CLAS/CLAS12: изучение структуры нуклона в экспериментах по рождению мезонов

Глеб Федотов

Structure of the Talk

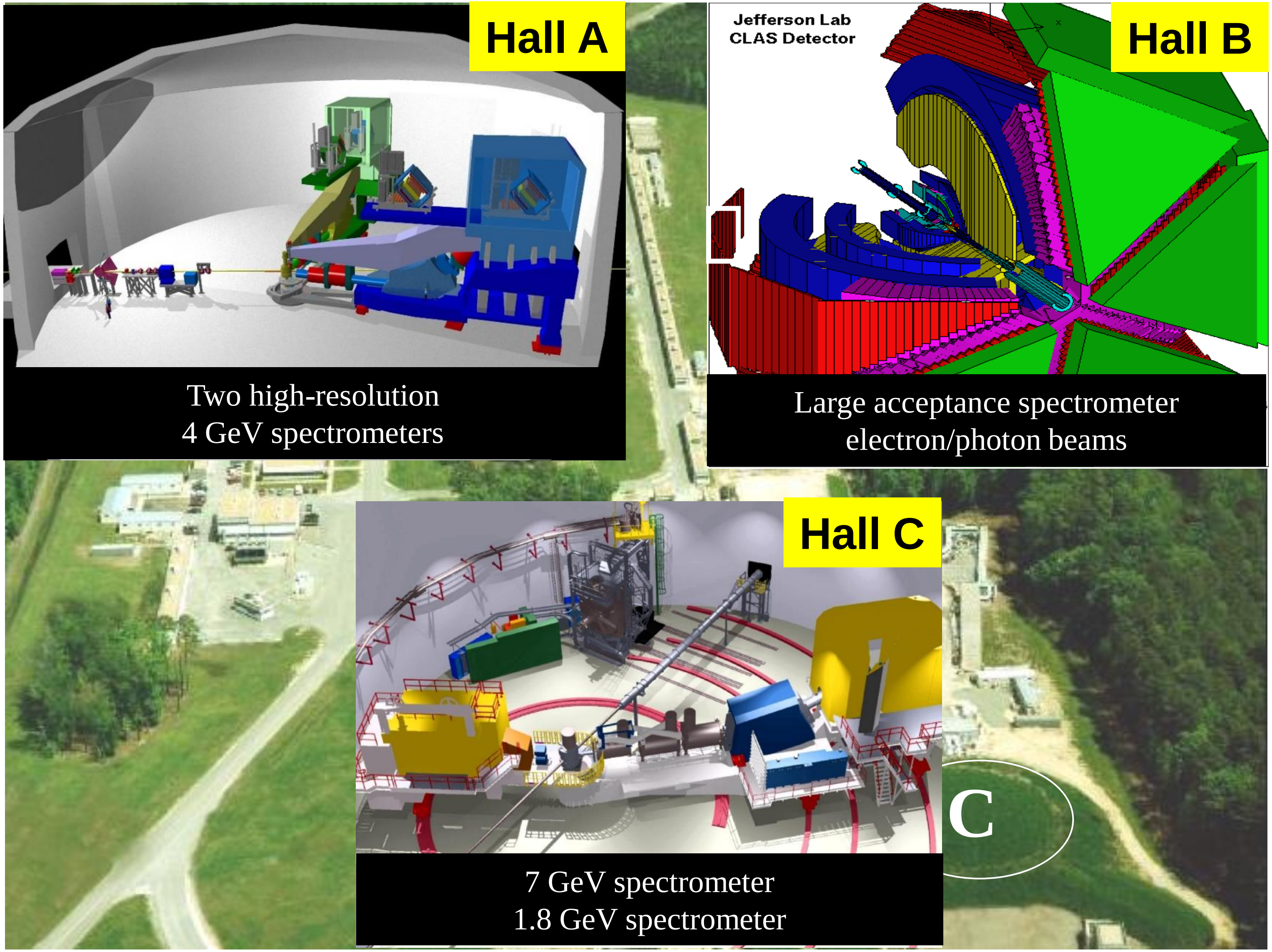
- The CLAS detector:

  - Thomas Jefferson Laboratory (Newport News, VA, USA)
  - Cross section measurements of the $\pi^+\pi^-$ electroproduction off proton
  - Physical interpretation of the extracted cross sections

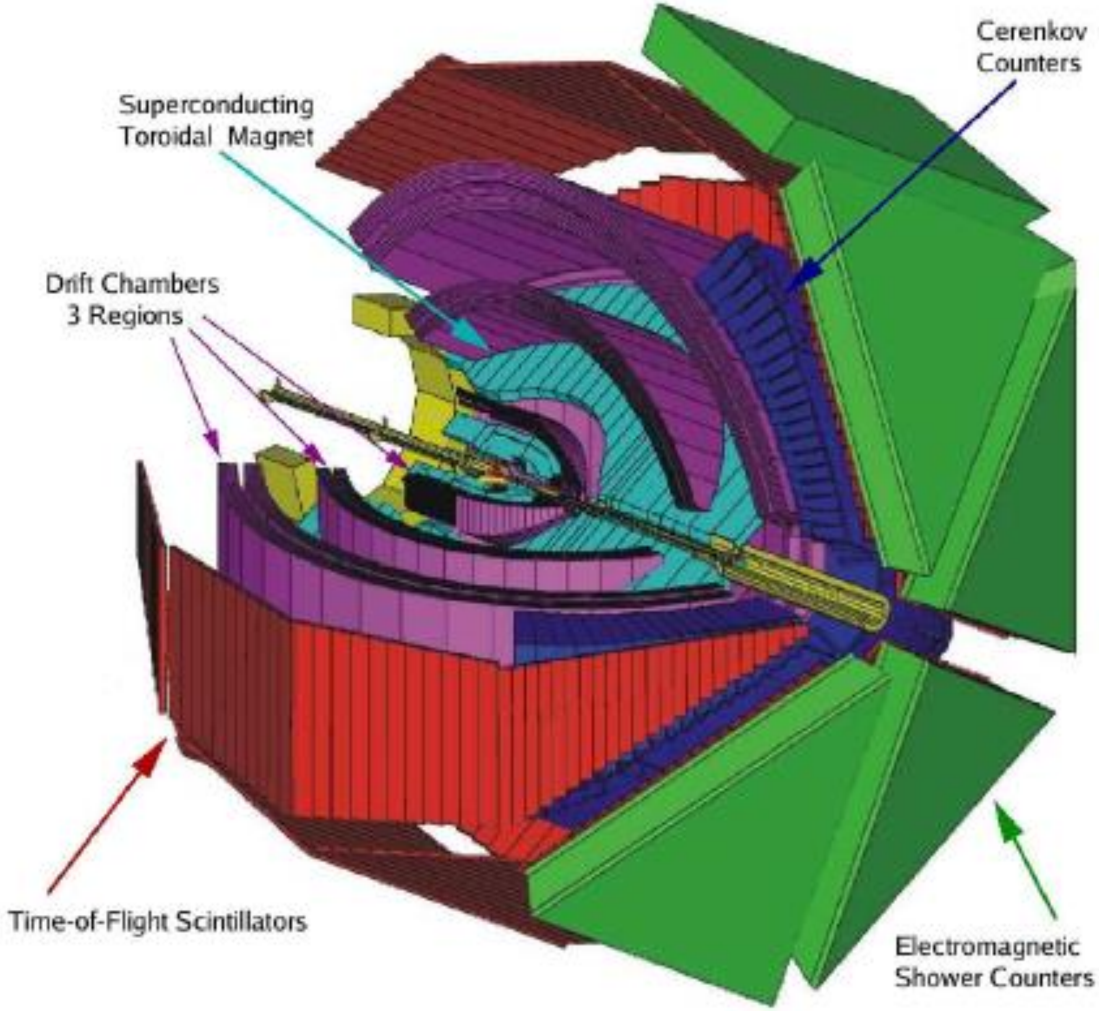
- The CLAS12 detector:

  - JLab upgrade. Transition from CLAS to CLAS12
  - Forward Time-of-Flight system for CLAS12
  - Further experiments

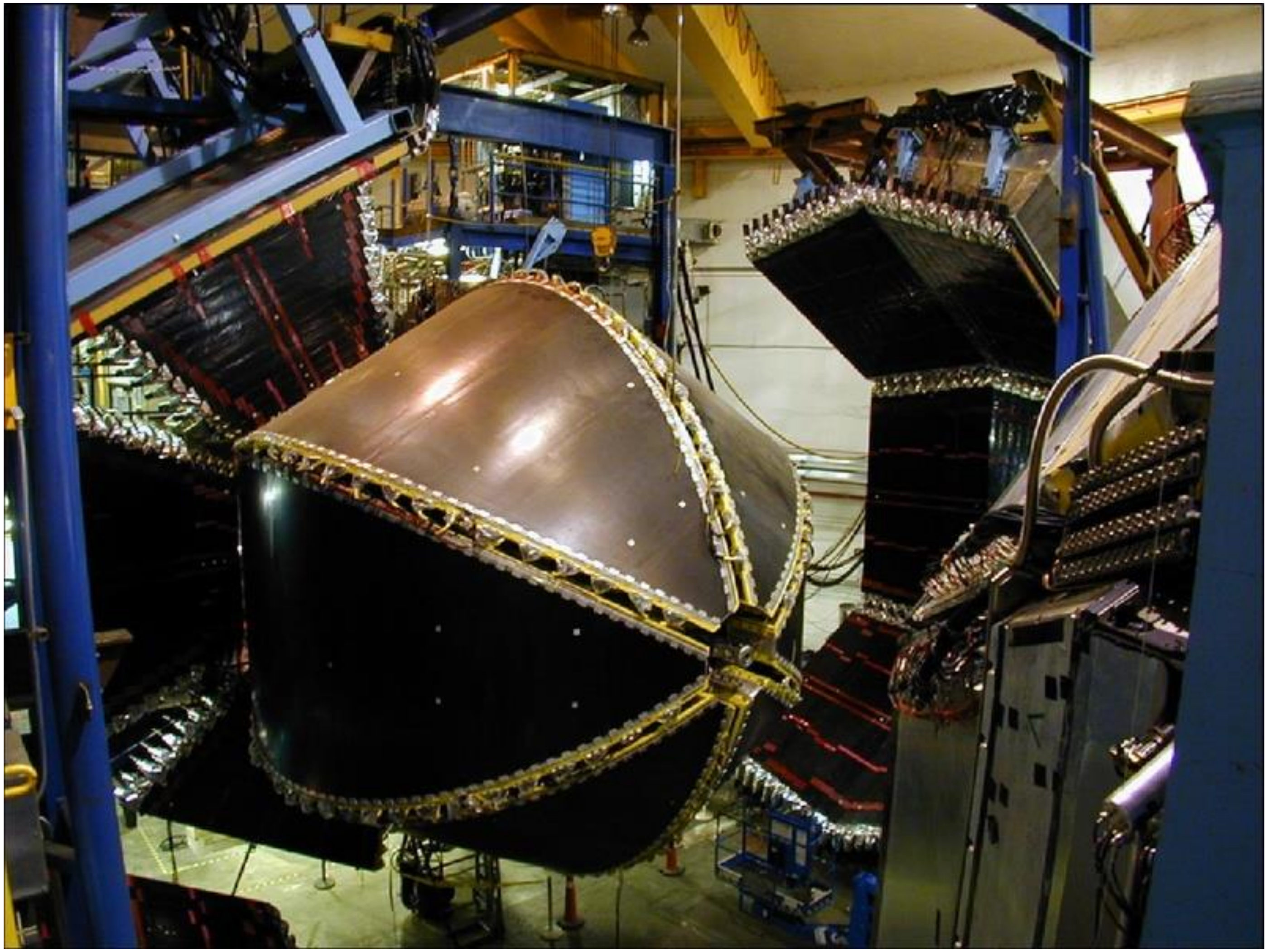
Experimental Halls



CEBAF Large Acceptance Spectrometer at Hall-B



Detector CLAS at Hall-B



Photoproduction Data from CLAS

Final state	Observables	W, GeV
$\pi^0 p$	$\frac{d\sigma}{d\Omega}$	1.5 - 2.5
$\pi^+ n$	$\frac{d\sigma}{d\Omega}$	2.1 - 2.5
ηp	$\frac{d\sigma}{d\Omega}$	0.8 - 2.8
$\pi^+\pi^- p$	Y_{ij}, $\frac{d^2\sigma_l}{dtdM_{\pi\pi}}$, $\frac{d^2\sigma_{10,\pm}}{dtdM_{\pi\pi}}$, ρ^0_{ij}, $\frac{d\sigma}{dt}$	2.6 - 2.8
	$Y(\pm)$	1.4 - 2.3
$K^+\Sigma^0, K^+\Lambda$	$\frac{d\sigma}{dcos(\theta^{cm}_K)}$	1.7 - 2.5 1.7 - 2.8
	C_x, C_z	1.8 - 2.5
	P_Λ, P_Σ	1.7 - 2.8

CLAS Physics Database: http://depni.sinp.msu.ru/cgi-bin/jlab/db.cgi

Electroproduction Data from CLAS

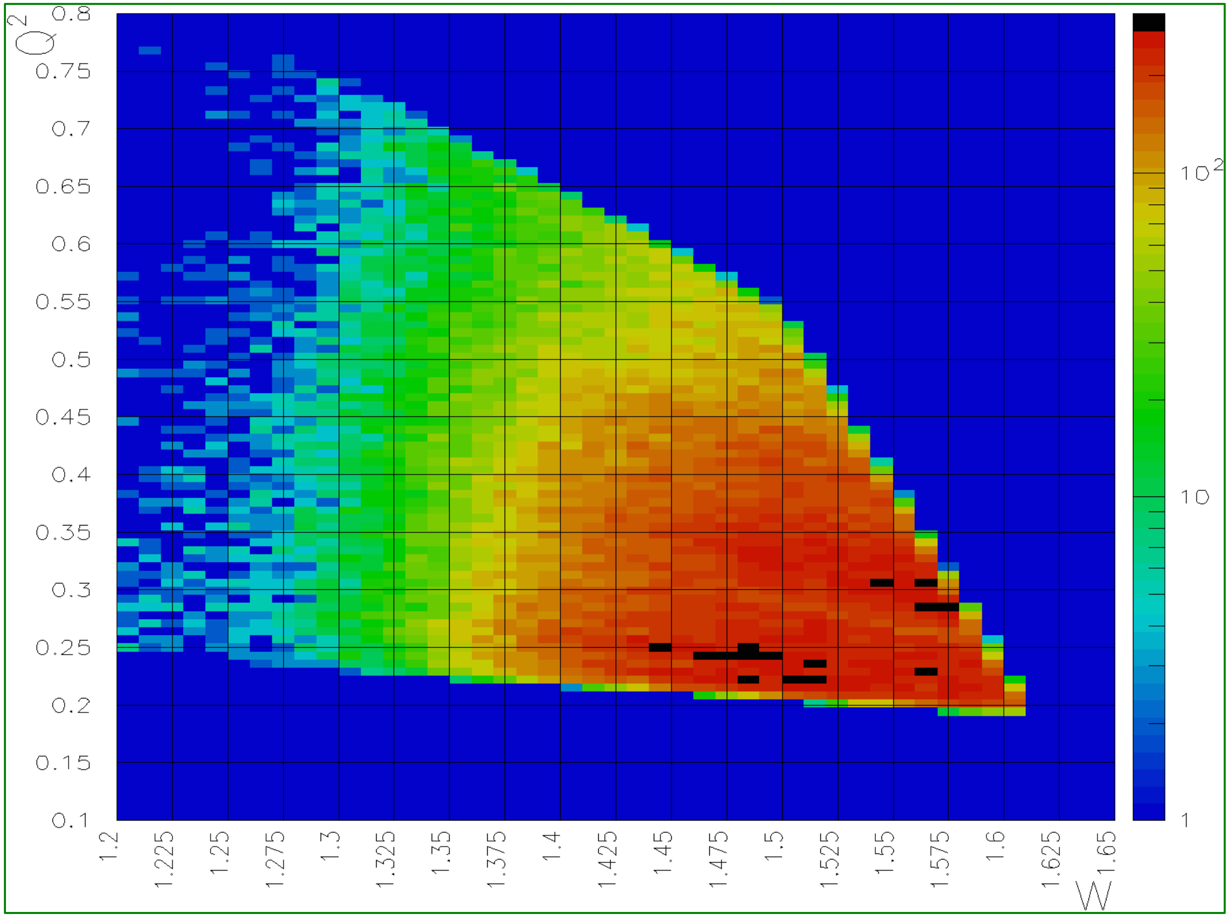
Final state	Observables	W, GeV	Q^2, GeV^2
$\pi^0 p$	$\frac{d\sigma}{d\Omega}$	1.1 - 1.4 1.1 - 1.7	0.16 - 0.36, 3.0 - 6.0 0.4 - 1.8
	$A_{LT'}$	1.1 - 1.7	0.4, 0.65
	A_t, A_{et}	1.1 - 1.6	0.25, 0.39, 0.61
$\pi^+ n$	$\frac{d\sigma}{d\Omega}$	1.1 - 1.4 1.1 - 1.6 1.1 - 1.7	0.16 - 0.36 0.3 - 0.6 1.7 - 4.5
	$A_{LT'}$	1.1 - 1.7	1.7 - 4.5, 0.4, 0.65
ηp	$\frac{d\sigma}{d\Omega}$	1.5 - 1.9 1.5 - 2.3	0.38 - 1.39 0.17 - 3.1
$\pi^+\pi^- p$	σ, $\frac{d\sigma}{dM_{inv}^{ij}}$, $\frac{d\sigma}{d\psi_i}$, $\frac{d\sigma}{d\varphi_i}$, $\frac{d\sigma}{d(-cos(\theta_i))}$	1.4 - 1.9 1.3 - 1.6 1.4 - 2.0	0.65, 0.95, 1.3 0.2 - 0.6 2 - 5
$K^+\Sigma^0$	σ, σ_{TT}, σ_{TL}, $\frac{\sigma_L}{\sigma_T}$, $\frac{d\sigma}{d\Omega}$	1.7 - 2.3	0.65 - 2.55
$K^+\Lambda$	σ, σ_{TT}, σ_{TL}, $\sigma_{LT'}$, $\frac{\sigma_L}{\sigma_T}$	1.7 - 2.4	0.65 - 2.55

CLAS Physics Database: http://depni.sinp.msu.ru/cgi-bin/jlab/db.cgi

Double Pion Electroproduction with CLAS

$$ep \rightarrow e' p' p^+ p^-$$

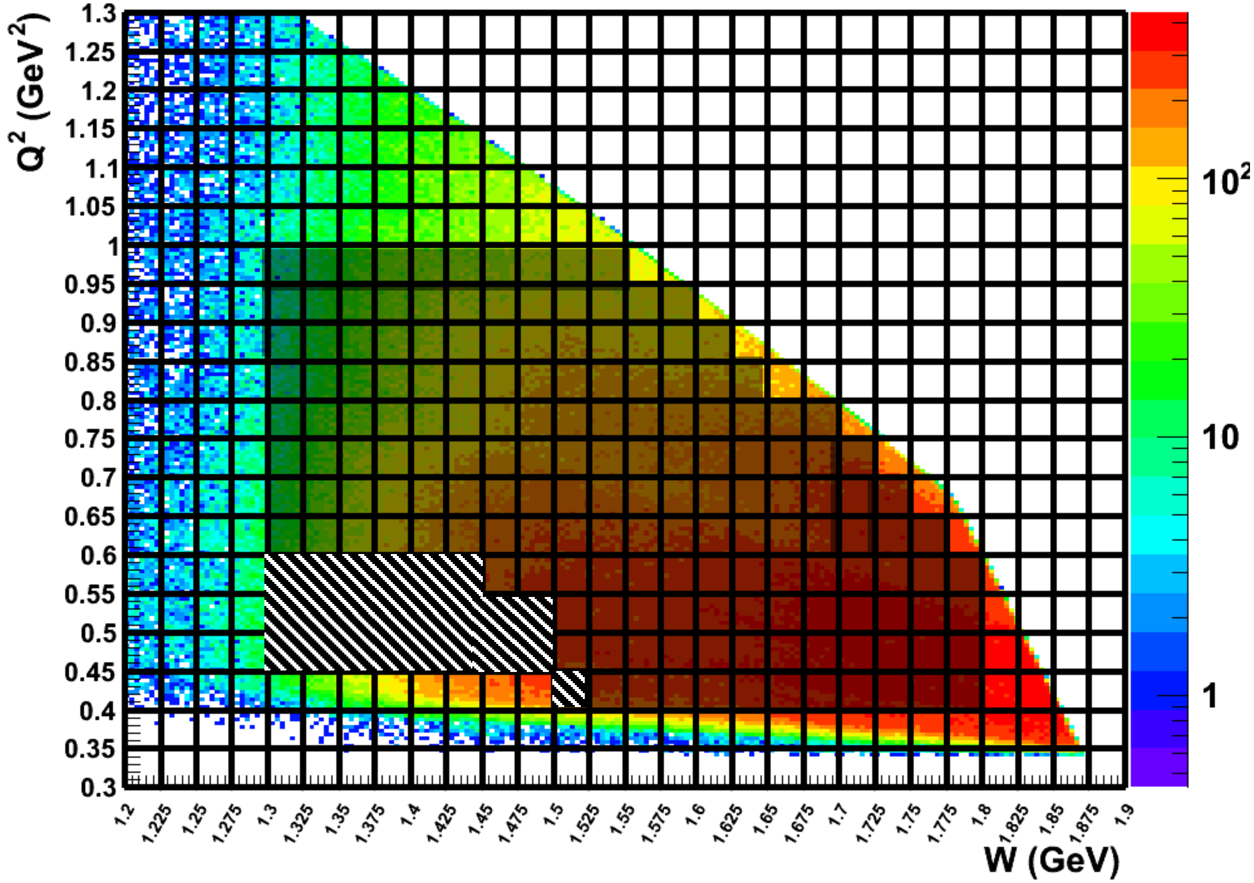
The dataset with E_{beam} = 1.5 GeV



Double Pion Electroproduction with CLAS

$$ep \rightarrow e' p' p^+ p^-$$

The dataset with E_{beam} = 2 GeV



Cross Section Calculation

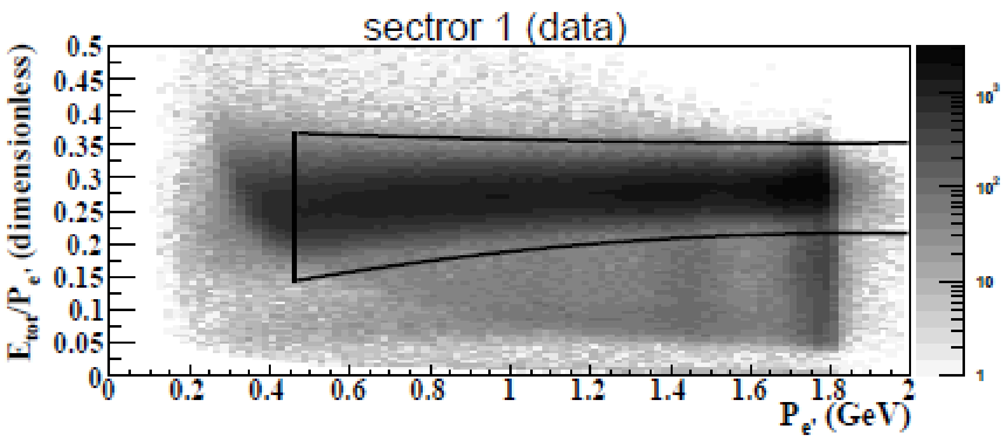
$$\frac{d^7\sigma}{dW\,dQ^2 d^5\tau} = \frac{1}{F \cdot R} \frac{\left(\frac{N_{full}}{Q_{full}} - \frac{N_{empty}}{Q_{empty}}\right)}{\Delta W \Delta Q^2 \Delta^5\tau \left(\frac{l\rho N_A}{q_e M_H}\right)}$$

