Decay of the free neutron to the atom of hydrogen

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General picture

Beta decay of the neutron. Main channel

$$n \to p + e^- + \bar{\nu}_e.$$

The energy $E_{fin} - E_{in} = 1.29$ MeV. The energy $T = 783 keV$ is shared between the proton, electron and antineutrino.
Another channel

$$n \to H + \bar{\nu}_e.$$

Here H in the ground state $1s$ or in an excited state. The energy $T = 783 keV$ is shared between the atom of hydrogen and antineutrino. In this process

$$\mathbf{p}_{\bar{\nu}} + \mathbf{p_H} = 0; \quad E_\nu + T_H = T; \quad E_\nu = p_\nu$$

Thus

$$E_\nu^2/2m_p + E_\nu = T$$

The kinetic energy is $T_H = 352$ eV, $E_\nu = 783$keV.

Four-fermion local interaction. V-A theory.

The four-fermion local interaction

$$L = \frac{G}{\sqrt{2}}\,\bar{\psi}_e(x)\,\Gamma^i\,\psi_\nu(x)\,\bar{\psi}_n(x)\,\Gamma_i\,\psi_p(x),$$

with $\Gamma_i = I,\ \gamma_5,\ \gamma_\mu,\ \gamma_5\gamma_\mu,\ \sigma_{\mu\nu}$ or their superpositions.

$$\gamma_5 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}.$$

For the lepton current

$$\Gamma = \gamma_\mu(1+\gamma_5) = V_\mu - A_\mu$$

$$L_\mu = \bar{\psi}_e\gamma_\mu(1+\gamma_5)\psi_\nu.$$

V-A theory.

Note that

$$\frac{1+\gamma_5}{2}\,\psi = \psi^L; \qquad \frac{1-\gamma_5}{2}\,\psi = \psi^R$$

$$\bar{\psi}^L = \bar{\psi}\,\frac{1-\gamma_5}{2}; \qquad \bar{\psi}^R = \bar{\psi}\,\frac{1+\gamma_5}{2}$$

Thus the factor $1+\gamma_5$ in the lepton current is the consequence of the equality

$$m_{\bar{\nu}} = 0.$$

The nucleon current

$$J_\mu = \bar{\psi}_n(\gamma_\mu + a\gamma_\mu\gamma_5)\psi_p; \quad a = |g_A/g_V| \approx 1.27.$$

Decay to the bound state

For the decay to continuum states

$$dW = \omega(p_e)\frac{d^3p_e}{(2\pi)^3} = N^2(p_e)\omega_e(p_e)\frac{d^3p_e}{(2\pi)^3}; \quad N(p_e) = \psi_{p_e}(r=0).$$

For decay to a bound state with the principle quantum number n

$$W_n^b = \omega_e(p_e = 0)|\psi_b(r=0)|^2.$$

Only decays to the ns states contribute, and

$$|\psi_{ns}(r=0)|^2 = \frac{(m\alpha)^3}{n^3\pi}.$$

Thus we expect $W_b/W \sim 10^{-6}$. Indeed

$$W_b/W = 3.9 \times 10^{-6}.$$

Why study?

Let $\mathbf{p_H}$ be the axis of quantization. We find

$$m_n = m_p + m_e + m_{\bar{\nu}}; \quad m_{\bar{\nu}} = -1/2.$$

Decays to the states with $F = 1$ are quenched. The probability is proportional to $(1 - g_A/g_V)^2 \approx 1/13$. In the nonrelativistic limit

$$\langle n|J_\alpha|p\rangle = g_V \psi_n^* \psi_p \delta_{\alpha 0} + g_A \psi_n^* \sigma_i \psi_p \delta_{\alpha i}; \quad i = 1, 2, 3.$$

The operator

$$1 - \sigma_p \sigma_e$$

kills the states with $F = 1$ in the atom of hydrogen.

Why study?

