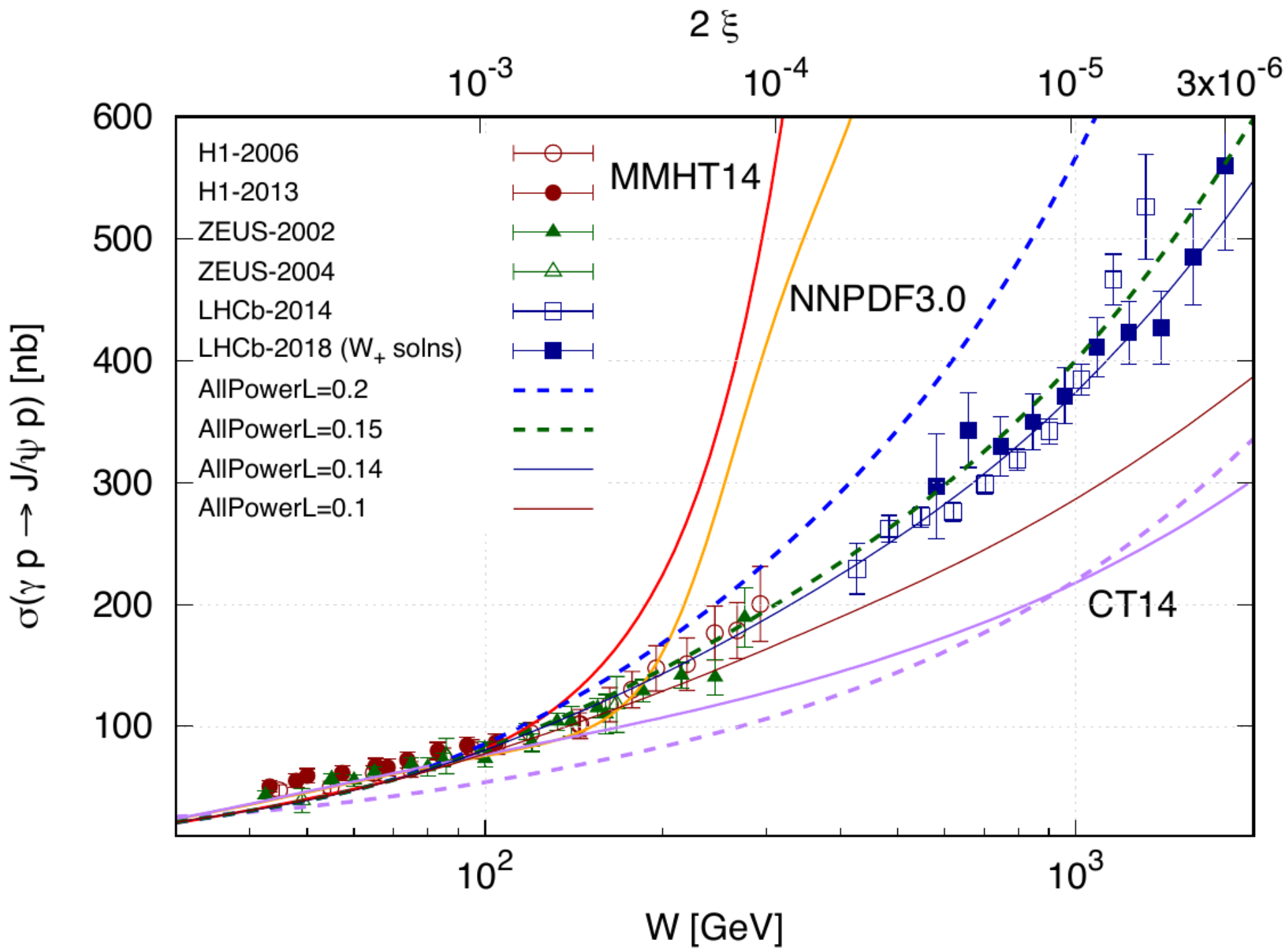
$$\langle \Delta y \rangle = \langle \ln x_i - \ln x_{i+1} \rangle \sim 1/\omega_0 \sim 3$$