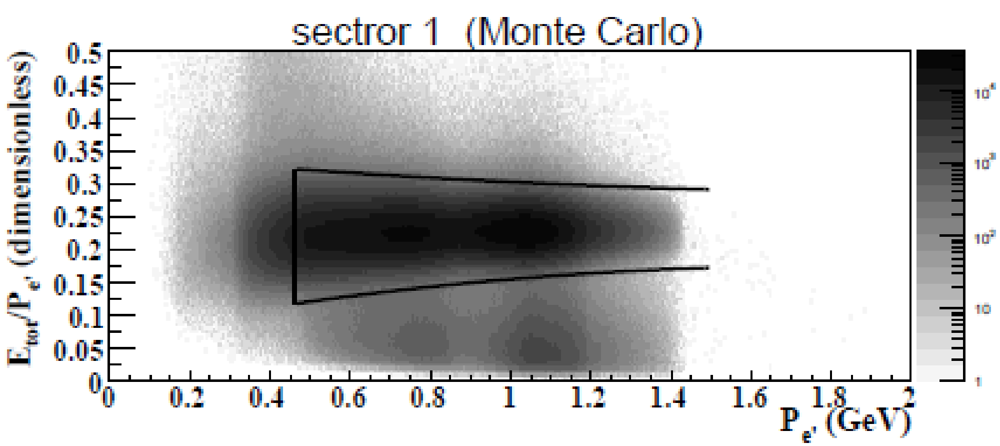
$$\Delta^5\tau = \Delta M_{h_3 h_2} \Delta M_{h_1 h_2} \Delta(-cos(\theta_{h_1}))\Delta\varphi_{h_1} \Delta\alpha_{h_1}$$

- N_{full} and N_{empty} are the numbers of double-pion events inside the seven-dimensional bin for runs with hydrogen and empty target, respectively.
- Q_{full} and Q_{empty} are the integrated Faraday cup charges for runs with hydrogen and empty target, respectively.
- $F(\Delta W, \Delta Q^2, \Delta^5\tau) = \frac{N_{rec}}{N_{gen}}$ is the efficiency determined by the Monte Carlo simulation.
- N_{rec}, N_{gen} are the numbers of reconstructed and generated events, respectively.
- $R(\Delta W,\Delta Q^2)$ is the radiative correction factor.

Electron Identification (EC cut)

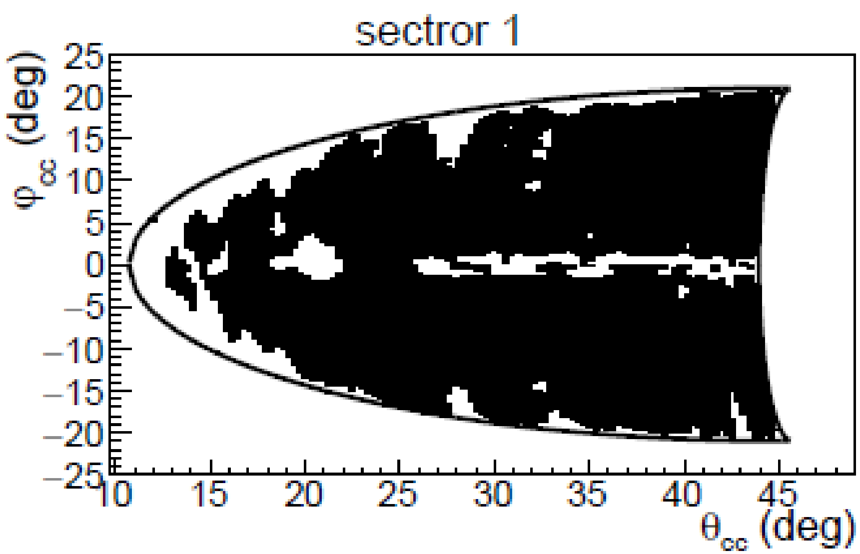
Electrons are identified by their coincident signals in the SC, DC, EC and CC detector elements as the first in time particles.

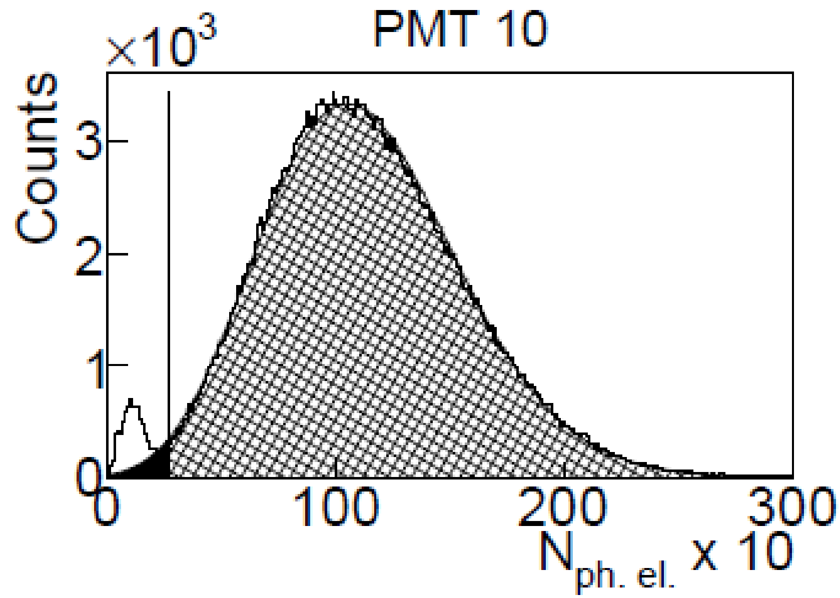




- Total energy deposited in the Electron Calorimeter (EC) divided by electron momentum versus electron momentum.
- We identify events enclosed by the black curves as electrons.

Electron Identification (CC cut)



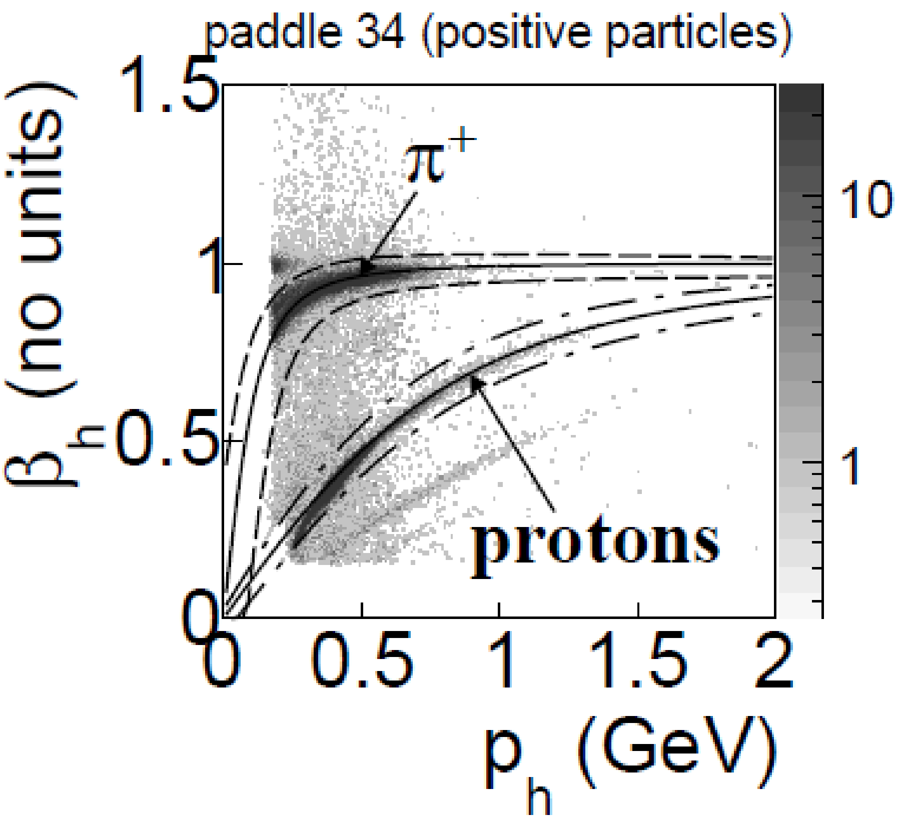


- Only efficient CC zones, shown in black in the left figure, were included into analysis.
- An additional correction factor was introduced to recover removed good events (shown by the black area in the right plot).

Hadron Identification

To identify hadrons, the information from the Drift Chambers (DC) and the Time-of-Flight system (ToF) is used.

- β versus momentum distribution for positively charged particles seen by ToF scintillator paddle 34 of CLAS sector one.
- Black solid curves are theoretical calculations with exact hadron mass assumptions.
- Events between the two dashed and two dot-dashed curves are selected as π^+ and proton candidates, respectively.



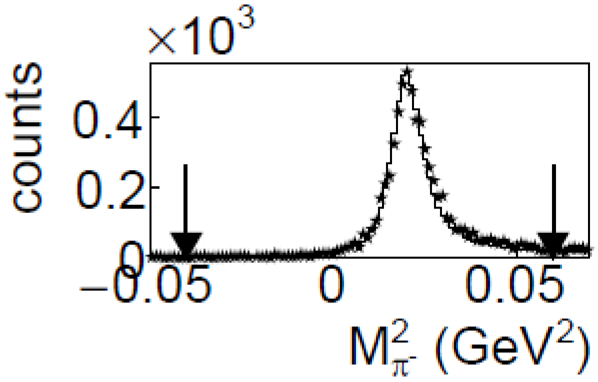
Event Selection

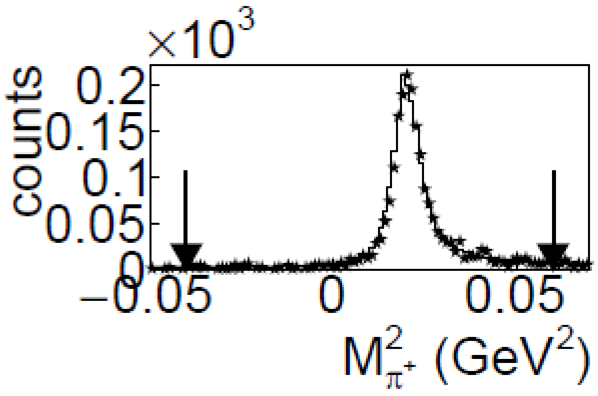
Cuts	Data	Simulation
Fiducial	yes	yes
EC-cut	yes	yes
CC-cut	yes	no/yes
β vs. p	yes	yes
θ vs. p	yes	yes
Electron momentum correction	yes	no
Proton energy loss correction	yes	yes
Exclusivity cut	yes	yes

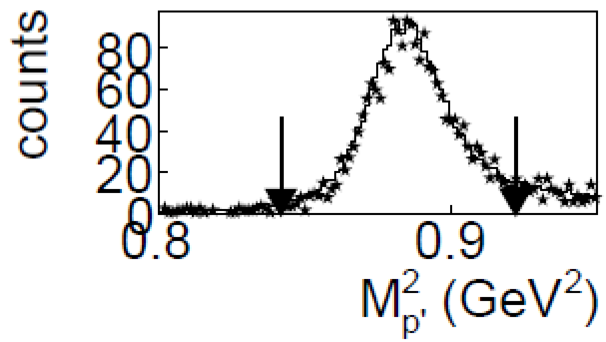
EC-cut, CC-cut → Electron identification

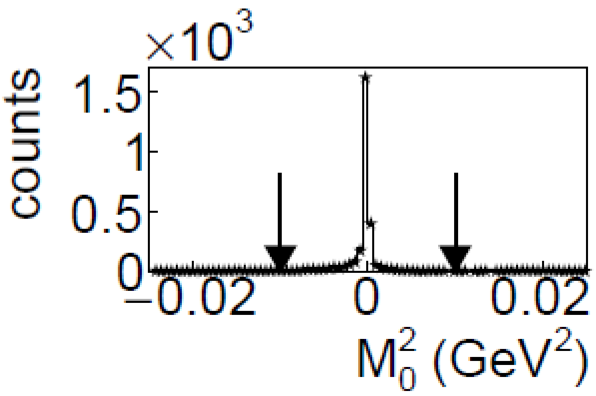
β vs. p → Hadron identification

Topologies and Exclusivity Cut









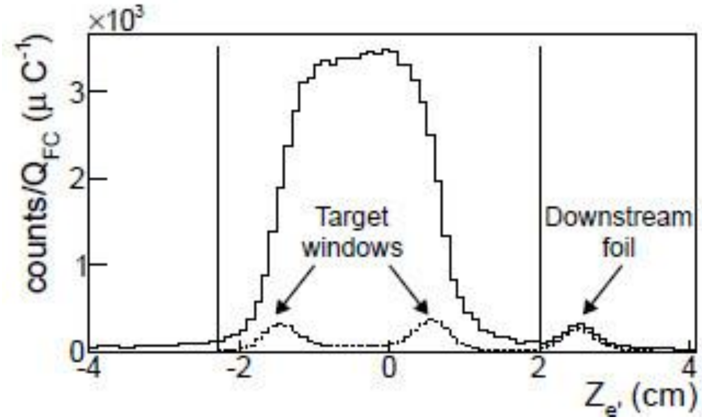
Topology	**Statistics (%)**
π^- missing	≈70
π^+ missing	≈10
Proton missing	≈10
Exclusive	≈10

For the first time for double-pion electroproduction all four topologies were combined together.

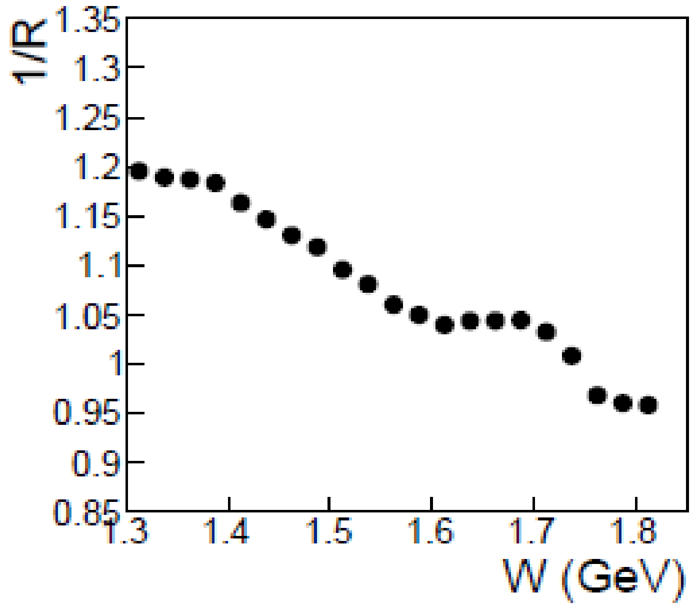
Three-pion background was taken into account according to the ratio of three-pion/double-pion cross sections taken from the paper: C. Wu et al., Eur. Phys. J. A23, 317 (2005).

Corrections to the Cross Section

- Empty target subtraction.



- Filling of the empty cells.



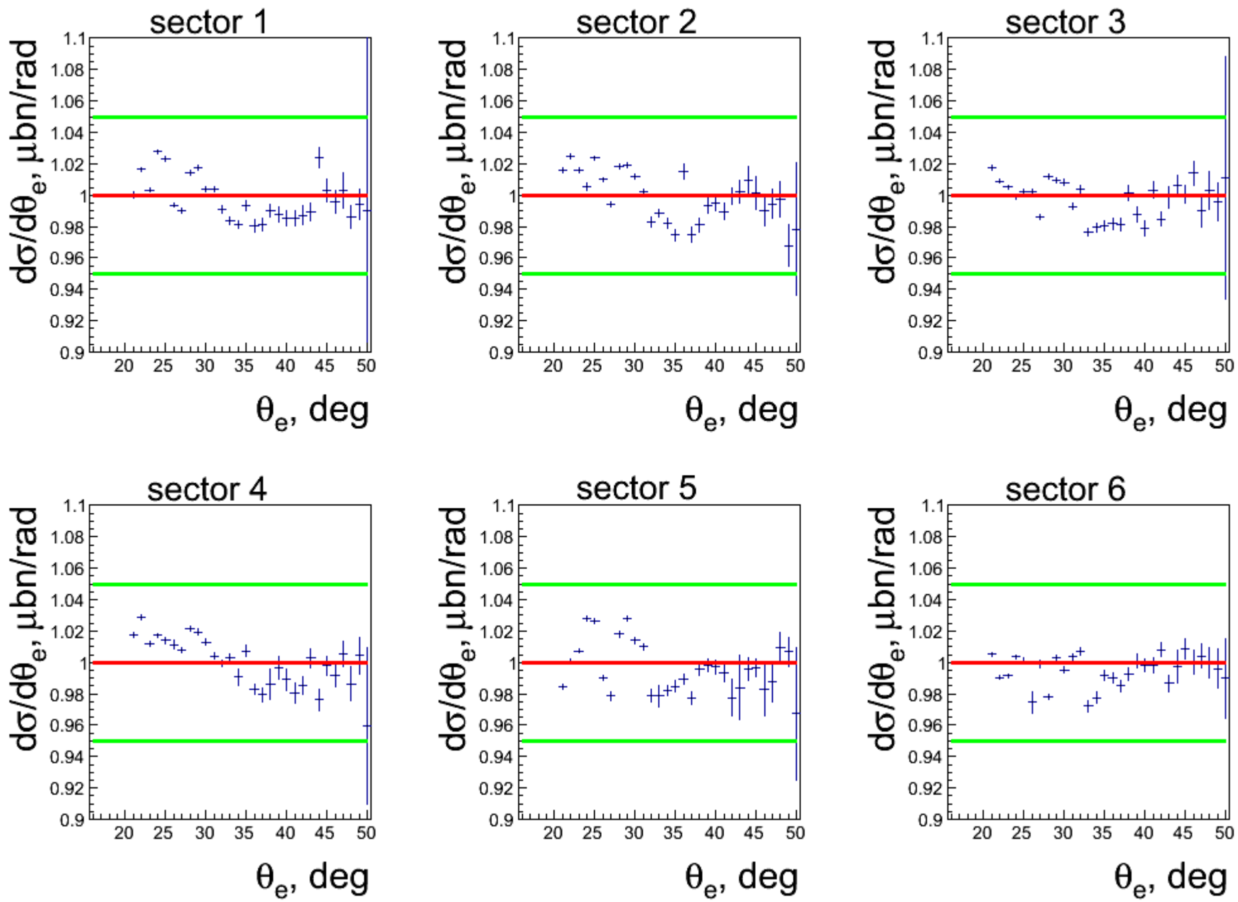
- Radiative corrections [1].

- Bin centering correction.

[1] L. W. Mo and Y.-S. Tsai, Rev.Mod.Phys. 41, 205 (1969).