Possible channels of the bound state decay

$m_{\bar{\nu}}$	m_n	m_p	m_e	F	M_F	W/W_{tot}
-1/2	-1/2	-1/2	1/2	0;1	0	0.44
-1/2	-1/2	1/2	-1/2	0;1	0	0.55
-1/2	1/2	1/2	1/2	1	1	0.006
-1/2	1/2	-1/2	-1/2	1	-1	0

It is interesting to measure the probability of the process in channel IV-a nonzero value signals about nonzero value of the antineutrino mass. The probability of the process in channel III is small. It is sensitive to possible interactions beyond the $V - A$.

$$F = F_{V-A} + F_S + F_{Ps} + F_T$$

$$F_i = \frac{G}{\sqrt{2}} \left[g_S\, \bar{\psi}_e \psi_\nu \bar{\psi}_n \psi_p + g_{Ps}\, \bar{\psi}_e \gamma_5 \psi_\nu \bar{\psi}_n \gamma_5 \psi_p + g_T\, \bar{\psi}_e \sigma_{\alpha\beta} \psi_\nu \bar{\psi}_n \sigma^{\alpha\beta} \psi_p \right].$$

We look for interference between F_{V-A} and F_i.

$$\psi_\nu = \frac{1+\gamma_5}{2}\,\psi_\nu + \frac{1-\gamma_5}{2}\,\psi_\nu$$

The probability of the decay to the state with $F = 1, M_F = 1$

$$W_3 \approx (1-a)\frac{1 - a + g_S - 2g_T + 4g_{Ps}v_H}{8}; \quad a = |\frac{g_A}{g_V}| \approx 1.275.$$

Note that

$$v_H = 8 \times 10^{-4} \sim 10^{-3}.$$

The probabilities W_i depend on g_S and g_T in terms of the function

$$\chi = \frac{1+g_S}{a+g_T}$$

The current limits

$$|g_S| < 0.06$$

(Adelberger, 1999)-from positron-neutrino correlations in $0^+ \to 0^+$ beta decay of argon.

$$-3 \times 10^{-2} < g_T < 6 \times 10^{-3},$$

(Wauters et al, 2010)-from the measurements of the beta-asymmetry parameter in the decay of ^{60}Co.

$$g_{PS} < 10^{-3},$$

(Herczeg,1995) from the decay $\pi^- \to e^- \bar{\nu}$.

We start with the massless leptons e^L and ν^L and two scalar fields ϕ_+ and ϕ_0.
The local invariance under the $U(1)$-transformation.

$$\psi'(x) = e^{ie\lambda(x)}\psi(x); \quad \bar{\psi}'(x) = e^{-ie\lambda(x)}\bar{\psi}(x).$$

One can not have a local $U(1)$ invariant theory of the free electron

$$L = \bar{\psi}(x)(i\gamma_\mu\partial_\mu - m)\psi(x); \quad L' = L + ie\partial_\mu\lambda(x)\cdot\bar{\psi}(x)(i\gamma_\mu\partial_\mu)\psi(x)$$

Thus we obtained a vector field.
There is a doublet of massless leptons e^L and ν^L in the Standard model

$$\begin{pmatrix} \nu^L \\ e^L \end{pmatrix}$$

The Standard model. The milestones

We request the local $SU(2)$ invariance. The transformation is $e^{ig\tau_i \cdot n^i}$. This provides three massless vector fields. We have also a doublet of scalar mesons

$$\begin{pmatrix} \varphi^+ \\ \varphi \end{pmatrix}$$

which enter the Lagrangian as

$$L = ... + (\partial_\mu \phi)^2 - \frac{\lambda^2(\varphi^2 - \eta^2)^2}{2}$$

It obtains the minima at $\varphi = \pm\eta$. We present $\varphi(x) = \eta + \chi(x)$.

Finally we have three massive vector mesons (W^+,W^- and Z) and massive scalar $\chi(x)$ (the Higgs boson).

The important parameter is $\eta = \langle 0|\varphi^*\varphi|0\rangle$.

How do the vector mesons obtain masses?

$$\partial_\alpha \varphi \partial^\alpha \varphi \to D_\alpha \varphi D^\alpha \varphi$$

with

$$D_\alpha \varphi(x) = (\partial_\alpha - \frac{i}{2} g \tau_\mu W_\alpha^\mu) \varphi(x)$$

Employing $\varphi(x) = \eta + \chi(x)$ we find the contribution

$$\frac{1}{4} g^2 \eta^2 W_\mu W^\mu$$

to the term

$$D_\alpha \varphi D^\beta \varphi$$

of the Lagrangian. This corresponds to mesons W with the masses $m_W^2 = g^2 \eta^2 / 4$.

