High Energy Physics(theory) in pre-QCD era

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1. **Mandelstam s, t, u variables**

2. **Analyticy - Dispersion relations**

3. **Unitarity**

4. **Regge approach**

$\sqrt{s} \sim 10$ **GeV**

No 'hard' inter^{ns}

k_T **are limited**

$$1 + 2 \to 3 + 4$$

$$s = (p_1 + p_2)^2 \quad t = (p_1 - p_3)^2 \quad u = (p_1 - p_4)^2$$

$$s + t + u = \sum_{i=1}^{4} m_i^2$$

$$s+t+u = 3p_1^2+p_2^2+p_3^2+p_4^2+2(p_1.(p_2-p_3-p_4))$$

$$\text{with } \; p_2 - p_3 - p_4 = -p_1$$

Analycity

$$S(t_1, t_2) \;\; ==> \;\; S(q) = \frac{1}{(2\pi)^4}\int_{-\infty}^{+\infty} e^{iEt} S(t)\theta(t)dt$$

converges at Im$E > 0$ **since** $t = t_2 - t_1 > 0$

Analitic function $==>$

$$\frac{dF}{dz} = \frac{F(z+dz) - F(z)}{dz} = const$$

does not depend on dz **'direction' for** $|dz| \to 0$**.**
Let $F = U + iV$ **and** $z = x + iy$ **then**

$$\frac{dU}{dx} = \frac{dV}{dy} \quad \text{and} \quad \frac{dU}{dy} = -\frac{dV}{dx}$$

$$\int_l (U dx - V dy) + i(U dy + V dx) = \int_\sigma dx dy [U'_y + V'_x + i(U'_x - V'_y)] = 0$$

Dispersion relation ($t = 0$)

$$\mathrm{Re}A(s) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{ds'}{s'-s}\mathrm{disc}A(s) = \frac{1}{\pi}\int_0^{\infty}\frac{ds'}{s'-s}\mathrm{Im}[A(s')+A(u')]$$

Signature $\quad A^{\pm}(u) = \pm A^{\pm}(s)$ **(Crossing symmetry)**

($\sigma^{+}(pp) = \sigma^{+}(p\bar{p}) \quad \sigma^{-}(pp) = -\sigma^{-}(p\bar{p})$**)**

$\mathbf{Im}A(s) = s\sigma_{tot}(pp)$, $\quad \mathbf{Im}A(u) = -s\sigma_{tot}(p\bar{p})$

(for $s \gg m^2$). Thus

$$\mathrm{Re}A(s) = \frac{2s}{\pi}\int_0^{\infty}\frac{ds'}{s'^2-s^2}\mathrm{Im}A(s')$$

$$\mathrm{Re}A(s, t=0) \propto d\sigma_{tot}/d\ln(s)$$

Subtraction

$$A(s) = A(s_0) + A'(s - s_0) + \frac{1}{2}A''(s - s_0)^2 + ... + a(s)$$

$$A(s) \; = \; A(s_0) + \frac{s - s_0}{\pi} \int_0^\infty \frac{ds'}{(s' - s_0)(s' - s)} \mathrm{Im}[A(s') + A(u')]$$

If No singularities $==>$ $a(s) = 0$

Max analycity $-- >$

ALL singularities have the physical nature

a) poles $1/(M_R^2 - s - i\Gamma M_R)$

b) cuts (due to $|\vec{p}| = \frac{1}{2}\sqrt{s - 4m^2}$ **in** $d\Omega = d^3p/2E$**).**

S-matrix unitarity $\sum prob^s = 1$, $S_{ik}S^+_{kj} = 1_{ij}$.

$$S = 1 + iT, \qquad SS^+ = 1 \;\; ==> \;\; 2\mathrm{Im}T = TT^+$$

2 particles elastic unitarity equation:

$$2\mathrm{Im}T_{11}(b,s) = |T_{11}(b,s)|^2 + G_{inel}(b,s) \;\; \text{with} \;\; G_{inel} = \sum_{k=2}^{\infty} T_{1k}T^+_{k1}$$

$$\mathrm{Im}T \leq 2 \quad |\mathrm{Re}T| \leq 1 \qquad l = b\sqrt{s}/2$$

Froissart limit

Assume: i) $T(s) < s^n$ **usually** $n = 2$

ii) no singularity in t-plane for $t < t_0 = 4m_\pi^2$

i.e. $T(b,s) < se^{-2bm_\pi}$ $==>$ $\mathbf{Im}T(b,s) \leq e^{(R-b)2m_\pi}$

small for $b > R = \ln s/2m_\pi$ **for** $b < R$ $\mathbf{Im}T < 2$

$$\sigma \leq \pi R^2 = \frac{\pi}{m_\pi^2} \ln^2 s$$

photon contribution (with $t_0 = 0$**) is negelected**

Regge theory, RFT (and multiperipheral model)

$$2mER(r) \; = \; \left[-\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dR}{dr} \right) \; + \; \frac{l(l+1)}{r^2} R + U(r)R \right]$$

$$E = E(l)$$

$$A_\pi = g^2_{\pi NN}/(t - m^2_\pi) \propto s^0, \; A_\gamma = s \cdot 8\pi\alpha/t \propto s^1 \; ....$$

$$A_J \propto s^J = s^{\alpha(t)}, \qquad A(s) = \int_{-\infty}^{+\infty} A(J) \left(\frac{s}{s_0} \right)^J dJ$$

Veneziano model – only resonances and Regge poles.

$$A(s,t) = \frac{\Gamma(1-\alpha_R(s))\Gamma(1-\alpha_R(t))}{\Gamma(1-\alpha_R(s)-\alpha_R(t))}$$

$$+\frac{\Gamma(1-\alpha_R(s))\Gamma(1-\alpha_R(u))}{\Gamma(1-\alpha_R(s)-\alpha_R(u))}+\frac{\Gamma(1-\alpha_R(u))\Gamma(1-\alpha_R(t))}{\Gamma(1-\alpha_R(u)-\alpha_R(t))}$$

RFT

$\mathbf{2Im}T = |T|^2 + G_{inel}$ − General solution

$$T(b,s) \;=\; i(1 - e^{-\Omega(b,s)/2}) = i\left[\frac{\Omega}{2} - \frac{1}{2}\left(\frac{\Omega}{2}\right)^2 + ...\right]$$

$$\Omega_P(b,s) = \int \frac{d^2q}{4\pi^2}\frac{A_P(q,s)}{is}e^{i\vec{q}\vec{b}}$$

Miltiperiphery

$$A \propto s^{g^2()-1} \qquad n \sim \ln(s/s_0) \qquad \Delta b \sim 1/ < k_t >$$

$$B_{el} \sim (\Delta b)^2 = B_0 + \alpha'_P \ln(s/s_0)$$

$$R < c \cdot \ln s \qquad \sigma < 2\pi R^2 < \tilde{c} \ln^2(s)$$