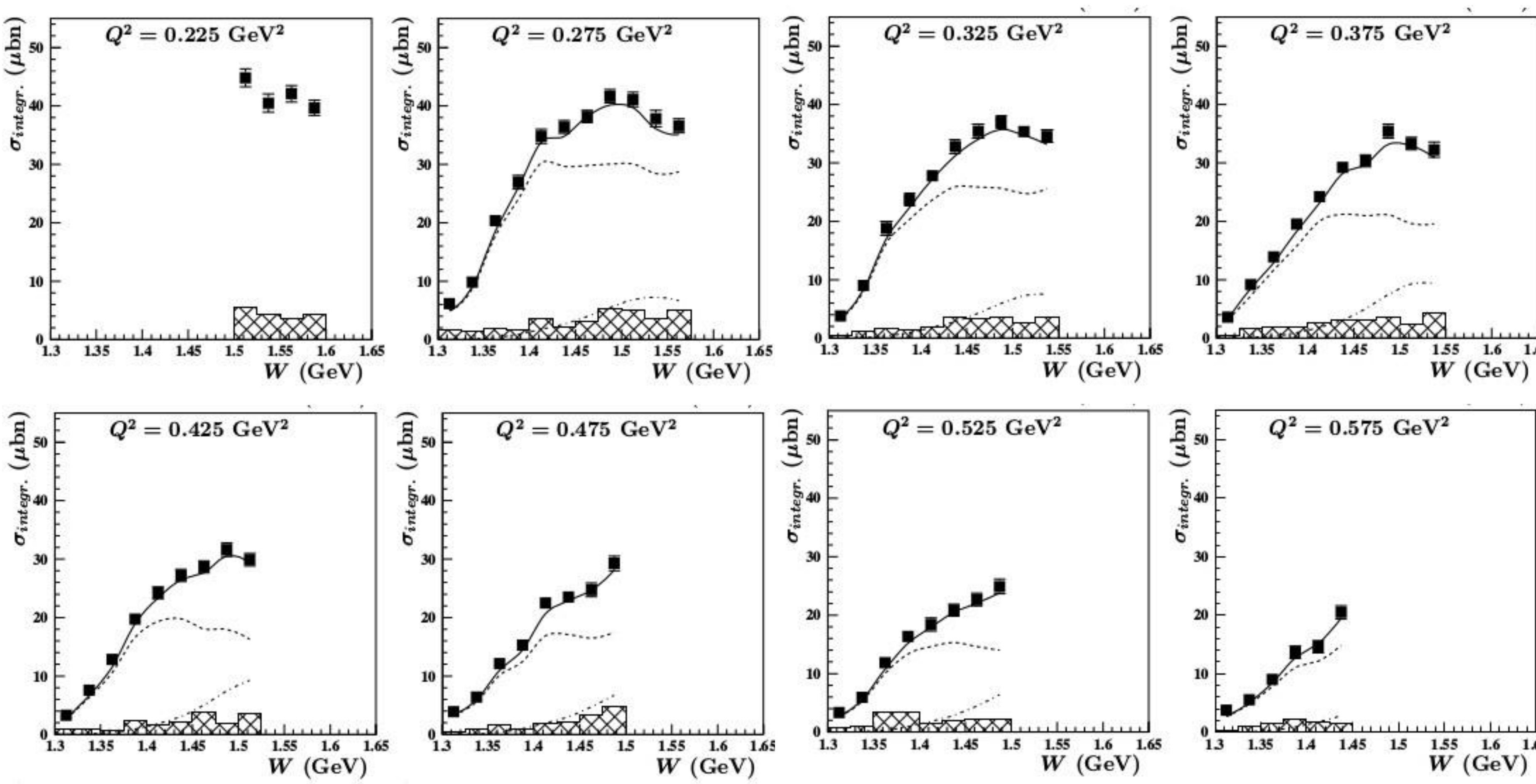
Cross Section Normalization Check

Ratio of the elastic cross section to the P. Bosted parameterization plotted versus θ_e. The parameterized cross sections are "radiated" and compared to the extracted elastic cross sections that are not corrected for radiative effects.



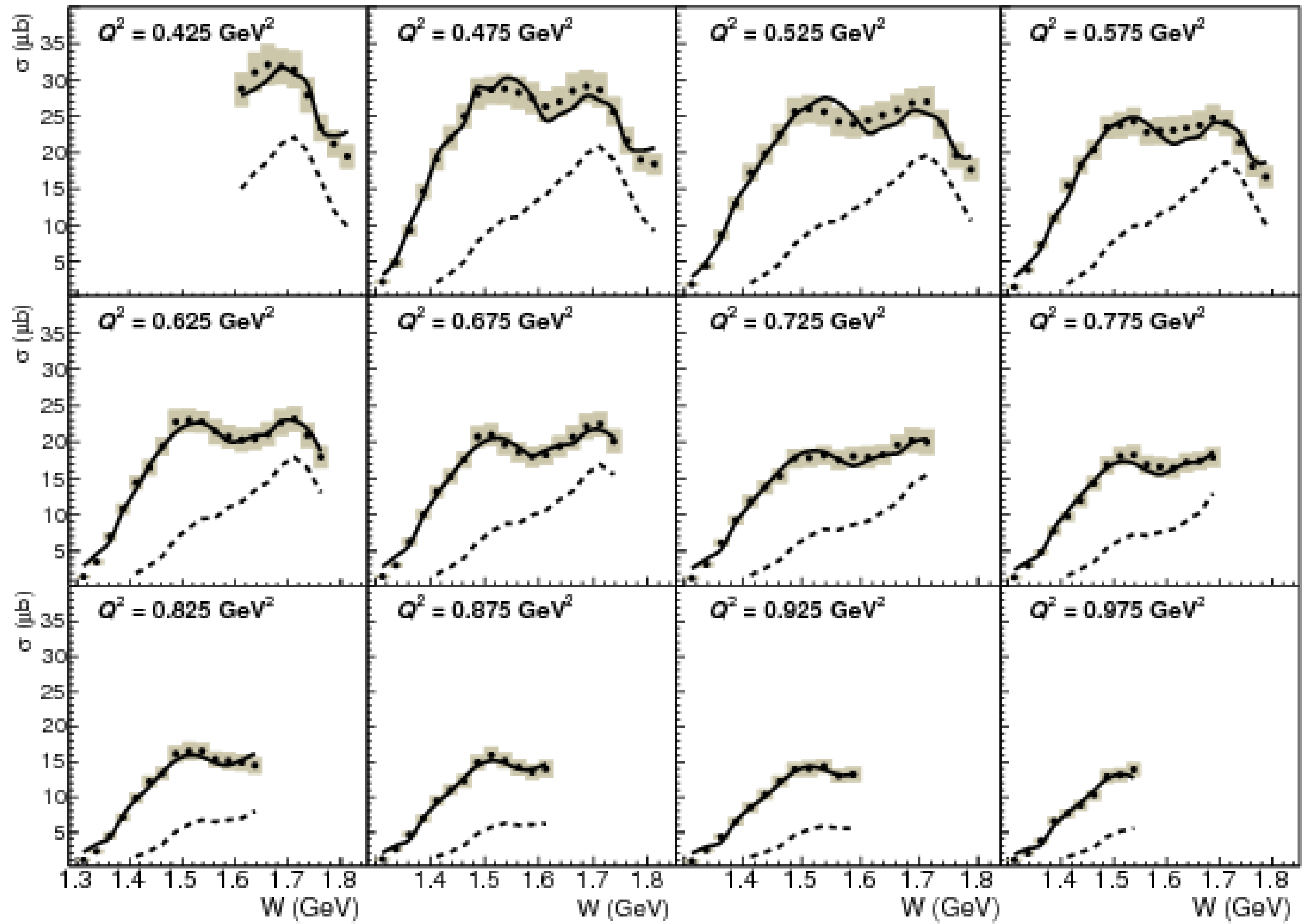
Good agreement in all 6 sectors

Cross Sections from Dataset with E_{beam} = 1.5 GeV



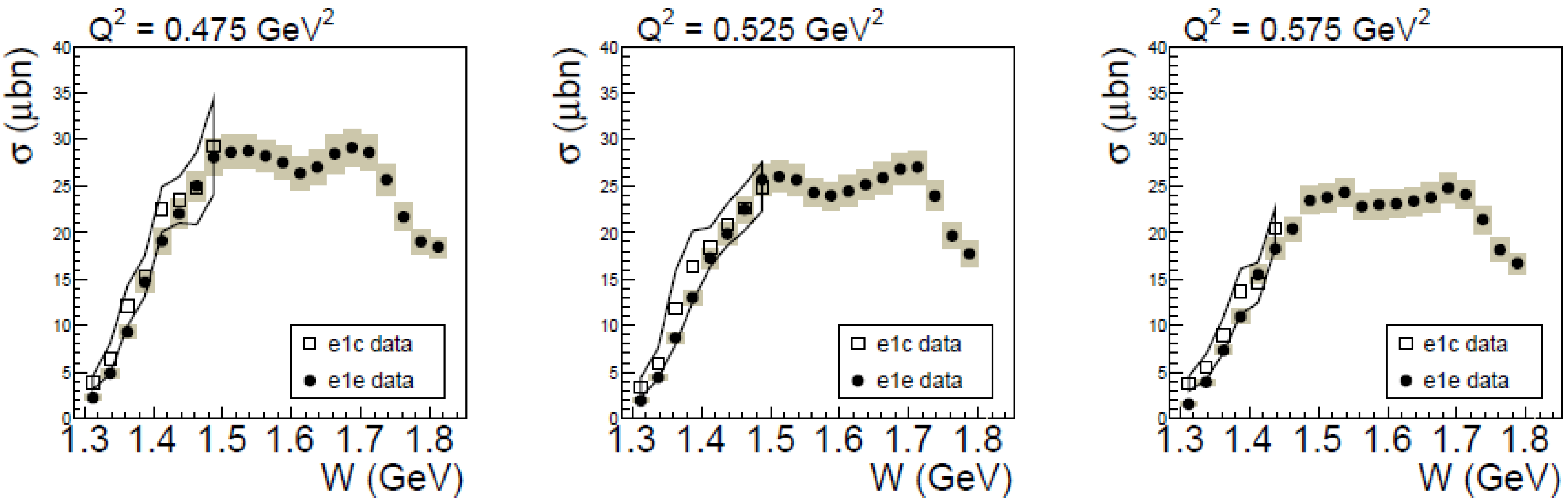
Symbols are for the extracted cross sections. Hatched area represents systematic uncertainties. Solid curves are for the model fit. The contribution from resonant and non-resonant mechanisms are shown by dot-dashed and dashed curves, respectively.

Cross Sections from Dataset with E_{beam} = 2 GeV



Symbols are for the extracted integral cross sections. Shadowed areas correspond to the total uncertainty. Solid curves are for the cross section model estimation, while dashed curves are for the resonant contribution.

Agreement Between Two Datasets



Open squares represent published data with E_{beam} = 1.5 GeV [1], while the black circles represent the new data with E_{beam} = 2 GeV [2]. Good agreement within systematic uncertainties.

[1] G. Fedotov *et al.*, PRC79, 015204 (2009).
[2] G. Fedotov *et al.*, under PRC review, arXiv:1804.05136.

For Each Point in *W* and Q^2

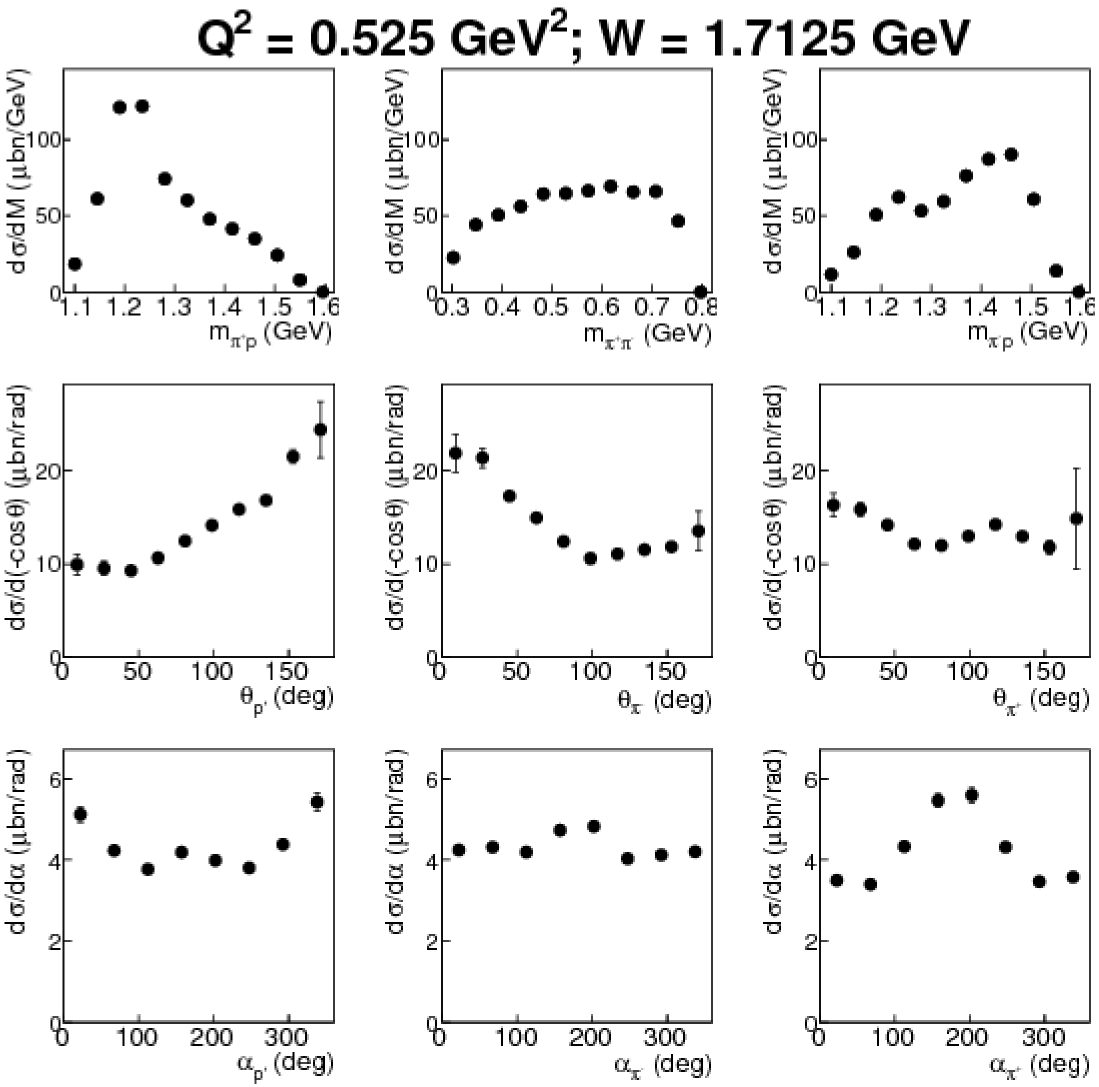
The following set of the single-differential cross sections is reported.

$$\frac{\mathrm{d}\sigma_v}{\mathrm{d}M_{p'\pi^+}} \qquad \frac{\mathrm{d}\sigma_v}{\mathrm{d}M_{\pi^-\pi^+}} \qquad \frac{\mathrm{d}\sigma_v}{\mathrm{d}M_{\pi^-p'}}$$

$$\frac{\mathrm{d}\sigma_v}{\mathrm{d}[-\cos\theta_{p'}]} \qquad \frac{\mathrm{d}\sigma_v}{\mathrm{d}[-\cos\theta_{\pi^-}]} \qquad \frac{\mathrm{d}\sigma_v}{\mathrm{d}[-\cos\theta_{\pi^+}]}$$

$$\frac{\mathrm{d}\sigma_v}{\mathrm{d}\alpha_{p'}} \qquad \frac{\mathrm{d}\sigma_v}{\mathrm{d}\alpha_{\pi^-}} \qquad \frac{\mathrm{d}\sigma_v}{\mathrm{d}\alpha_{\pi^+}}$$

An Example of Single-Differential Cross Sections

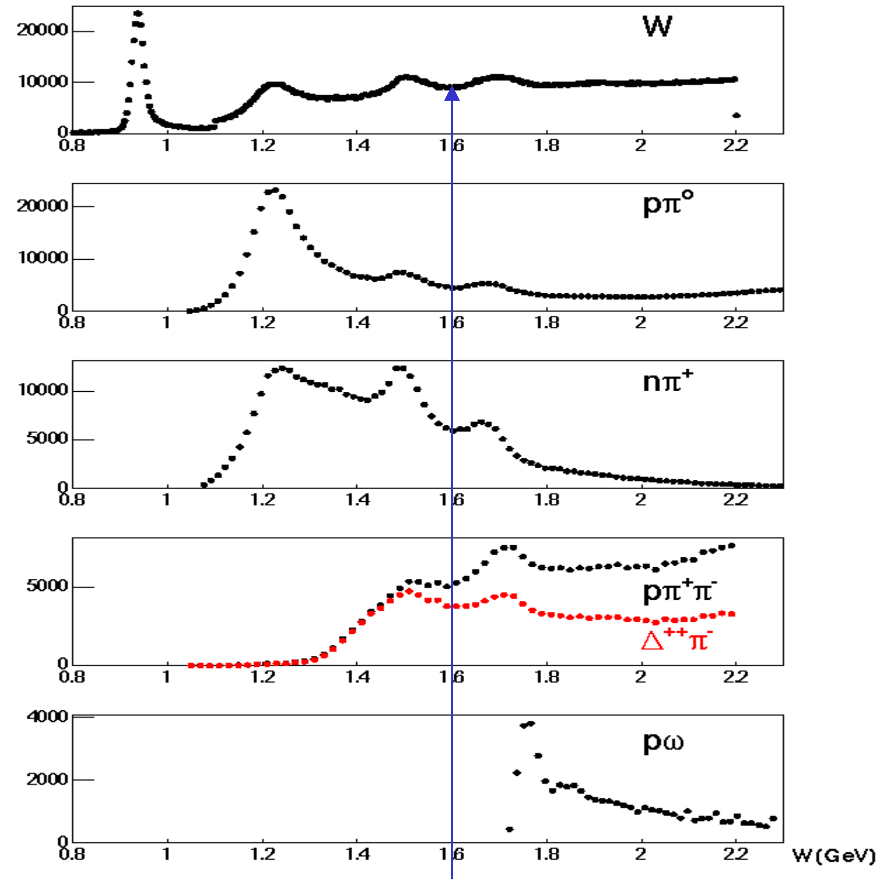


Importance for N* Physics

Hadronic decays of prominent N*s at W < 1.8 GeV.

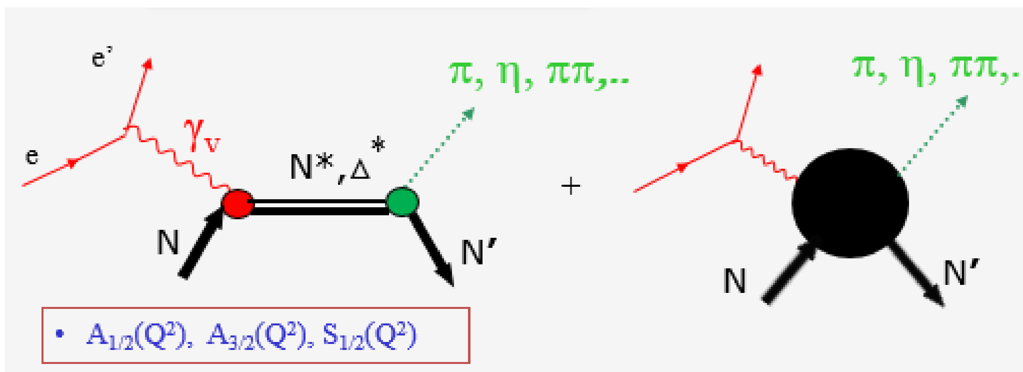
State	Bran. Fract. to Nπ.	Bran. Fract. to Nη	Bran.Fract. Nππ
Δ(1232)P_{33}	0.995		
N(1440)P_{11}	0.55-0.75		0.3-0.4
N(1520)D_{13}	0.55-0.65		0.4-0.5
N(1535)S_{11}	0.48±0.03	0.46±0.02	
Δ(1620)S_{31}	0.20-0.30		0.70-0.80
N(1650)S_{11}	0.60-0.95	0.03-0.11	0.1-0.2
N(1685)F_{15}	0.65-0.70		0.30-0.40
Δ(1700)D_{33}	0.1-0.2		0.8-0.9
N(1720)P_{13}	0.1-0.2		> 0.7

CLAS data on yields of meson electroproduction at $Q^2 < 4$ GeV^2



Physical Data Analysis

Data → Observables (cross sections, asymmetries, etc.) → Reaction models, partial wave analysis → Electrocouplings $A_{1/2}$, $A_{3/2}$, $S_{1/2}$ → QCD Hadron models



- **Separation of resonant/non-resonant mechanisms.**
- **Parameterization of resonant amplitudes.**
- **Fit to the data.**
- **Extraction of resonance parameters.**

P_{11}(1440) Electrocouplings

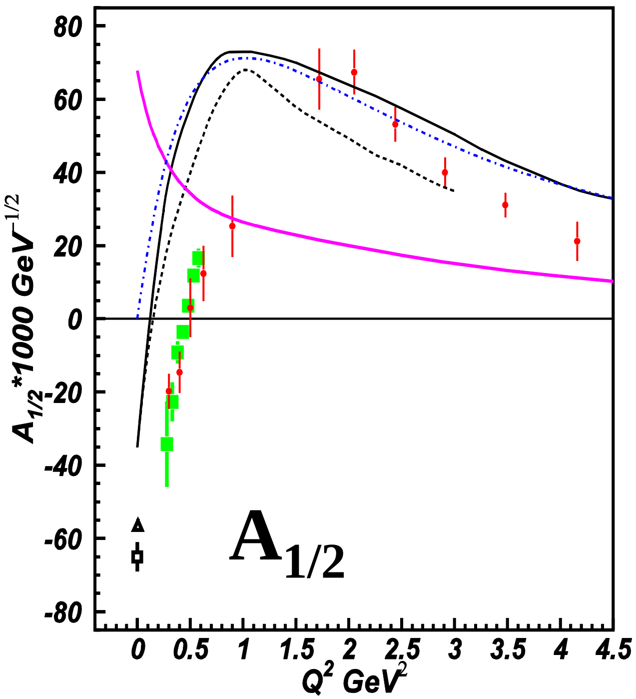
Quark models:

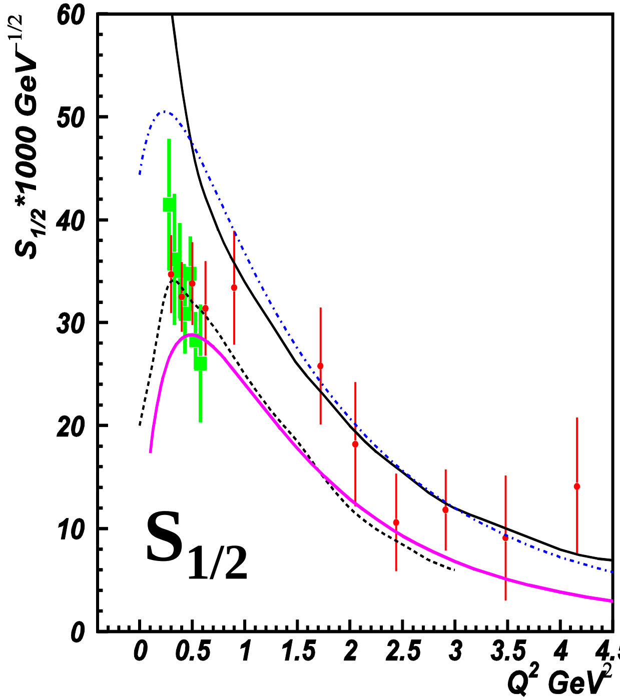
—— I. Aznauryan LC

------ S. Capstick LC

-·-·-·- G. Ramalho QM based on relat. cov. approach by F.Gross

—— MB dressing abs. val. (EBAC)



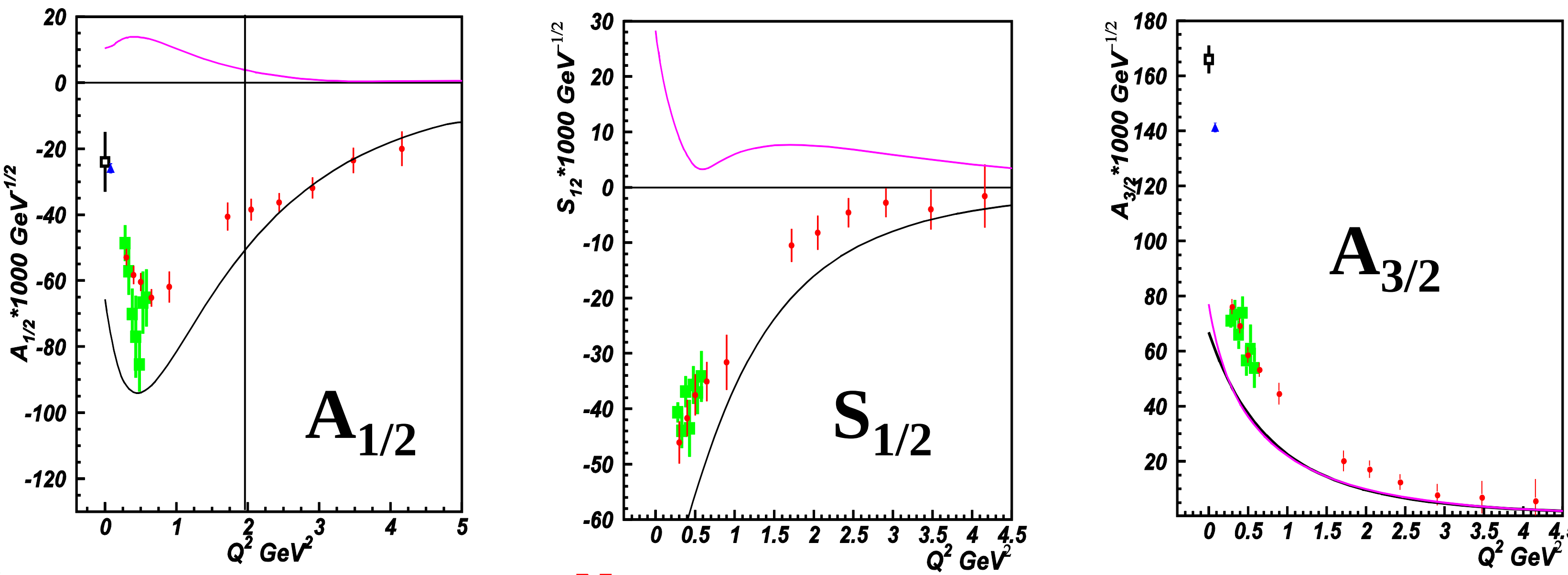


■ $N\pi\pi$ V. I. Mokeev et al., PRC 86, 035203 (2012).

● $N\pi$ I.Aznauryan,V.Burkert, et al., PRC 80,055203(2009).

- **The electrocouplings are consistent with *P_{11}(1440)* structure as a superposition of the contributions from: a) quark core as a first *radial excitation of the nucleon as a 3-quark ground state and b) meson-baryon dressing*.**

$D_{13}(1520)$ Electrocouplings



■ Nππ V. I. Mokeev et al., PRC 86, 035203 (2012).

● Nπ I.Aznauryan,V.Burkert, et al., PRC 80,055203(2009).

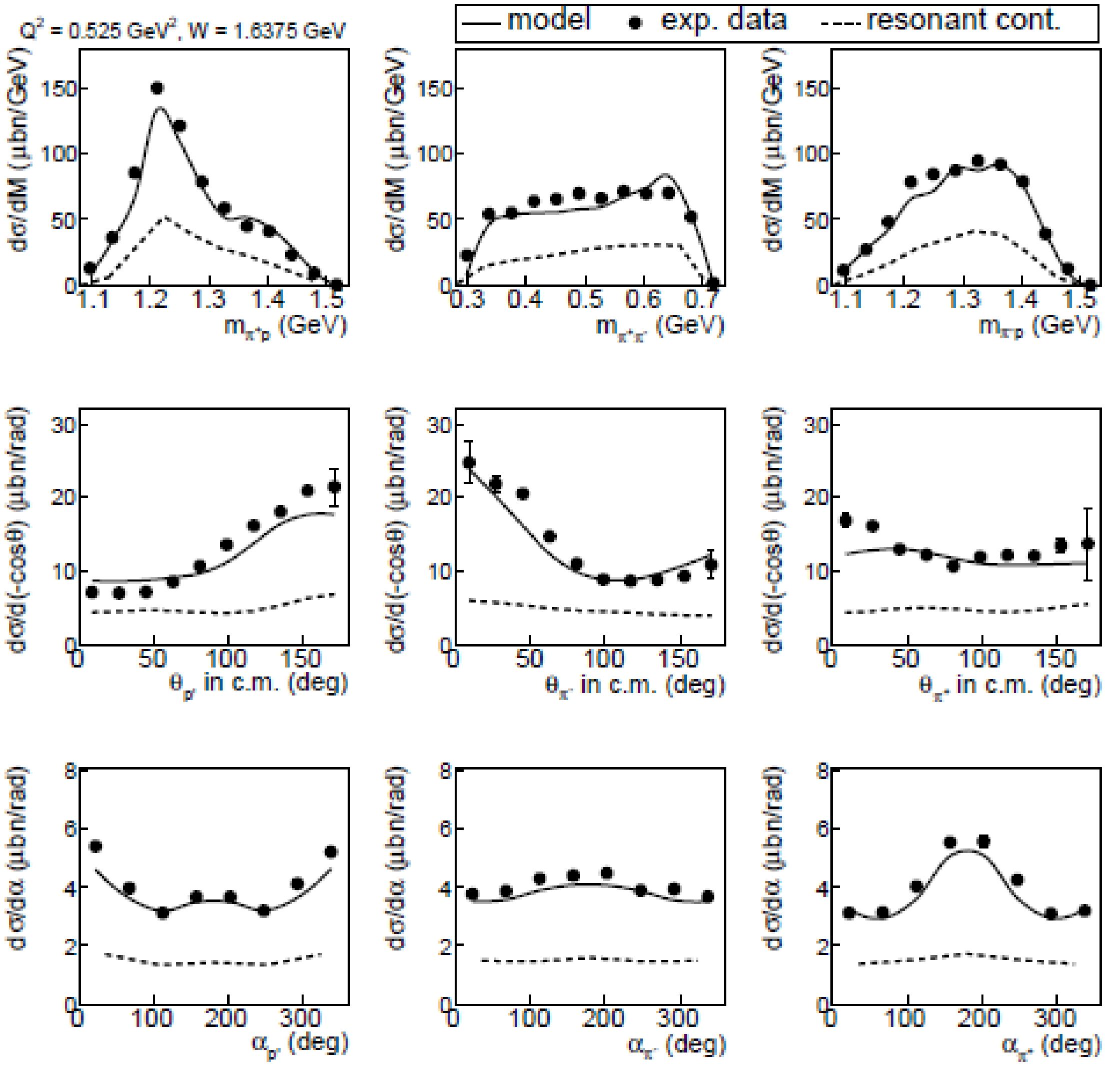
—— M.Giannini/E.Santopinto hCQM

—— MB dressing abs val. (EBAC

- good agreement between the results from Nπ & Nππ channels
- substantial contributions from quark core at $Q^2 > 2.0$ GeV^2 and sizable MB cloud at $Q^2 < 1.0$ GeV^2

The results on $A_{1/2}$ electrocouplings at $Q^2 > 2.0$ GeV^2 for the first time offer almost direct access to the quark core!

Physical Results from Dataset with E_{beam} = 2 GeV



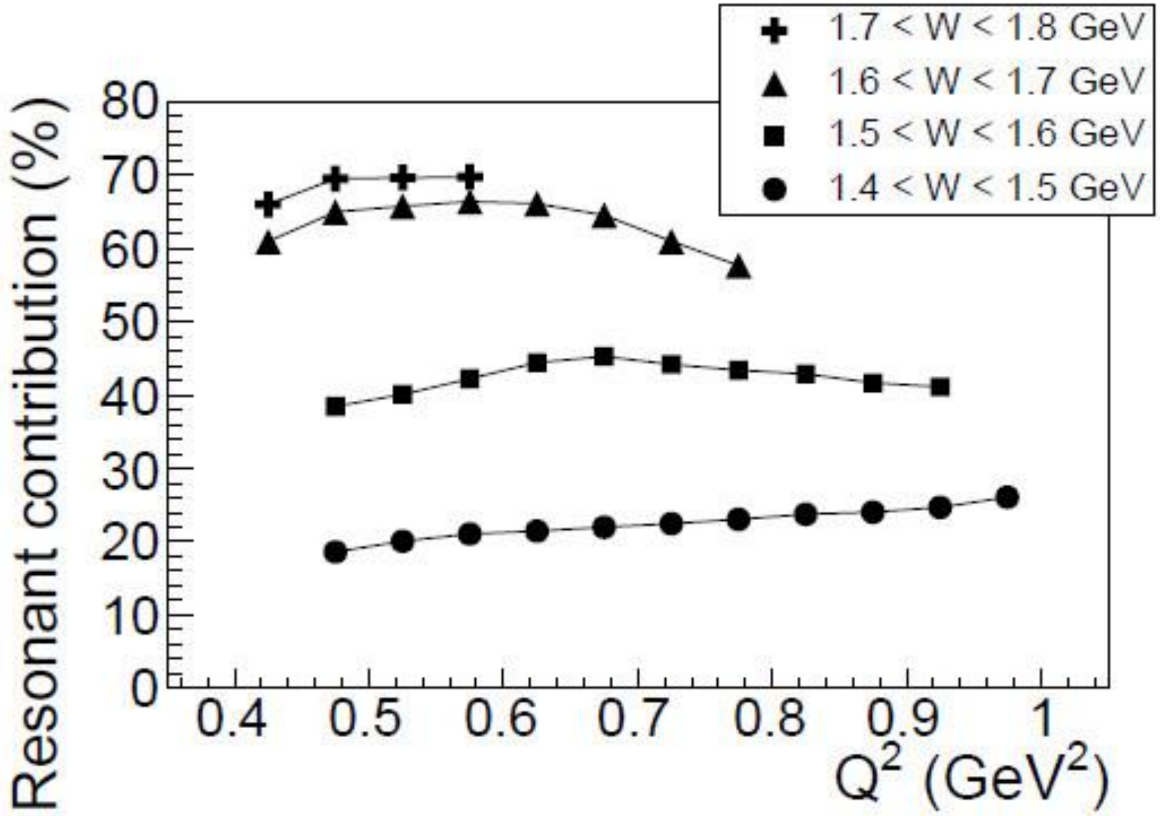
Symbols are for the extracted cross sections. Error bars correspond to the statistical uncertainty.

Solid curves are for the model cross section estimation.

Dashed curves are for the resonant contribution predicted by JM model.

Example is given for W = 1.6375 GeV and Q^2 = 0.525 GeV^2.

Physical Results from Dataset with E_{beam} = 2 GeV



The relative resonant contribution to the cross section is found to range from 20% to 60% (depending on kinematical region) that advantages the extraction of resonance electrocouplings.

Summary (CLAS Part of the Talk)

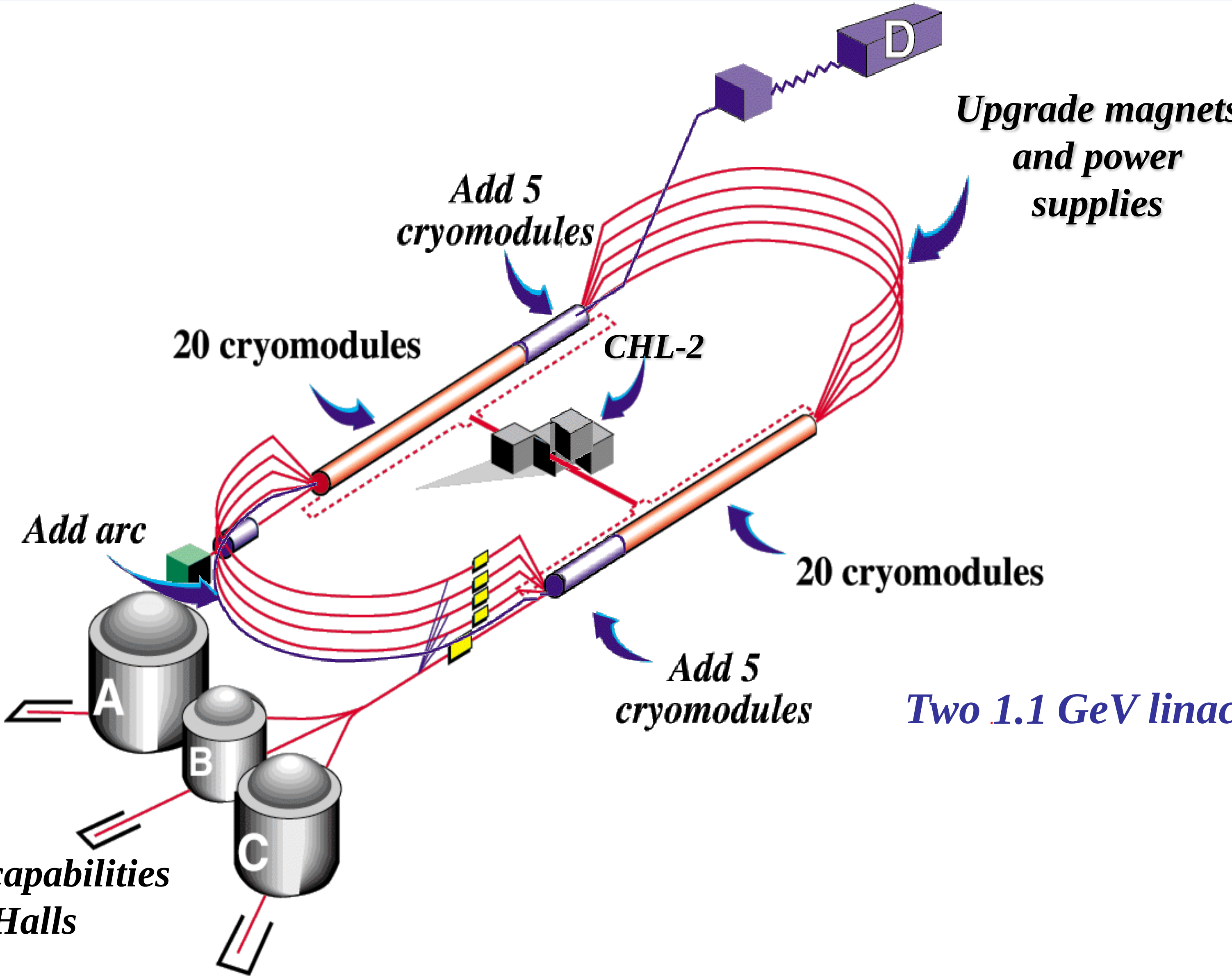
- Cross sections of the $\gamma_v p \rightarrow \pi^+ \pi^- p$ reaction from dataset with E_{beam} = 1.5 GeV at low W < 1.55 GeV were obtained and published

- Obtained cross sections were analyzed within the phenomenological model and electrocouplings of $P_{11}(1440)$ and $D_{13}(1520)$ resonances were obtained and published

- Cross sections of the $\gamma_v p \rightarrow \pi^+ \pi^- p$ reaction from dataset with E_{beam} = 2 GeV for W from 1.3 GeV to 1.8 GeV and Q^2 from 0.4 GeV^2 to 1.0 GeV^2 were obtained, the corresponding paper is currently under PRC review

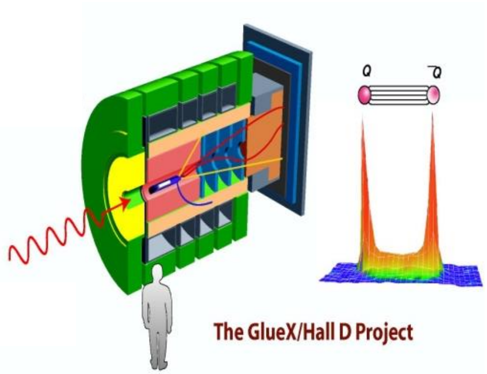
- The phenomenological analysis of this data will extend considerably the available information on the Q^2-evolution of the high-lying N* electrocouplings

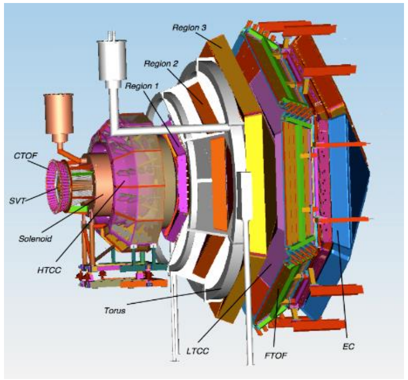
Jefferson Lab 12 GeV Upgrade

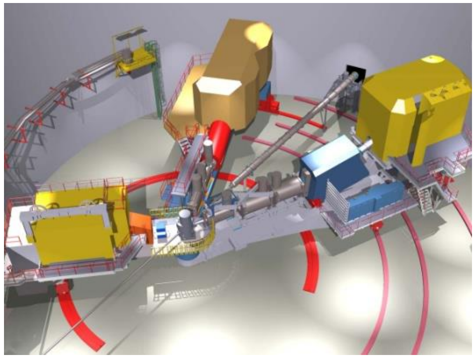


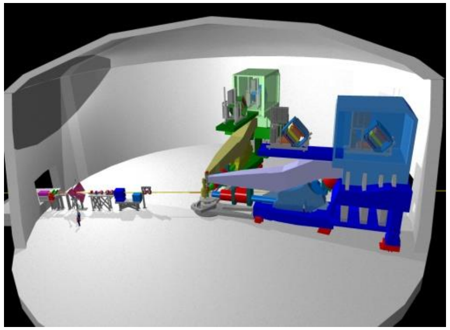
Two 1.1 GeV linacs

Experimental Halls After Upgrade



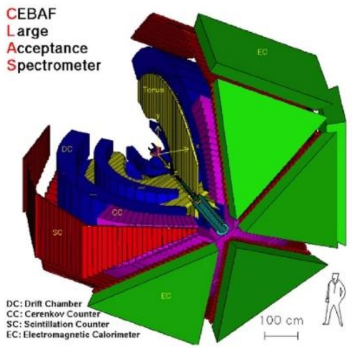


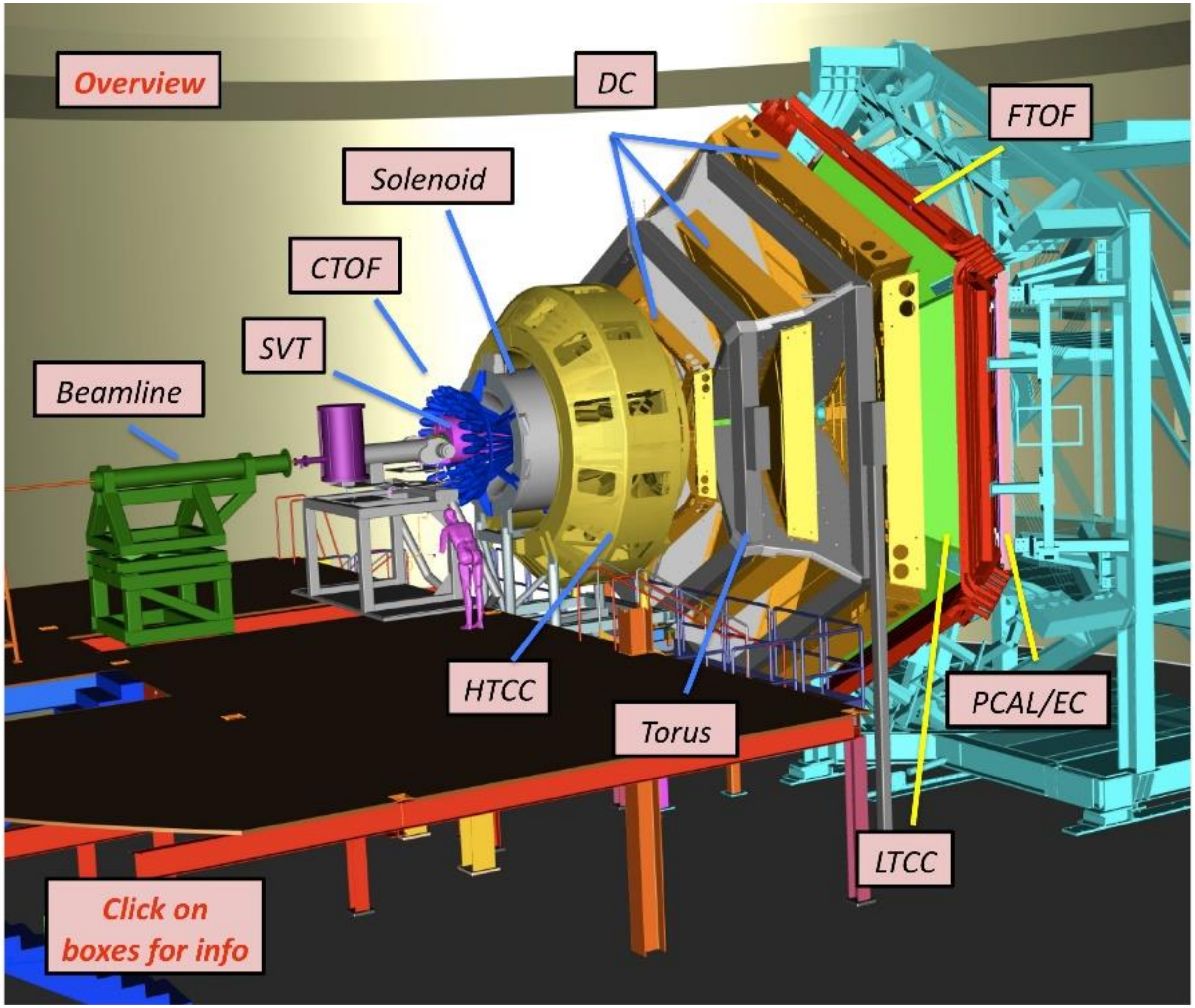




Hall D	Hall B	Hall C	Hall A
4π hermetic detector GlueEx	luminosity 10^{35} CLAS12	High Momentum Spectrometer SHRS	High Resolution Spectrometer HRS
E_γ ~ 8.5-9.0 GeV	11 GeV beamline		
10^8 photons/s	target flexibility		
good momentum/angle resolution		excellent momentum resolution	
high multiplicity reconstruction		luminosity up to 10^{38}	