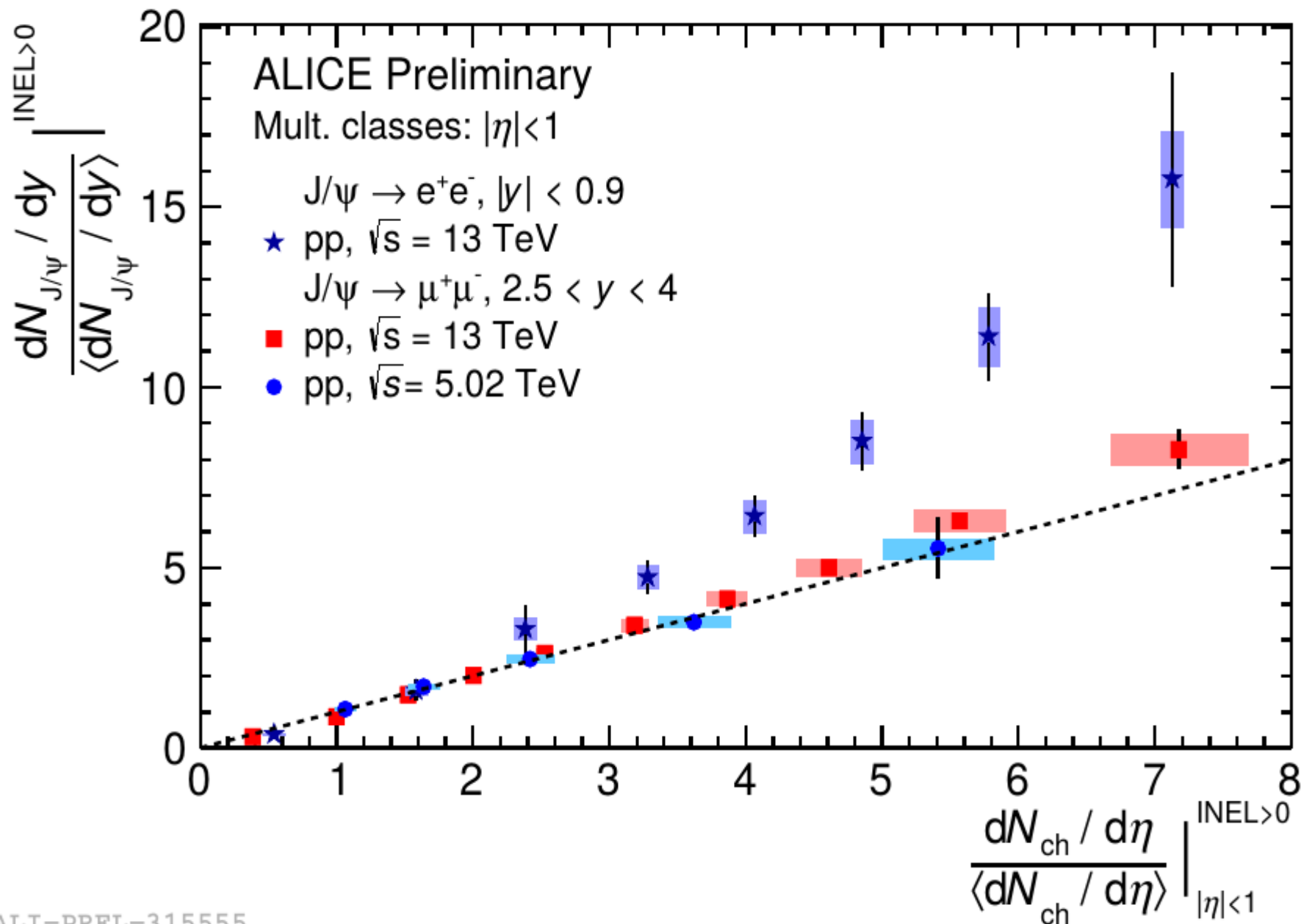
Dipole representation

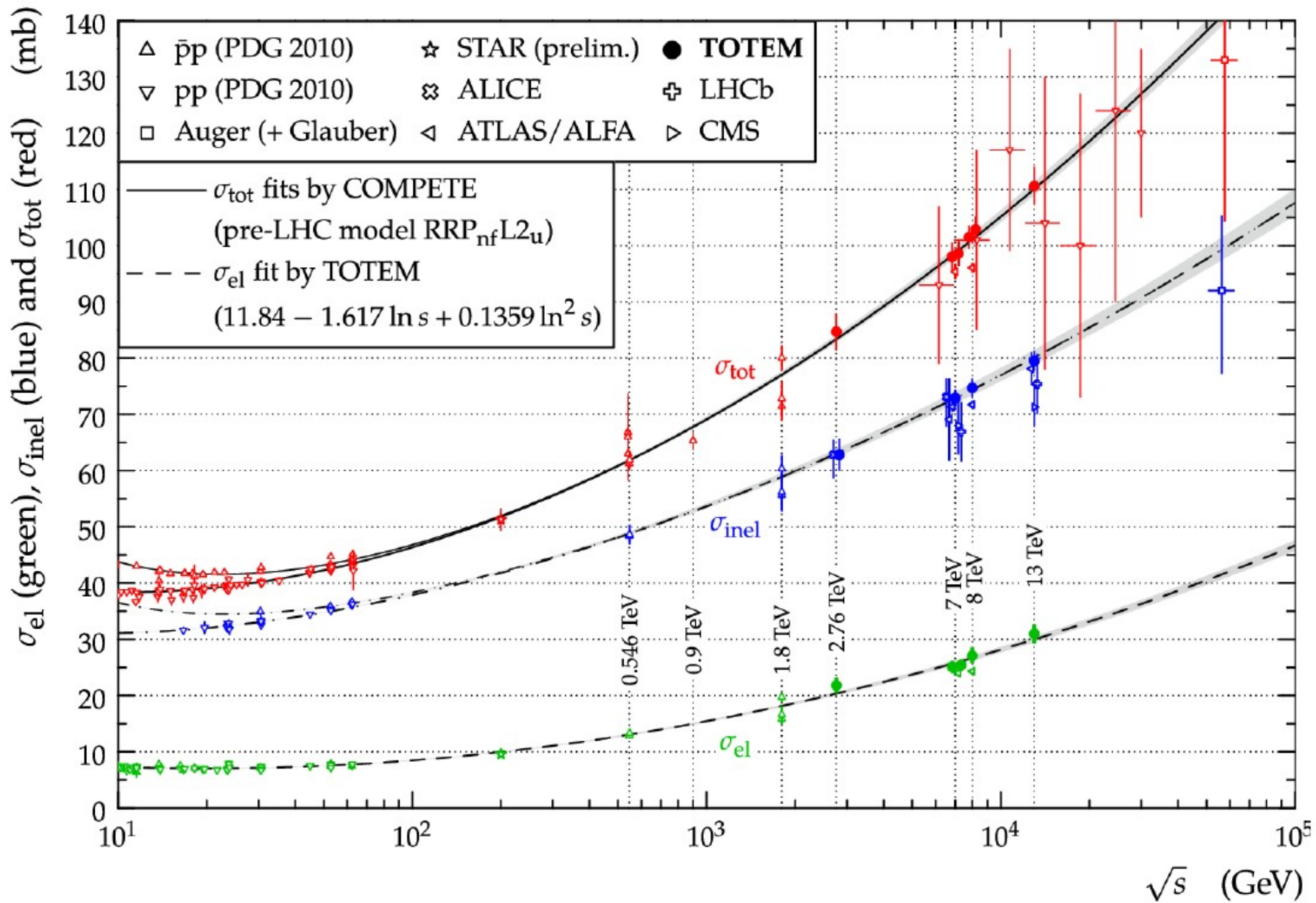
$$\begin{aligned} \frac{d}{dY} N(x, y; Y) &= \frac{\alpha_s N_c}{2\pi^2} \int d^2z \frac{(x-y)^2}{(x-z)^2 (y-z)^2} \\ &\times [N(x, z; Y) + N(y, z; Y) - N(x, y; Y) \\ &\quad - N(x, z; Y) N(y, z; Y)] \end{aligned}$$