Beta decay in the Standard model

$$F = g^2 \langle n|\bar{d}_L \gamma_\alpha D_{\alpha\beta} u_L|p\rangle \bar{\psi}_{eL}\gamma^\beta \psi_{\nu L}.$$

This is written for leptons and quarks.

$$\frac{g^2}{8m_W^2} = \frac{G}{\sqrt{2}}; \quad g = g_V \approx 1$$

$$J_\alpha = \langle n|\bar{d}_L \gamma_\alpha u_L|p\rangle = \bar{\psi}_n \gamma_\alpha (1 + g_A \gamma_5)\psi_p$$

One can write other Lorentz structures for the nucleon matrix elements but only $V - A$ contribute to interaction with the leptons. In framework of the standard model

$$a_S = a_T = 0.$$

However,there are also the recoil terms. For the vector current

$$\langle n|J_\alpha|p\rangle_{rec} = \psi_n^*(b_S\frac{q_\alpha}{m_p} + b_T\frac{\sigma_{\alpha\beta}q_\beta}{2m_p})\psi_p$$

Similar terms for the axial current are quenched by the factor v_H One immediately finds $b_S = 0$ while b_T describes the modification of the vector current by the strong interactions.

In the models with several doublets of scalar mesons there are the charged Higgs scalars.
There are two versions

$$a_S \sim (1-a)\frac{m_W^2}{m_{Higgs}^2}; \quad m_{Higgs} > 600\,GeV.$$

Hence

$$a_S < 10^{-3}$$

Interaction of the neutral Higgs scalars with fermions

$$f_e \bar{e}_R \varphi e_L = f_e \bar{e}_R e_L \eta + f_e \bar{e}_R \chi e_L = m_e \bar{e}_R e_L + \frac{m_e}{\eta} \bar{e}_R \chi e_L$$

$$a_s \sim \frac{m_e}{\eta}\frac{m_N}{\eta}\frac{m_W^2}{m_{Higgs}^2}; \quad m_{Higgs} > 80\,GeV$$

Recall that $\eta > 250$ GeV.

$$|a_S| < 4 \times 10^{-6}$$

Beyond the Standard model. Leptoquarks

They can carry both the baryon and lepton quantum numbers. The LQ contribute to the exchange terms for the beta decay

$$F_{exch} = g_{LQ}^2 \langle n|\bar{u}J_\alpha \nu \bar{e} J_\beta d|p\rangle D_{\alpha\beta}^{LQ} +$$

$$g_{LQ}^2 \langle n|\bar{u}J_\alpha e \bar{\nu} J_\beta d|p\rangle D_{\alpha\beta}^{LQ}$$

The LQ carry $S = 0$ or $S = 1$.
The only limitation is $m_{LQ} > 1$ TeV.
Thus we can not exclude

$$|a_S| \sim 0.01; \quad |a_T| \sim 0.01$$

Left-right symmetric extension of the Standard model.

$$SU(2)_L \times SU(2)_R \times U(1)$$

A finite contribution to W_4 ($m_F = -1$).
In this model

$$\begin{pmatrix} \nu^L \\ e^L \end{pmatrix}; \quad \begin{pmatrix} \nu^R \\ e^R \end{pmatrix},$$

In the simplest version there are two distinct vector mesons

$$F = \frac{g_L^2}{2m_{WL}^2} J_{\alpha,L}^{Lept} J_{\alpha,L}^{quark} + \frac{g_R^2}{2m_{WR}^2} J_{\alpha,R}^{Lept} J_{\alpha,R}^{quark}$$

Left-right symmetric extension of the Standard model.

$$W_4 \sim \frac{g_R^4}{g_L^4}\frac{m_{WL}^4}{m_{WR}^4}$$

From experiments on the muon decay $m_{WR}/g_R < 576$GeV(R. Bayes et al, 2011) This provides

$$W_4 < 3.7 \times 10^{-4}.$$

If $g_{AR}/g_{VR} \approx g_{AL}/g_{VL}$

$$W_4 < 3 \times 10^{-5}.$$

Left-right symmetric extension of the Standard model.

The fields W_R and W_L are linear combinations of the mass-eigenstates W_1 and W_2

$$W_L = W_1 \cos\zeta + W_2 \sin\zeta$$

$$W_R = -W_1 \sin\zeta + W_2 \cos\zeta; \quad \zeta \ll 1.$$

In this model

$$W_4 \sim \frac{g_R^4}{g_L^4} \frac{m_{WL}^2}{m_{WR}^2} g_R \zeta.$$

From experiments on the muon decay $|g_R \zeta| < 0.020$(R. Bayes et al, 2011) Thus

$$W_4 < 4 \times 10^{-4}$$

THANK YOU FOR LISTENING!