Transition from CLAS to CLAS12

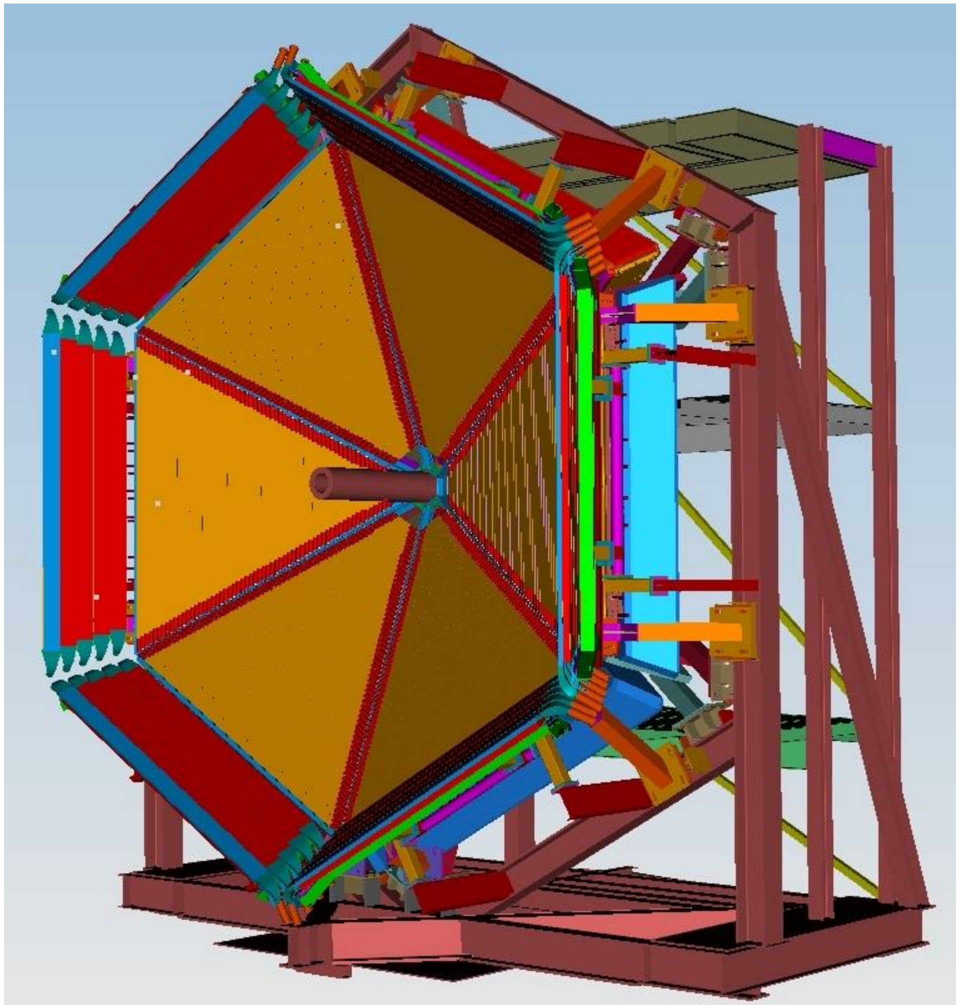


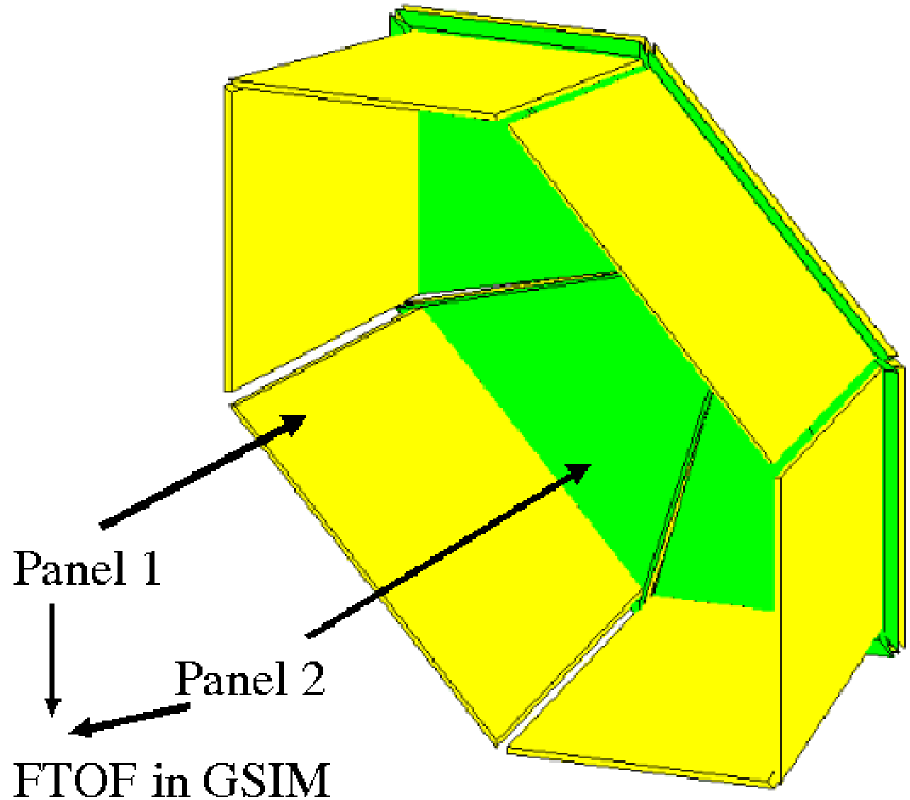


Shifts started on December 2017!

Forward Time-of-Flight Upgrade

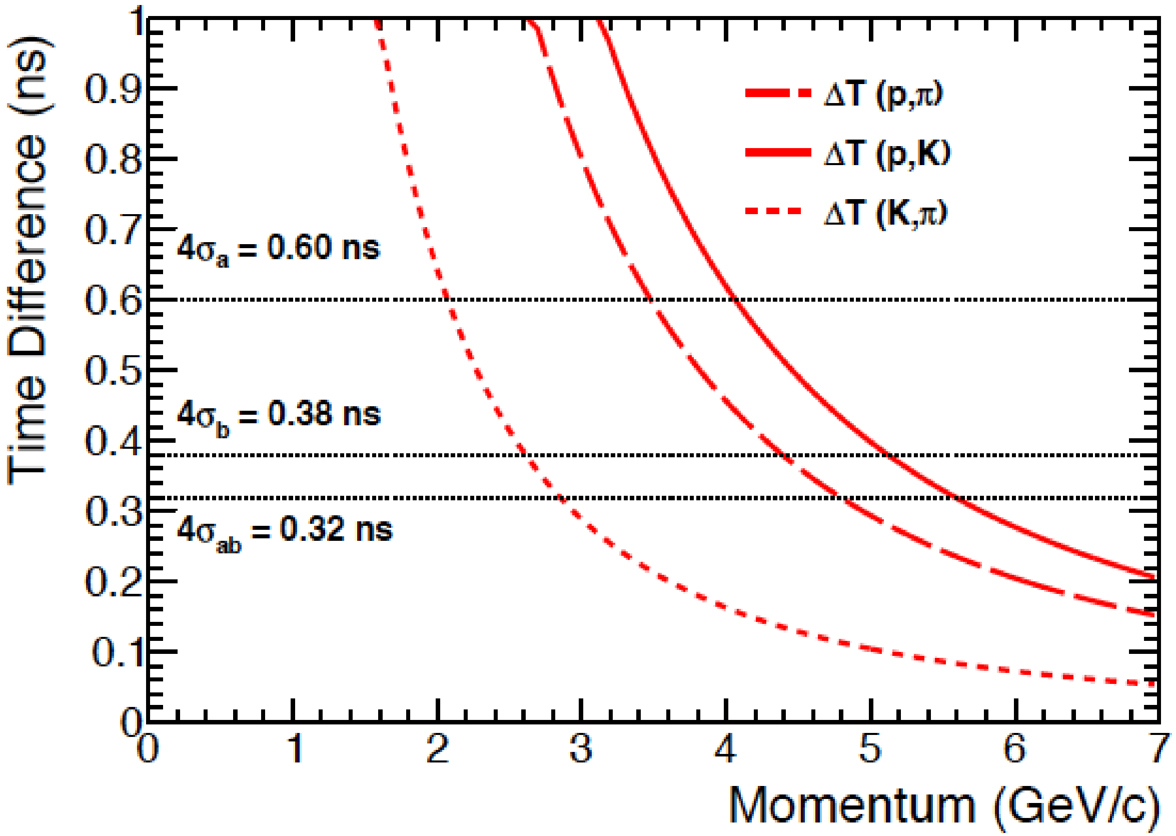
JLab Designer





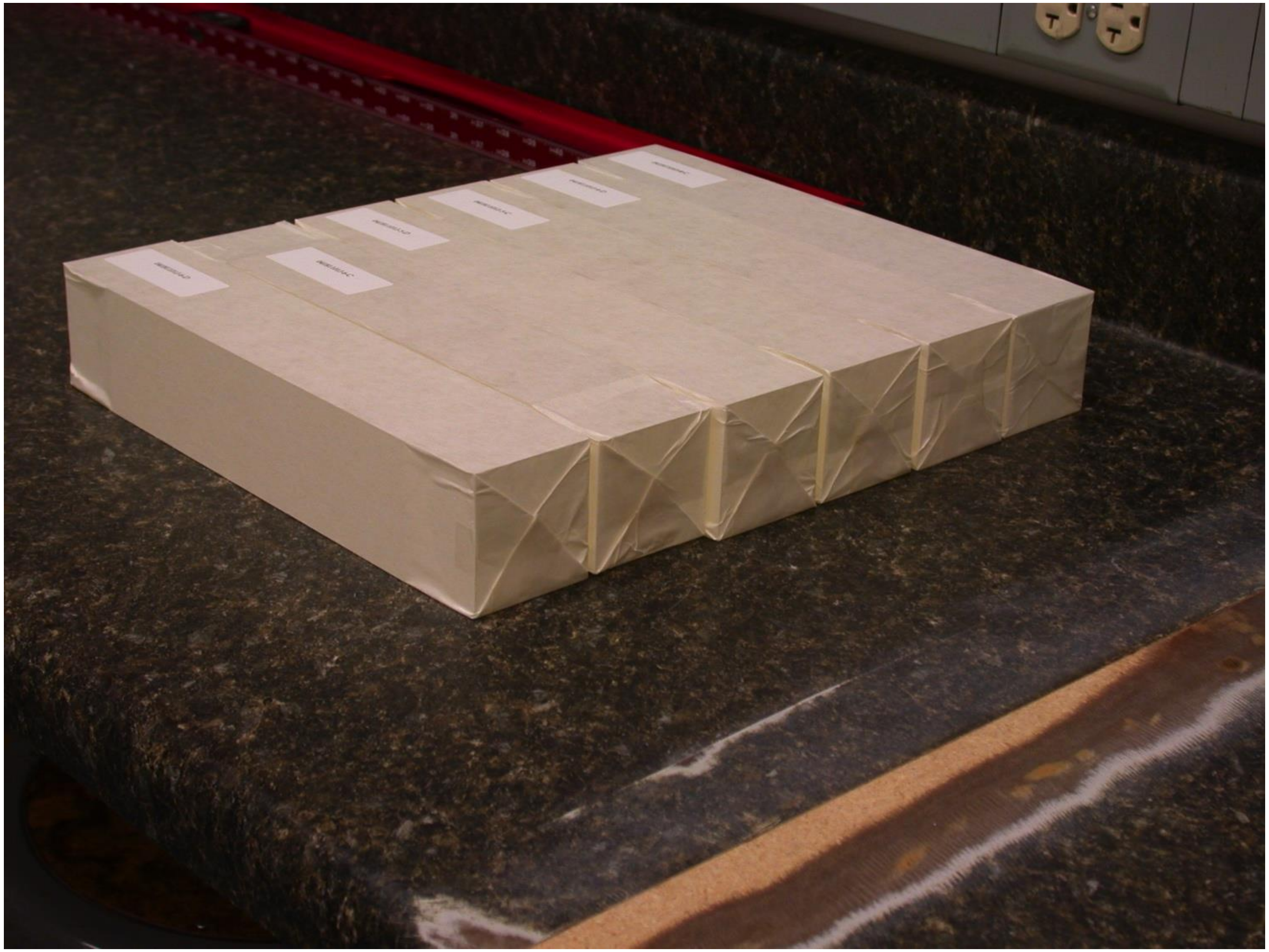
The panel 2b was built at the University of South Carolina

Design Requirements

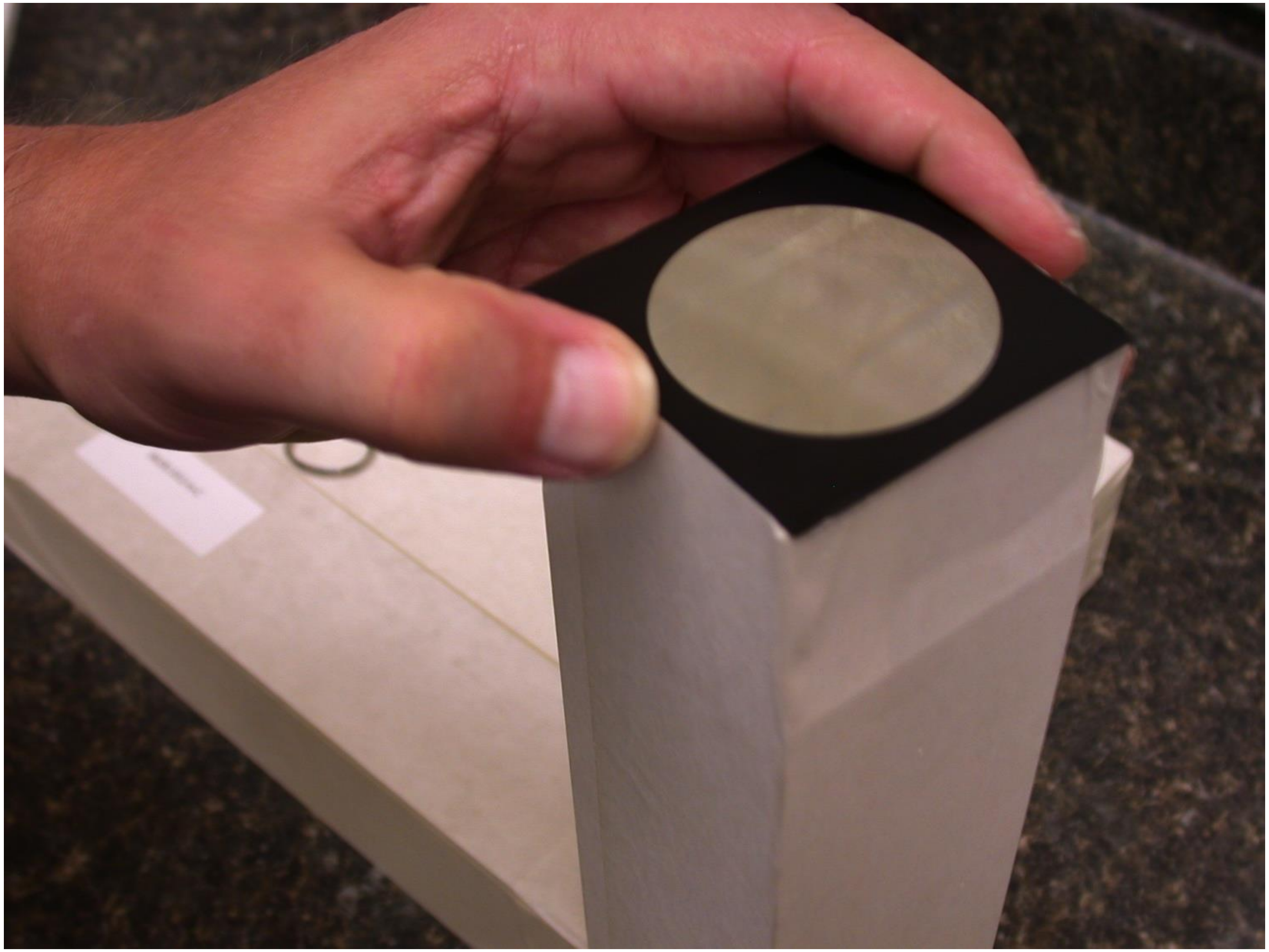


To achieve Proton-Pion separation up to 5.3 GeV, Proton-Kaon separation up to 4.5 GeV and Kaon-Pion separation up to 2.6 GeV an improved timing resolution of $\sigma \sim 80$ ps is needed.

Scintillators at Arrival



Reflection Protection



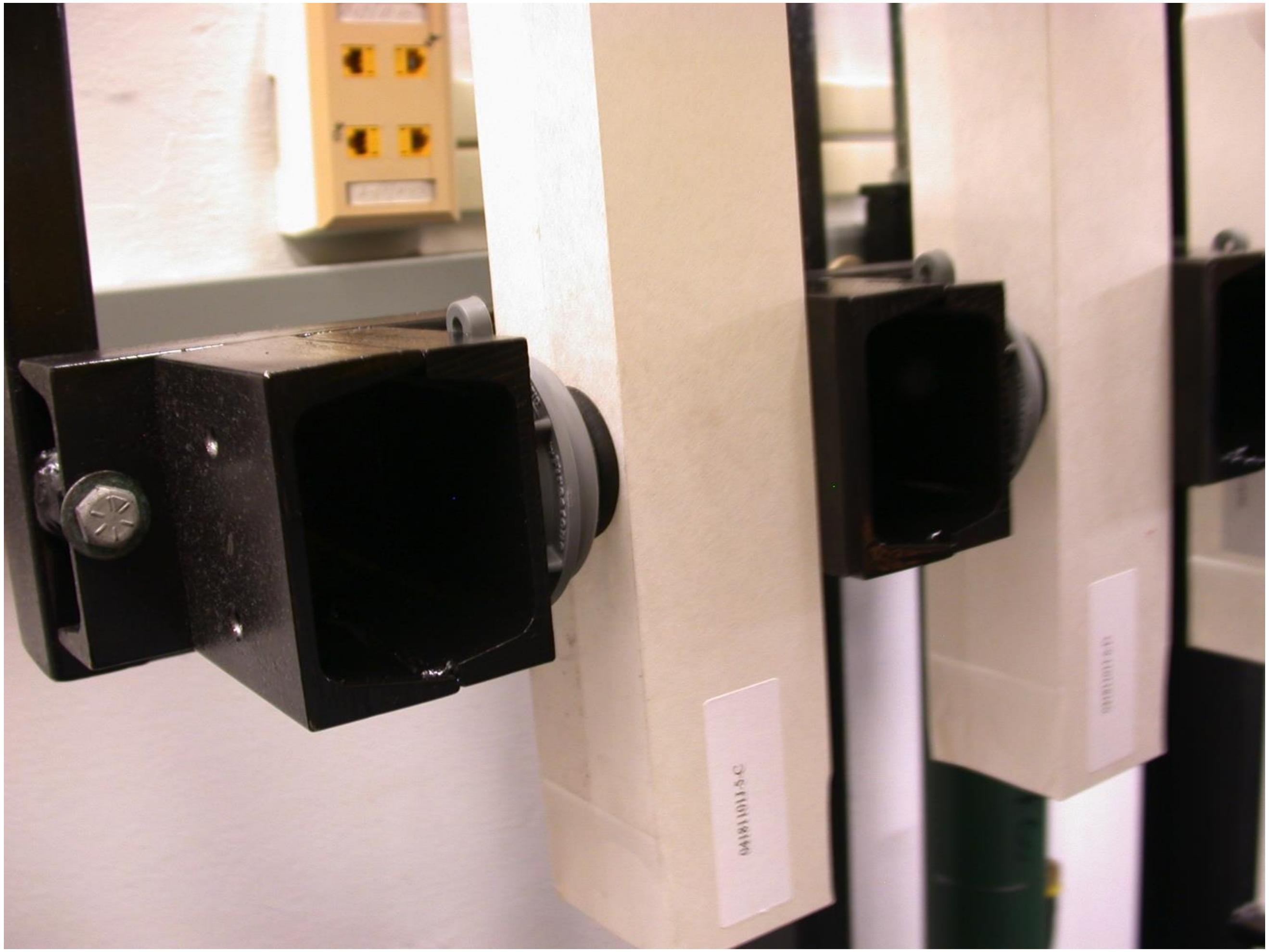
Anti-cookie were attached to avoid reflections from the areas not covered by PMT