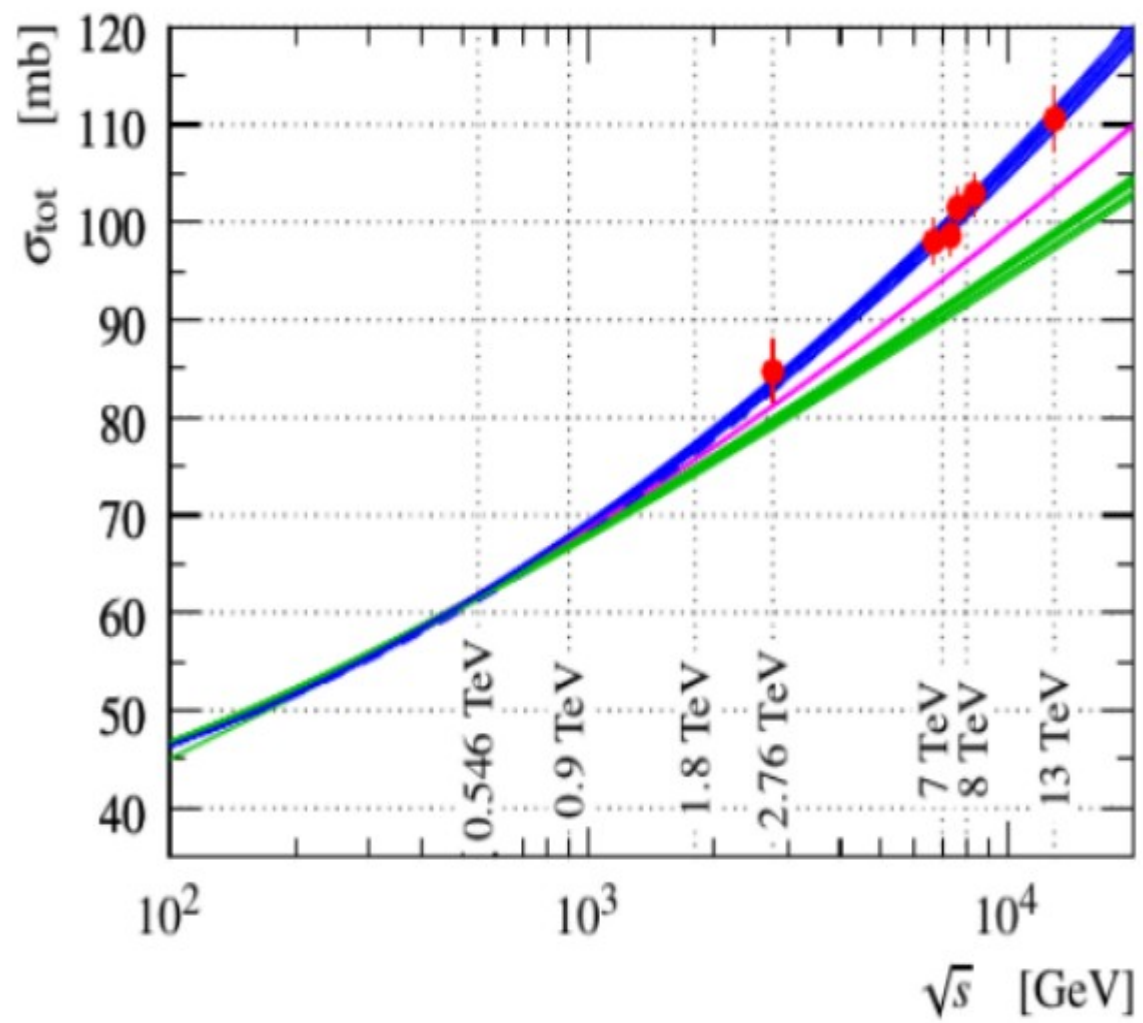
For a small density N the last term in square brackets can be neglected and equation reproduces the conventional BFKL equation in coordinate representation. However for a large $N \rightarrow 1$ the r.h.s. vanishes and we reach the saturation $N = 1$.