Gluing Process

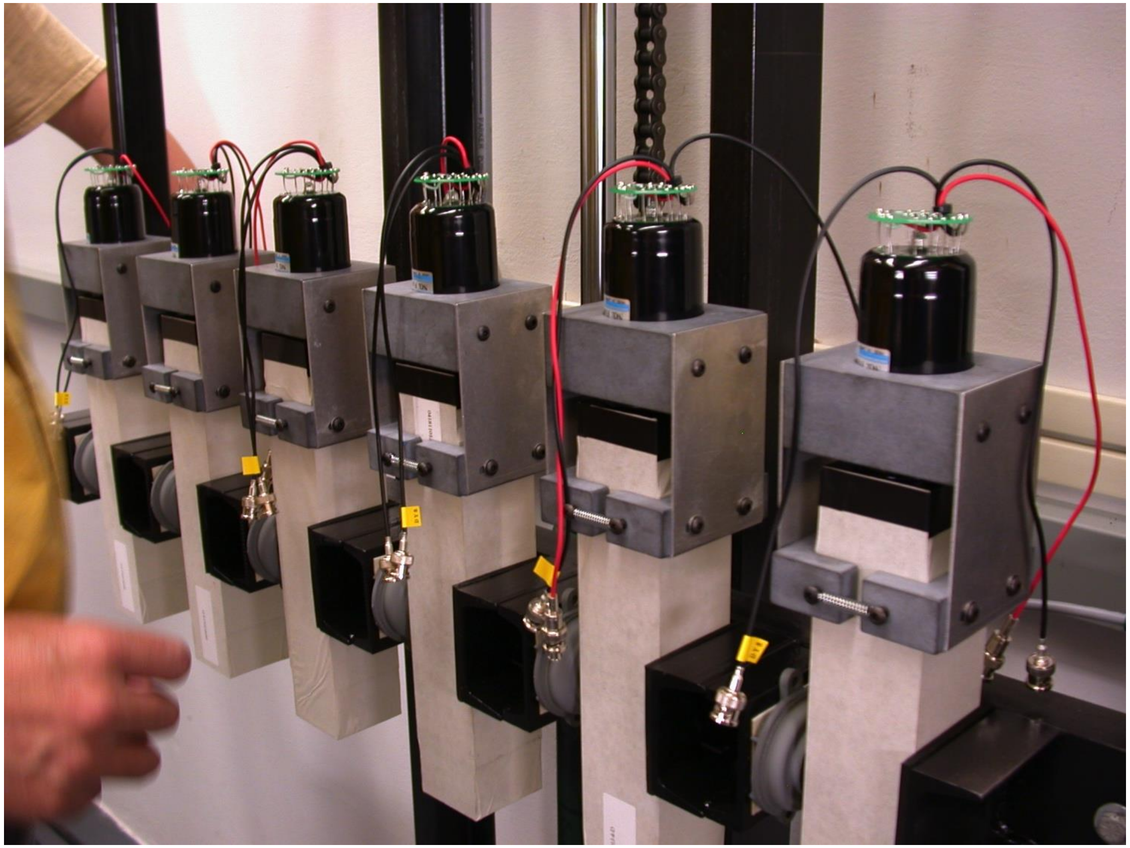
Special rotating supporting structure (windmill) was build at USC

Gluing Process (Continued)

Pressure Handling System

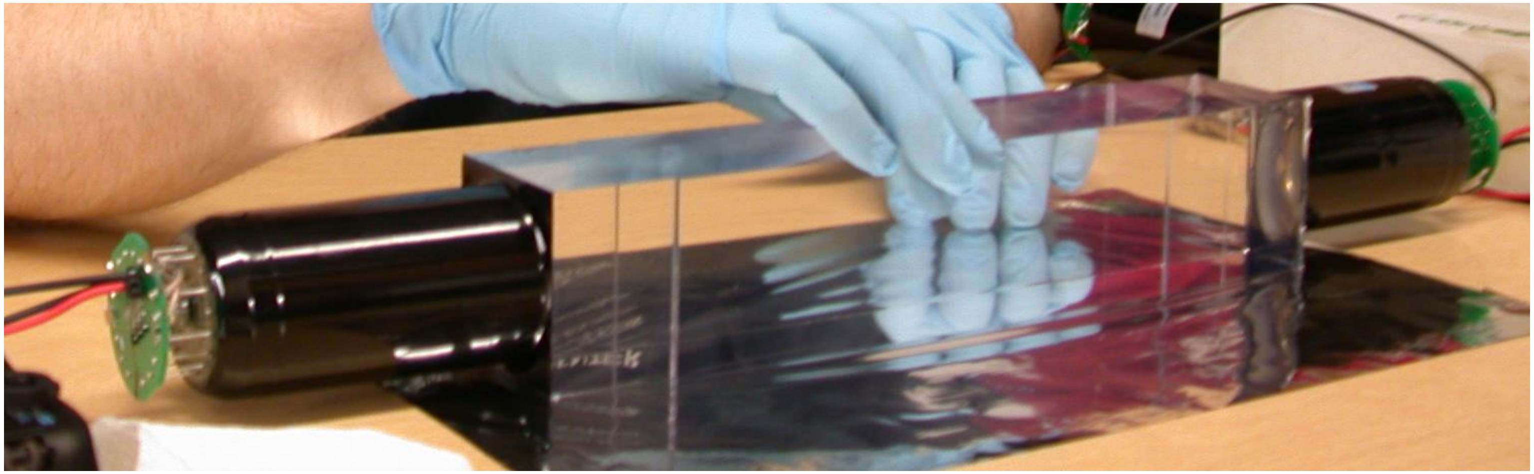


Centering Devices



Needed to hold PMTs in position to prevent swimming on the glue

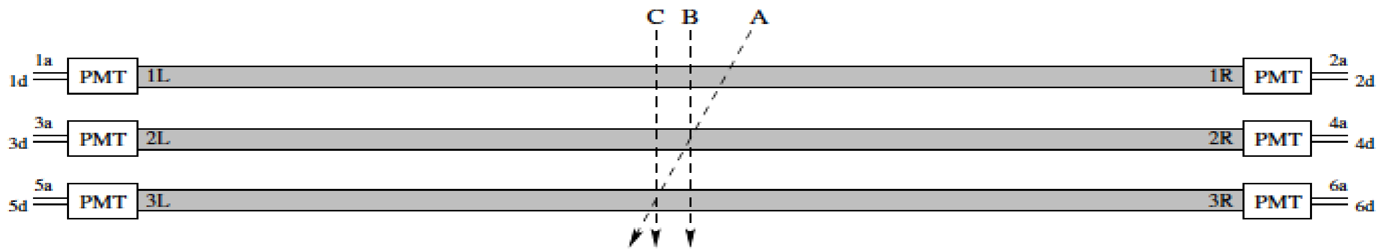
Scintillator Wrapping



Two layers of wrapping:

- reflective wrapping in order to maximize internal reflection to get better time resolution
- three layers of Tedlar to avoid external light penetration

Three-Bar Method



The time at which the particle interacts with i^{th} scintillator $t_i = \frac{t_{iL}+t_{iR}}{2} - \frac{L}{2v_{eff}}$, where $t_{iL,R} = t_{raw} - t_{ref}$ with t_{raw} is the raw time reported by TDC and t_{ref} is the reference time.

Counter interaction time for each of the three counters (t, m, and b) is finally given by

$$t_t = \tau + \varepsilon_t$$
$$t_m = \tau + \varepsilon_m + \delta$$
$$t_b = \tau + \varepsilon_b + 2\delta$$

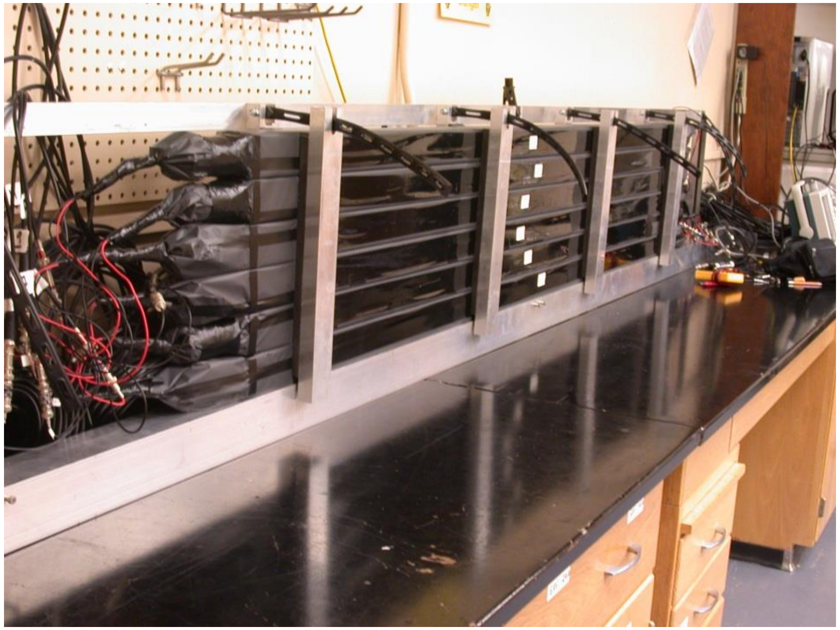
τ is the actual time at which the particle interacts with top counter.
δ is the time that particle travels between adjacent counters.
ε_i are all uncertainties that contribute to the measured time.
σ_ε is the resolution of the corresponding counter.

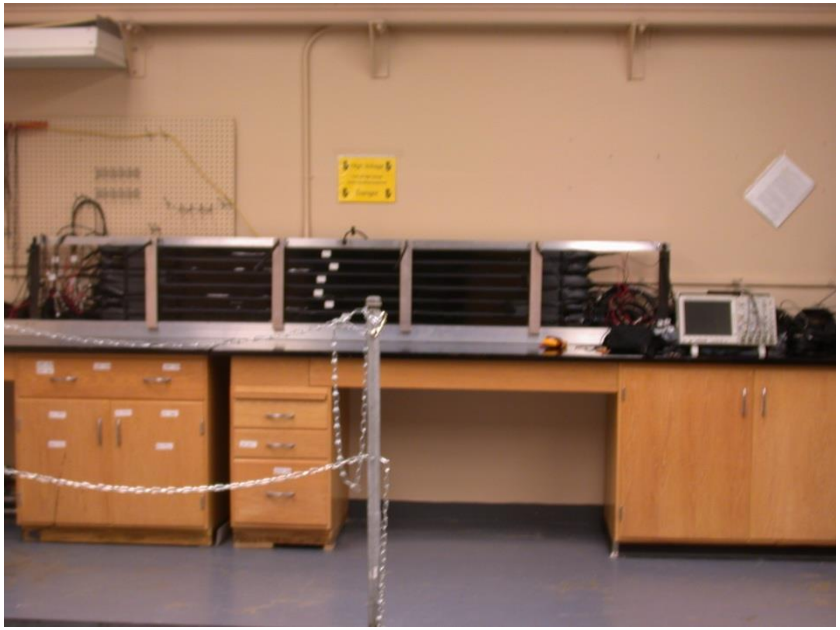
The quantity $T = \frac{t_t+t_b}{2} - t_m = \frac{\varepsilon_t+\varepsilon_b}{2} - \varepsilon_m$ is composed to be constant, only smeared by ε_i

The statistical uncertainty in T is defined by $\sigma_T^2 = (\sigma_{\varepsilon_t}^2 + \sigma_{\varepsilon_b}^2)/4 + \sigma_{\varepsilon_m}^2$

Assuming all counters are identical, the resolution is $\sigma_\varepsilon = \sqrt{\frac{2}{3}}\sigma_T$

Six-Bar Method



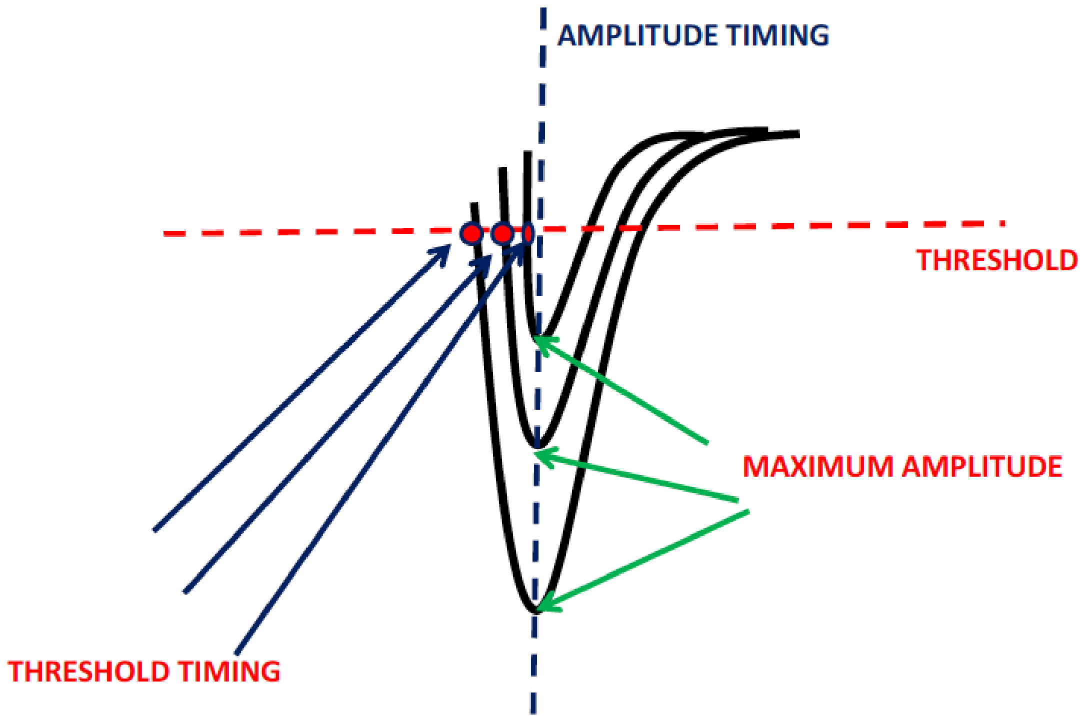


Resolution analyses were performed using the three-bar cosmic ray method. To speed-up the analysis process we run with six bars on top of each other and then select equidistant combinations of three bars for the analysis (6 combinations are possible).

Then the test is repeated for the complimentary order of the bars.

The pictures show the experimental setup for 210 cm long bars.

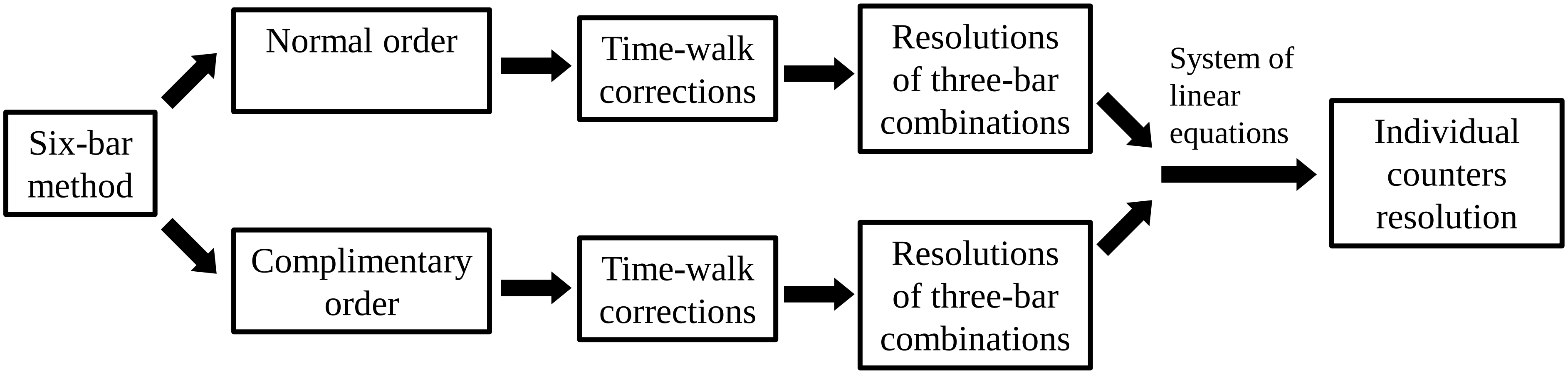
Time-Walk Corrections



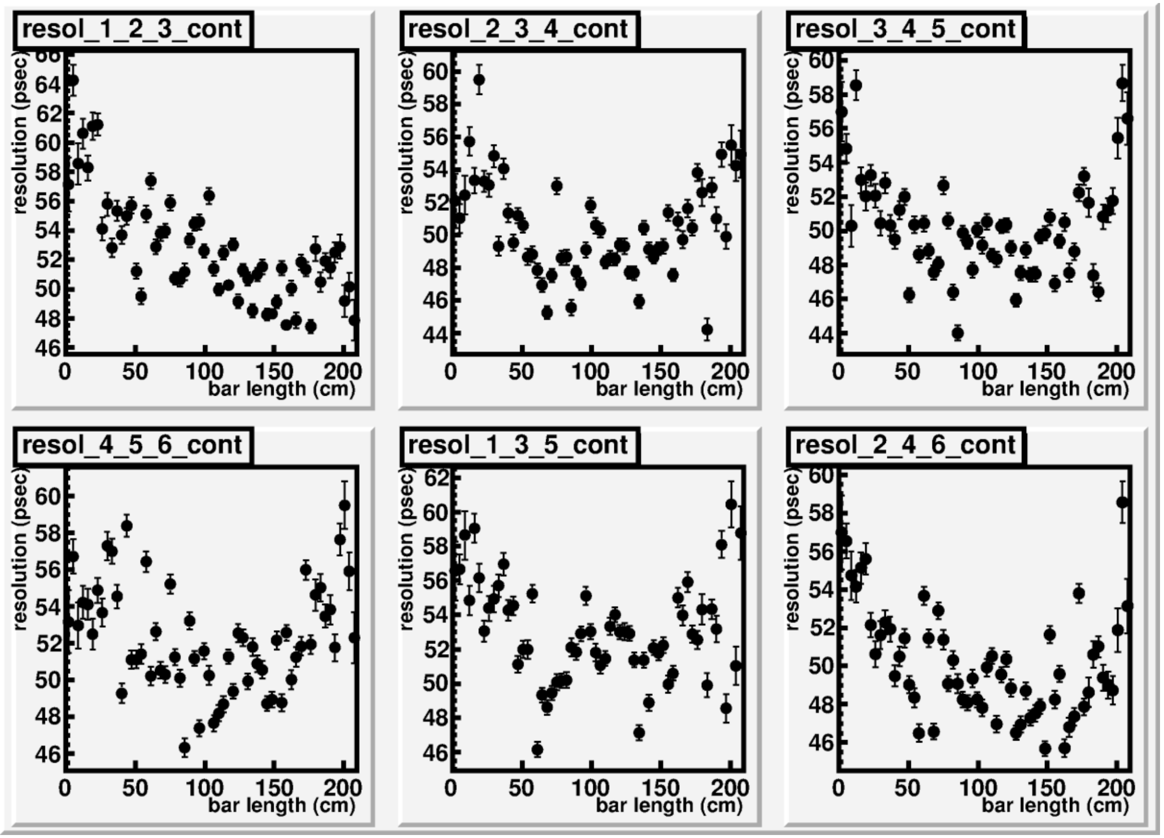
Signals with the same maximum position cross the threshold at different times.

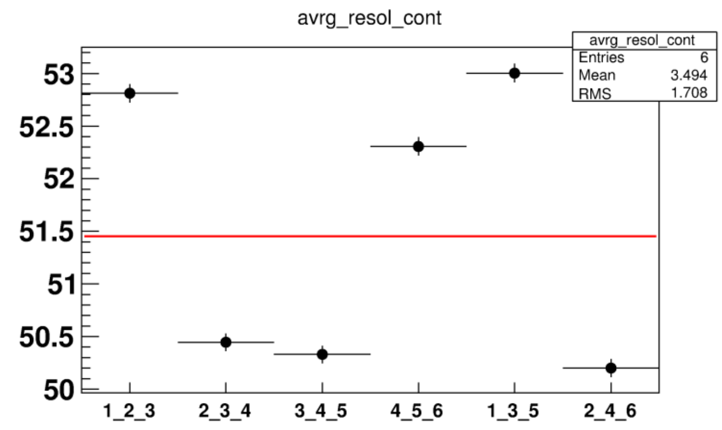
In CLAS time-walk corrections were performed using laser system, for CLAS12 the idea is to use data itself.

Steps of the Testing Procedure



Resolution of Three-Bar Combinations



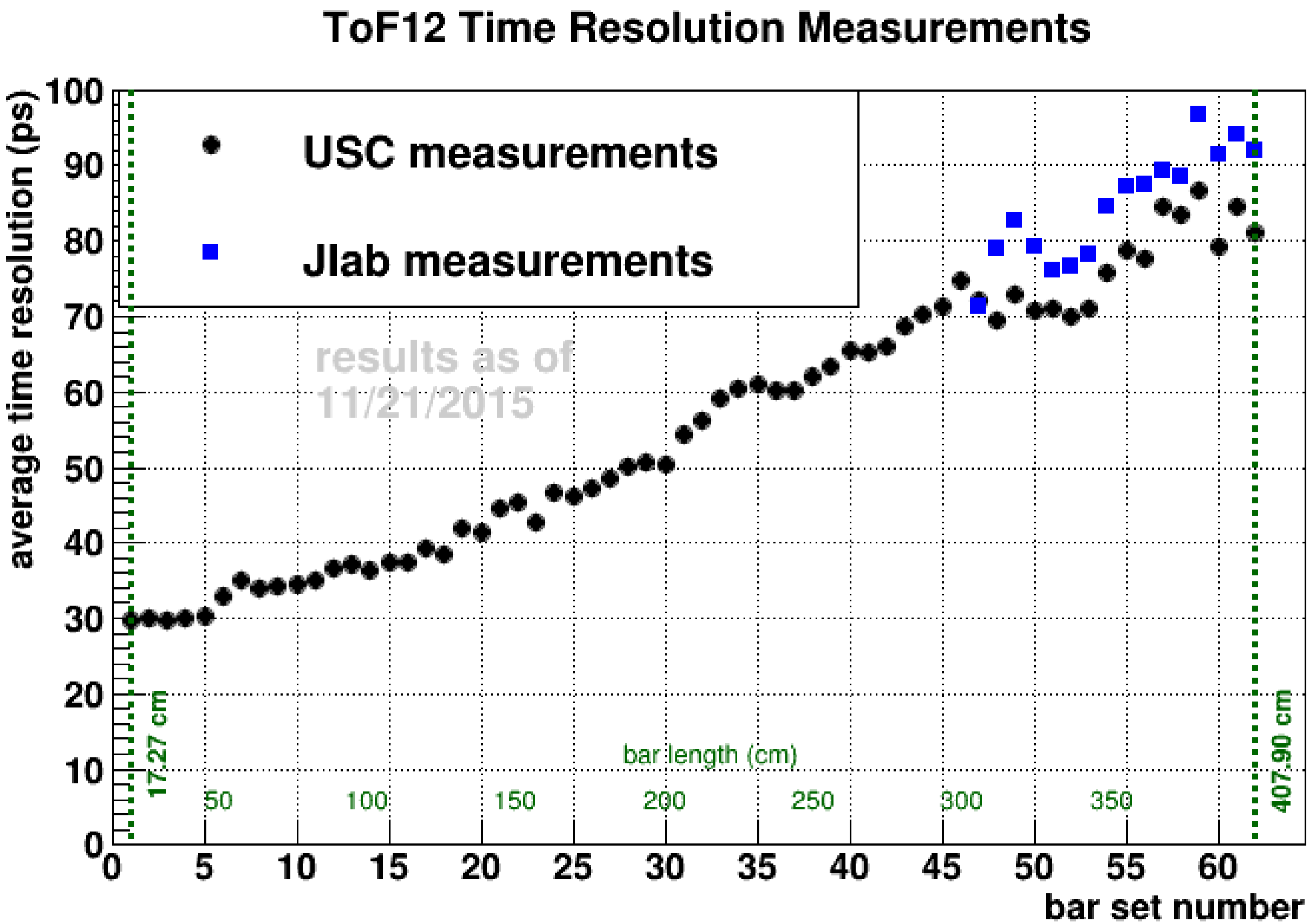


Example for 210 cm bars set

Average time resolution < 52 ps

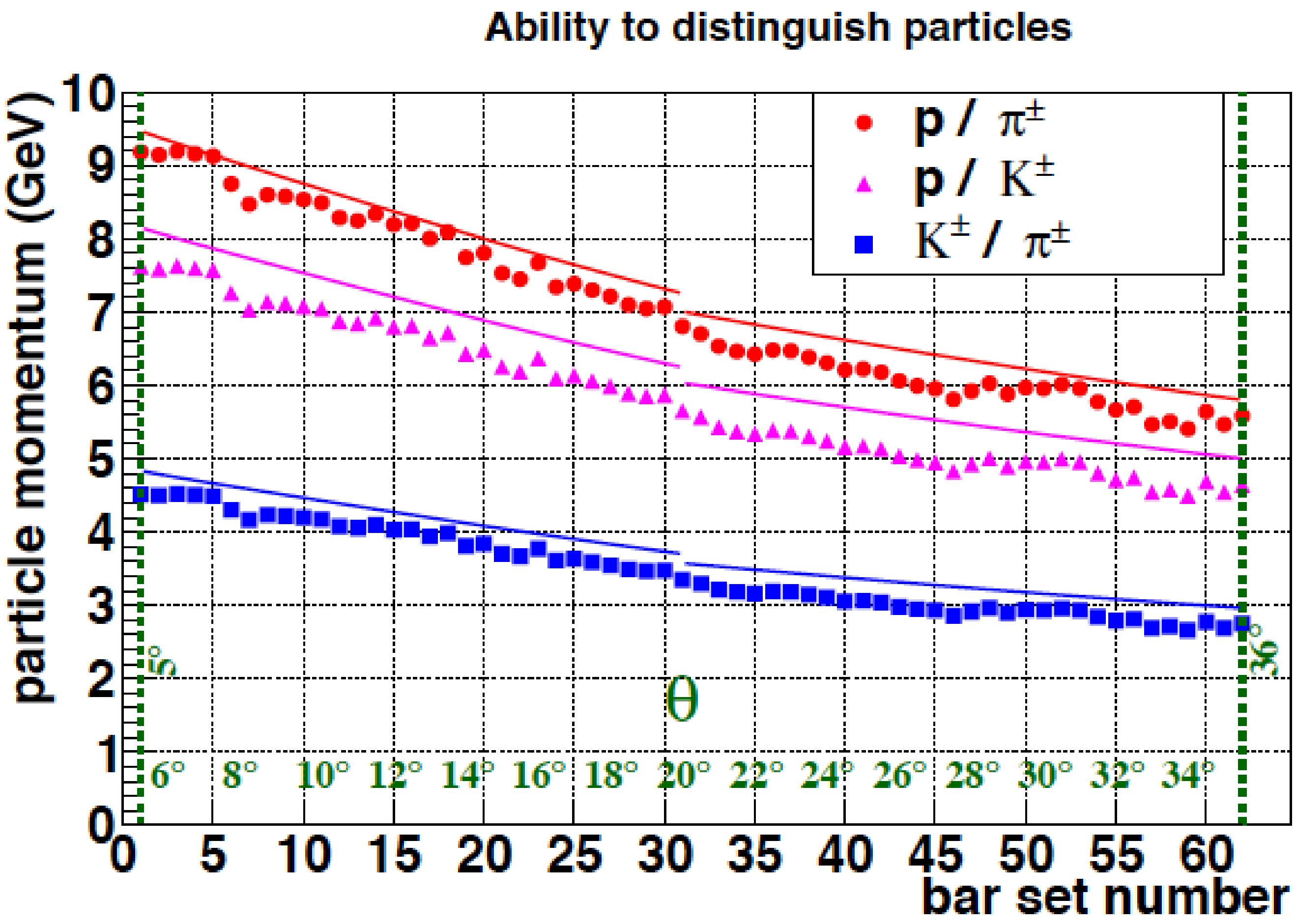
World record time resolution!

Average Time Resolution



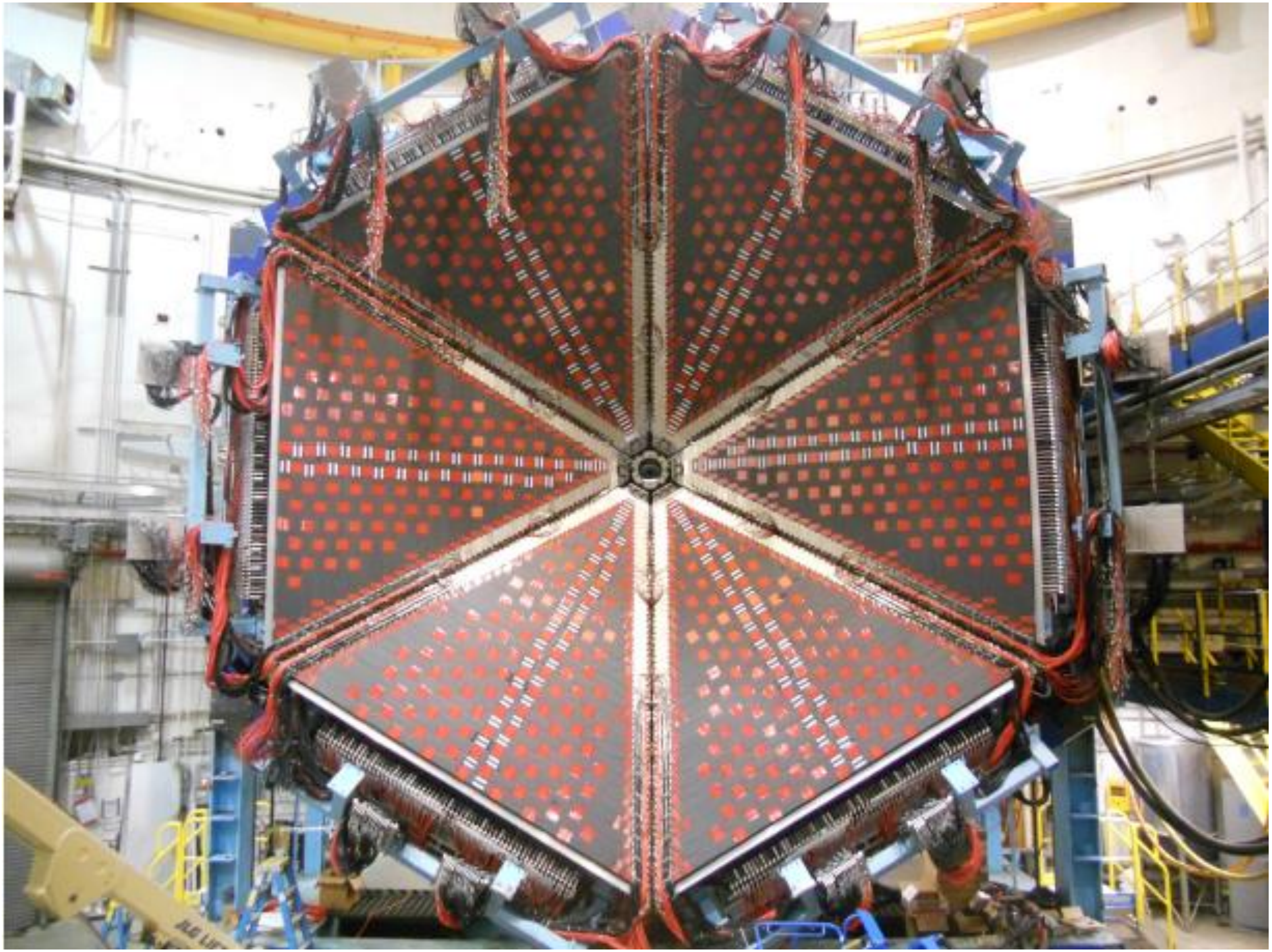
The time resolution better than the design requirement was achieved

Ability to Separate Particles



Maximum momentum up to which particles can be separated as function of bar set number and associated theta angle. Calculation were made in assumption of 700 cm path length from target to Panel 1-B.

FToF Installed at JLab Hall-B



Further Experiments

- "Nucleon Resonance Studies with CLAS12" proposal approved by PAC34.
- "Search for Hybrid Baryons with CLAS12 in Hall-B" proposal approved by PAC44.

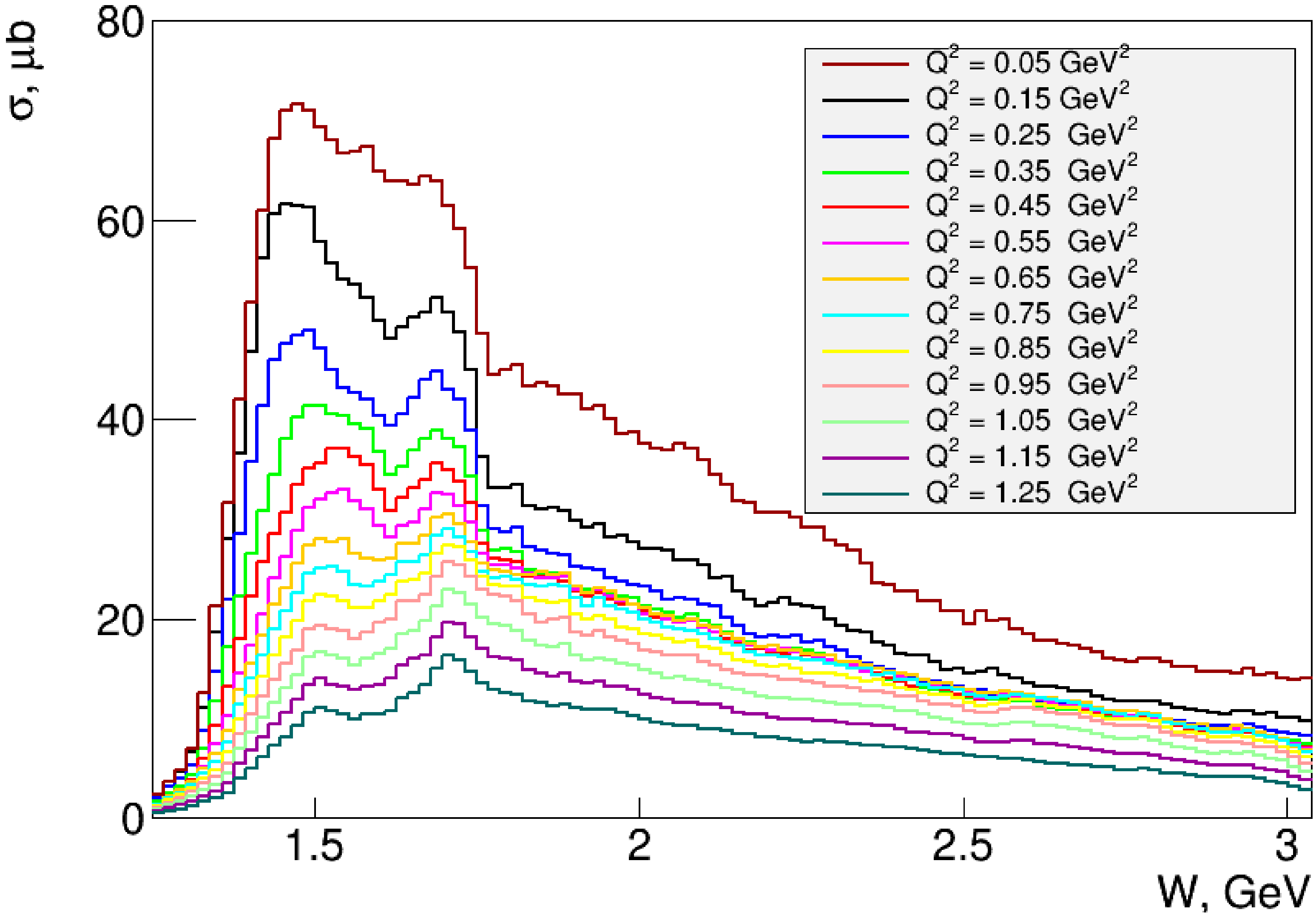
⬇

Event generator that can cover extremely low Q^2 (≈ 0.05 GeV^2)
and high W (> 2GeV) is needed

⬇

- Iu. Skorodumina, G. Fedotov et al., "TWOPEG: An Event Generator for Charged Double Pion Electroproduction off Proton", CLAS12-Note-2017-001, arXiv:1703.08081
- Iu. Skorodumina, G. Fedotov et al., "TWOPEG-D: An Extension of TWOPEG for the Case of a Moving Proton Target", CLAS12-Note-2017-014, arXiv:1712.07712

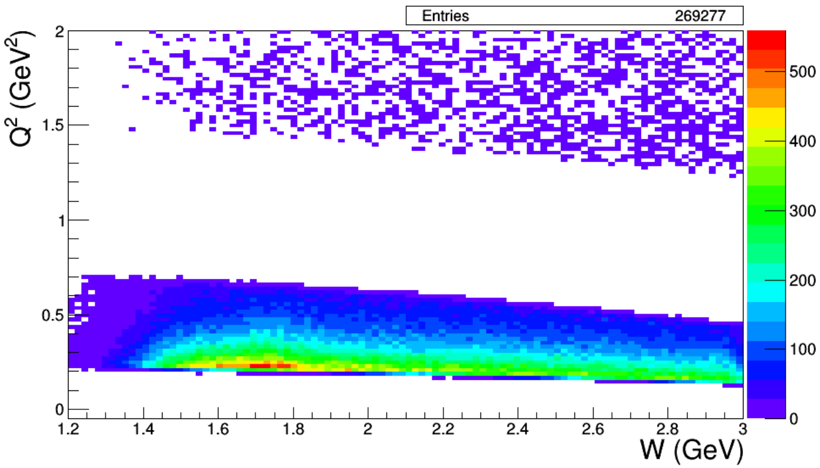
W-Dependences of Integral Cross Sections at Different Q^2-bins for E_{beam} = 11 GeV



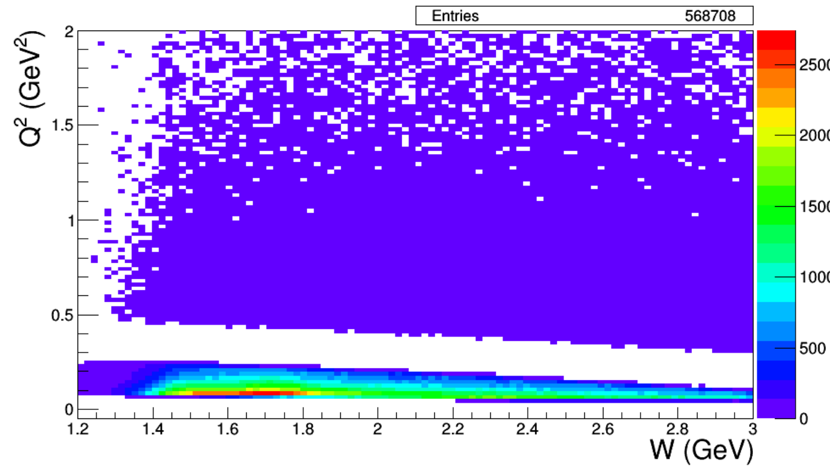
Iu. Skorodumina, G. V. Fedotov, et al., CLAS12-NOTE-2017-001, arXiv:1703.08081.

Run Conditions for Hybrid Baryon Search

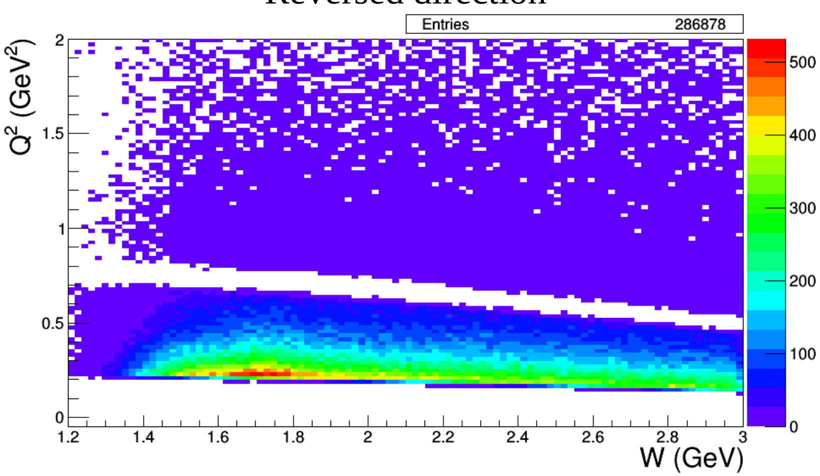
E_{beam} = 11 GeV, I_{torus} = 3375 A
Normal direction



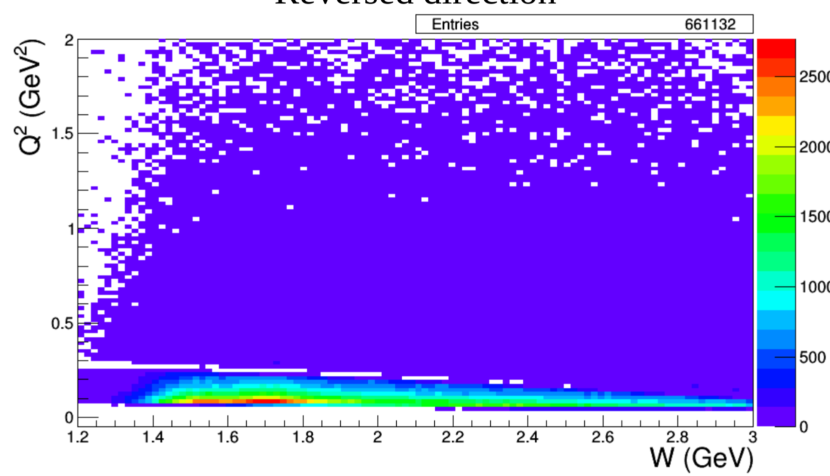
E_{beam} = 6.6 GeV, I_{torus} = 1500 A
Normal direction



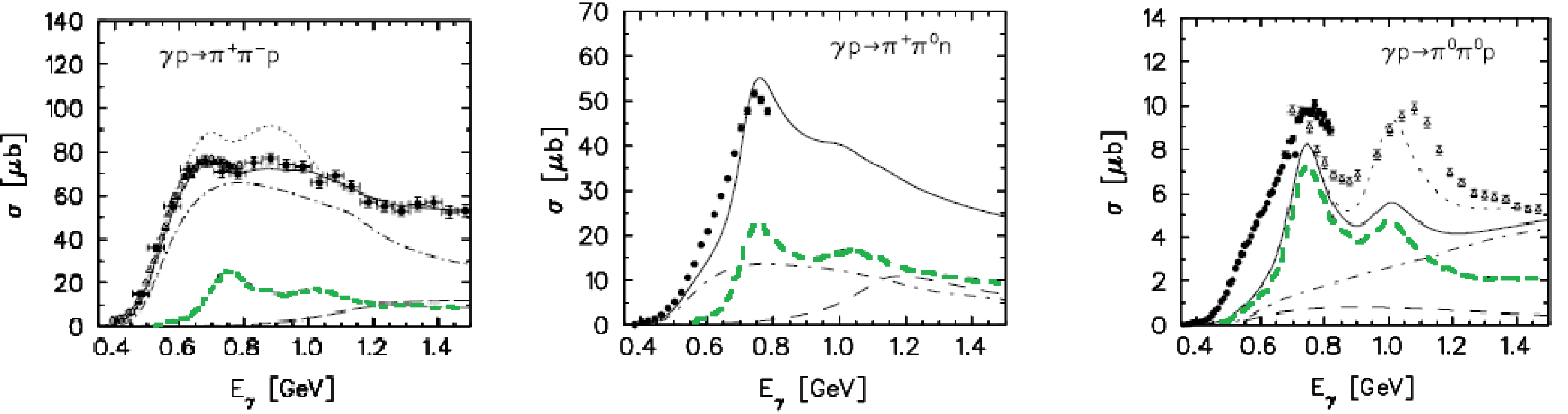
E_{beam} = 11 GeV, I_{torus} = 3375 A
Reversed direction



E_{beam} = 6.6 GeV, I_{torus} = 1500 A
Reversed direction



A New Promissing $n\pi^{+}\pi^{0}$ Channel



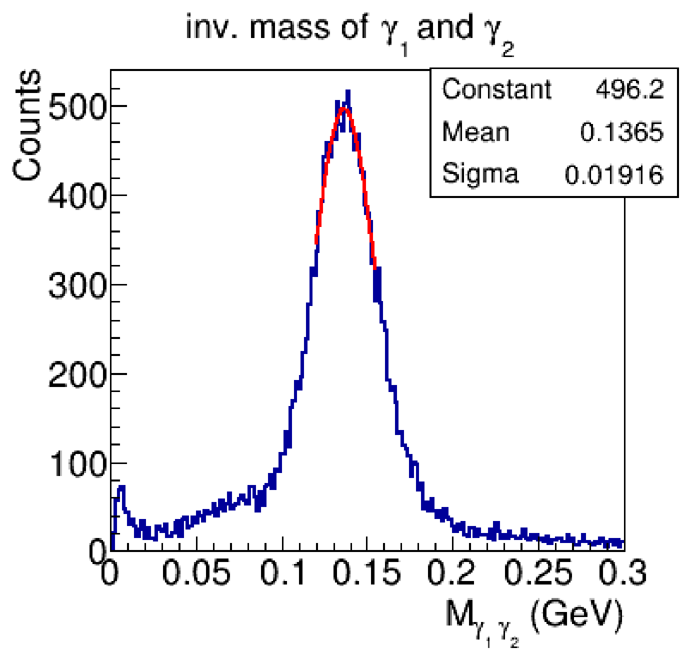
A. Fix, H. Arenhoevel Eur.Phys.J. A25 (2005) 115-135.