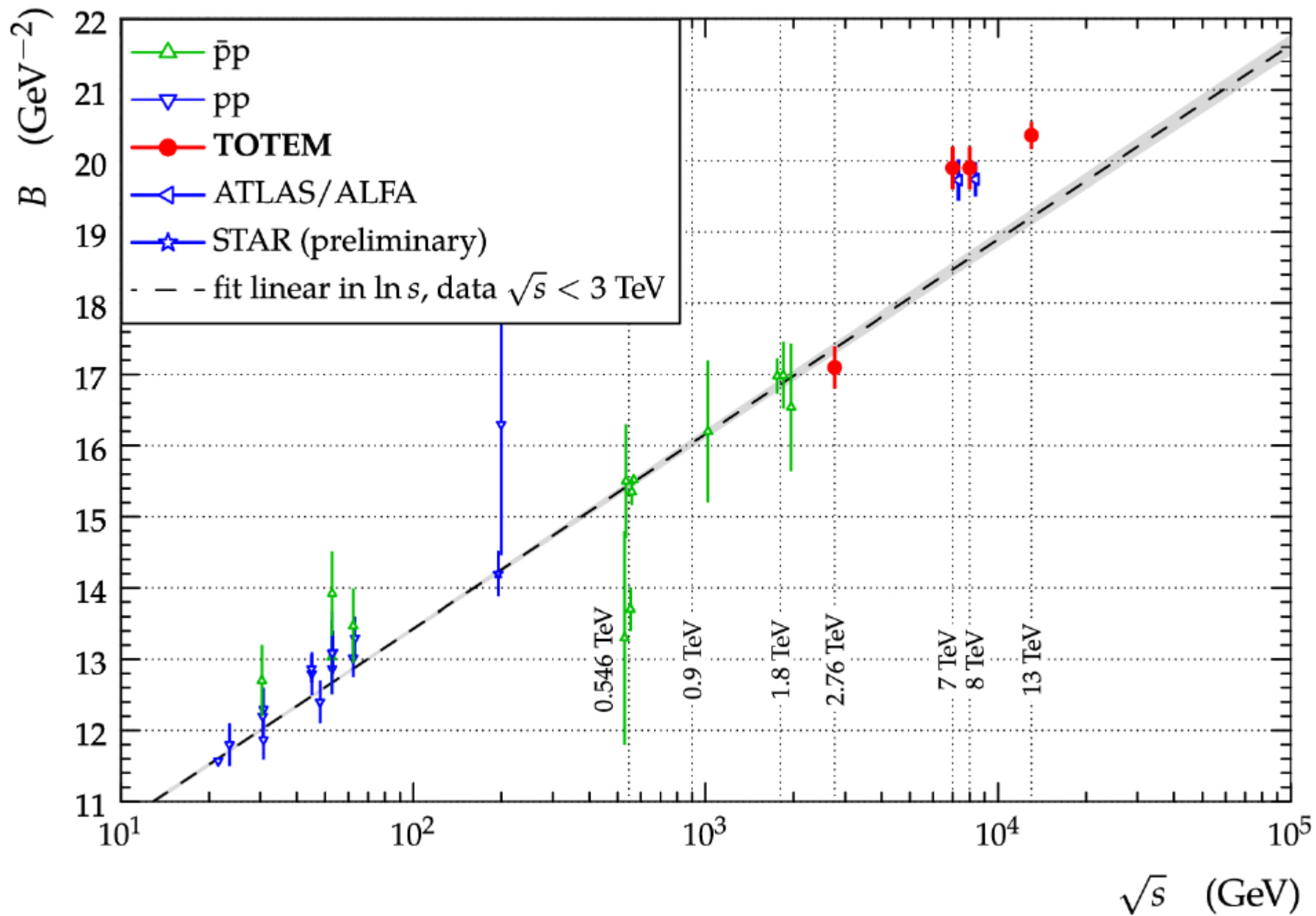
Colour re-connection











two high E_T jets ($\Delta y > \text{few}/\omega$)

a) inclusive — Mueller-Navelet

b) gap between 2 jets - Mueller-Tang

Jet-Jet correlat^s die out for $\Delta y \gg \omega_0$

$$(\omega_{n \geq 1} \leq 0 \quad \omega_{colour} \leq 0)$$

Secondary Reggeons

($q\bar{q}$ state in t -channel)

No explicit q_{Ti} ordering

$$\int_{Q_0^4/s}^s \frac{dq_T^2}{q_T^2} \sim \ln^2(s/Q_0^2)$$

DLogs $\implies \sum_n C_n (\alpha_s \ln^2 s)^n$

$$\implies \sigma \sim s^{\omega_R - 1} \quad \omega_R = \sqrt{2C_F \alpha_s / \pi}$$

($\omega_R \sim 0.5$ **for** $\alpha_s = 0.3$)