- An adaptation of the experimental analysis tools previously established for $p\pi^{+}\pi^{-}$channel .
- Monte Carlo simulation of the new $\gamma_v p \to n\pi^{+}\pi^{0}$channel is needed to determine the particle registration efficiency.
- An adaptation of the phenomenological reaction model to the $\gamma_v p \to n\pi^{+}\pi^{0}$ channel.

$n\pi^+\pi^0$ Production from Dataset with E_{beam} = 2 GeV

- The study of the $\gamma_v p \rightarrow n\pi^+\pi^0$ requires the registration of the π^0 by detecting of its decay products (2γ) in the electromagnetic calorimeter.

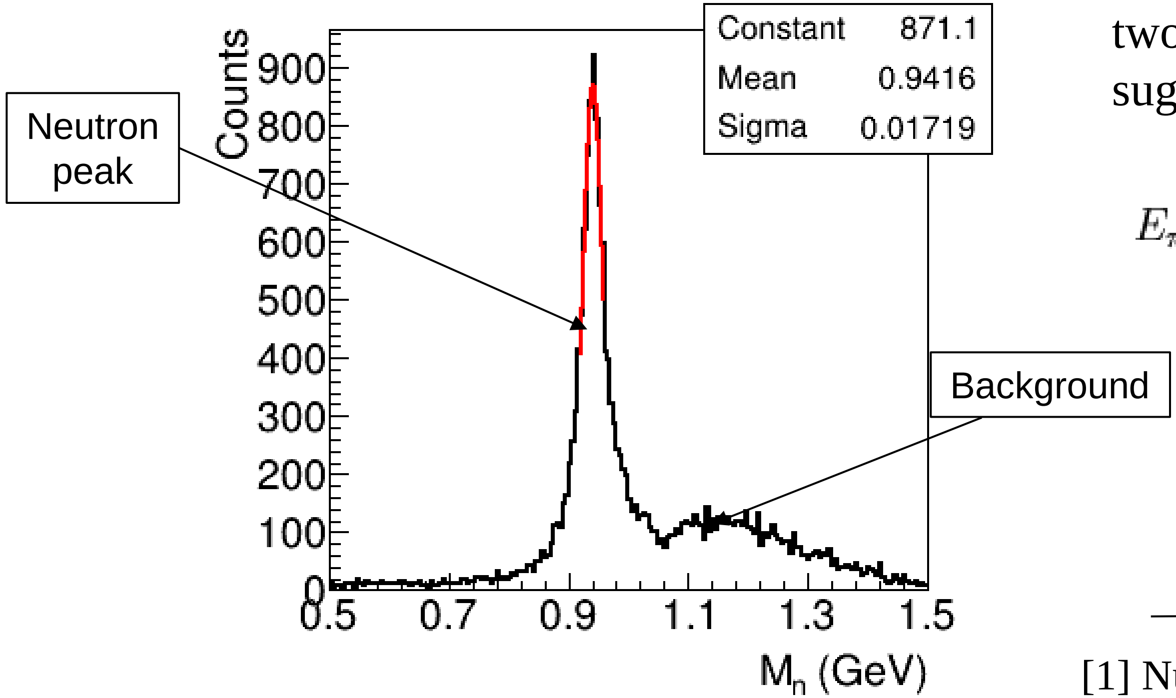
- The plot below shows the invariant mass of 2γ with the peak position at the π^0 mass obtained from CLAS dataset with E_{beam} = 2 GeV.



$n\pi^+\pi^0$ Production from Dataset with E_{beam} = 2 GeV

- A neutron in the reaction $\gamma_v p \rightarrow n\pi^+\pi^0$ can not be registered with CLAS detector and needs to be reconstructed as a missing particle.
- For that purpose the four-momentum of the π^0 needs to be known.
- A common way of the π^0 four-momentum calculation via the sum of the momenta of two photons gives a poor missing mass resolution.

$n\pi^+\pi^0$ production from dataset wit E_{beam} = 2 GeV



The π^0 energy was calculated using two photons opening angle $\phi_{\gamma_1\gamma_2}$ as suggested in [1].

$$E_{\pi^0} = E_{\gamma 1} + \frac{m^2_{\pi^0}}{2E_{\gamma_1}(1 - cos\phi_{\gamma_1\gamma_2})}$$

On this way a missing mass plot with a peak at the neutron mass position was produced.

[1] Nucl.Instrum.Meth. A269 (1988) 568-579.

Summary (CLAS12 Part of the Talk)

- Time-of-Flight system for CLAS12 detector was built and tested at USC and successfully delivered and mounted at JLab.

- Measured time resolution exceeds the design requirements. By now the world best resolution for given scintillator length was achieved.

- “Nucleon Resonance Studies with CLAS12” and “Search for Hybrid Baryons with CLAS12 in Hall-B” experiments are approved and will start soon.

- The new event generators for double-pion production off proton and deuteron targets (TWOPEG and TWOPEG-D) were developed and successfully used in the preparation of those experiments.

- The possibility to isolate $\gamma_v p \rightarrow n\pi^+\pi^0$ exclusive channel with CLAS12 was investigated. The adaptation of data analysis tool and the phenomenological model was initiated.

Спасибо за внимание!

Conclusions

- Cross sections of $\gamma_v p \rightarrow \pi^+\pi^- p$ reaction at low W < 1.55 GeV were obtained and published.

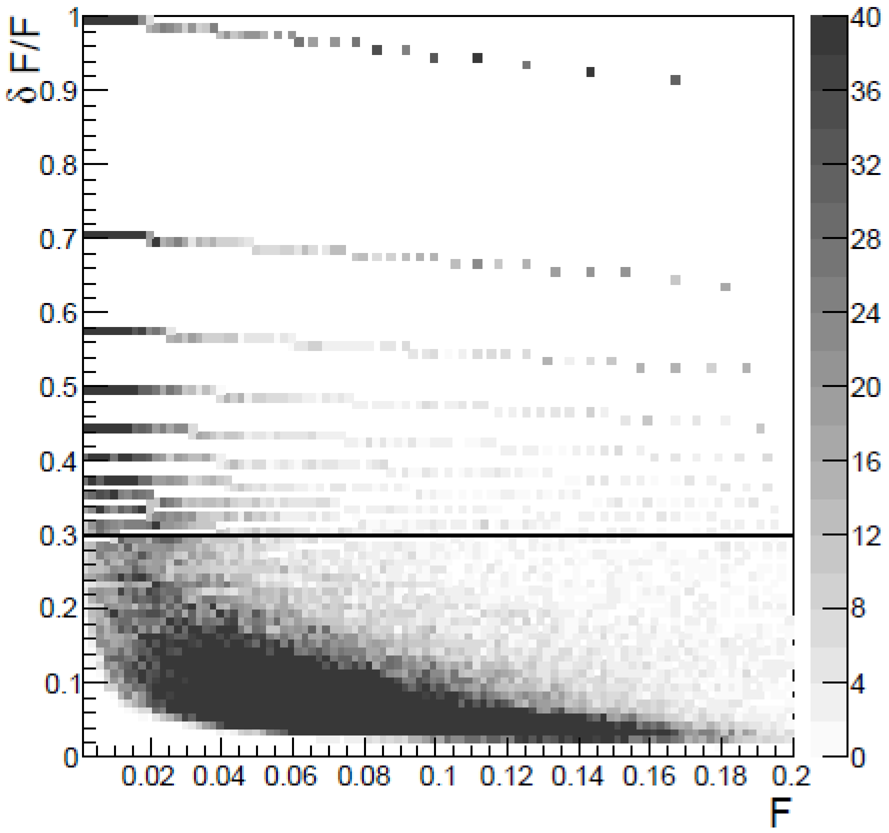
- Cross sections of $\gamma_v p \rightarrow \pi^+\pi^- p$ reaction were obtained for W from 1.4 GeV to 1.8 GeV and Q^2 from 0.4 GeV^2 to 1.1 GeV^2, analysis note is in preparation

- Kinematic coverage and statistics exceed the previously available CLAS data and allow to achieve six times finer binning in Q^2

- The phenomenological analysis of this data will extend considerably the available information on the Q^2 evolution of the high-lying N* electrocouplings

- Time-of-Flight system for CLAS12 detector with world record time resolution was built and tested at USC and successfully delivered and mounted at JLab

Empty Cells and Efficiency Error Cut



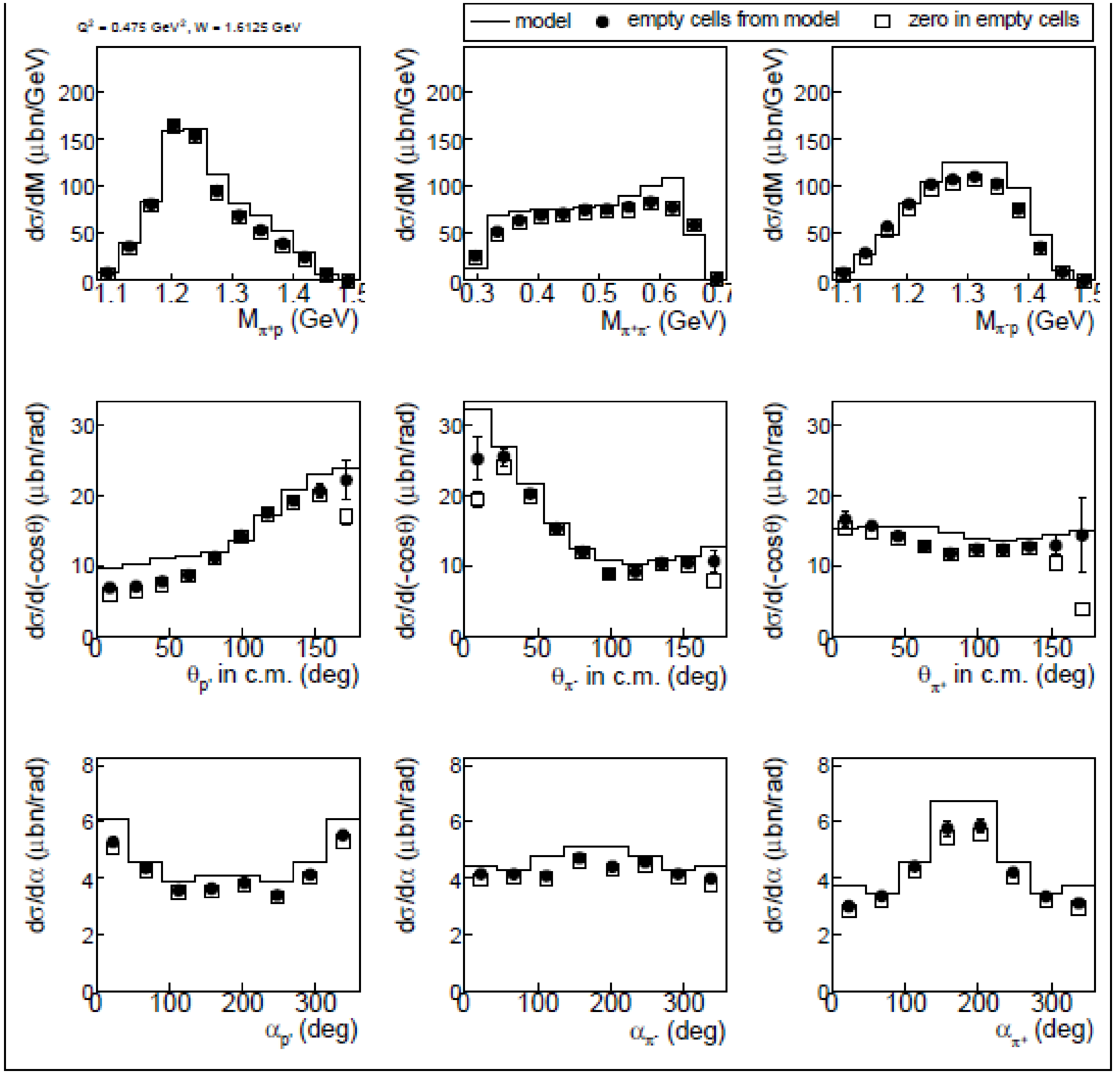
A kinematic cell is treated as empty if $N_{gen} > 0$ and $N_{rec} = 0$.

In addition to that the cells with relative efficiency error >30% also treated as empty.

The efficiency error was calculated according to the study B. Laforge and L. Schoeel, Nucl. Instrum. Meth. A394, 115 (1997).

Filling Cells with Zero Acceptance

$Q^2 = 0.475$ GeV^2, W = 1.6125 GeV



- Combination of all available topologies allows to minimize contributions from zones with zero acceptance.
- New advanced method to fill zones with zero acceptance based on TWOPEG .

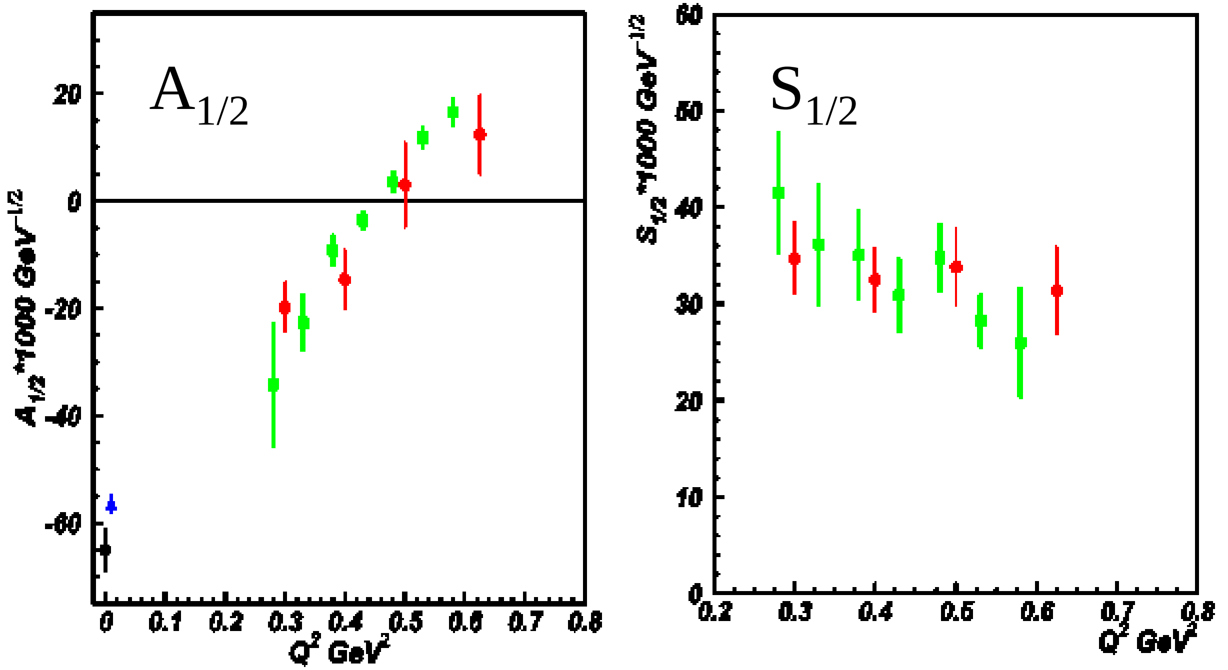
$P_{11}(1440)$ electrocouplings from the CLAS data on Nπ/Nππ electroproduction

Nππ

V. I. Mokeev et al., PRC 86, 035203 (2012).

Nπ

I.Aznauryan,V.Burkert, et al., PRC 80,055203 (2009).



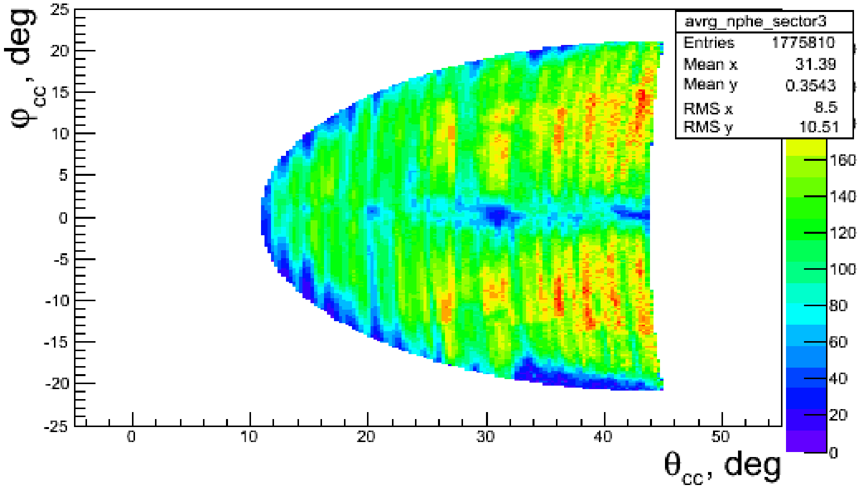
- **Good *agreement* between the electrocouplings obtained from the *Nπ* and *Nππ* *channels*: Reliable measurement of the electrocouplings.**

TOF Conclusions

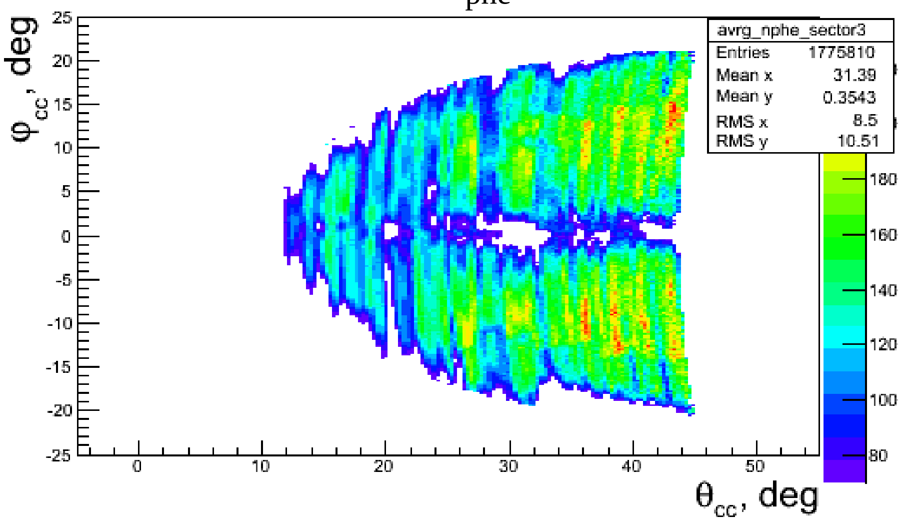
- Time-of-Flight system for CLAS12 detector was built and tested at USC and successfully delivered and mounted at JLab.
- Measured time resolution exceeds the design requirements. By now world best resolution for given scintillator length was achieved.
- New time-walk correction procedure was developed. It could be helpful to establish time-walk corrections during CLAS12 operation.

Cut on average number of photoelectron

Average number of photoelectrons as function of θ and φ angles in the Cherenkov counter plain for one particular CLAS sector



We geometrically remove areas with $<N_{phe}> < 70$



Conclusions (future experiments)

- “Nucleon Resonance Studies with CLAS12” and “Search for Hybrid Baryons with CLAS12 in Hall-B” experiments are approved and will start soon

- The new event generator for double-pion production (CLAS12-NOTE-2017-001, arXiv:1703.08081) was developed and successfully used in preparation of those experiments

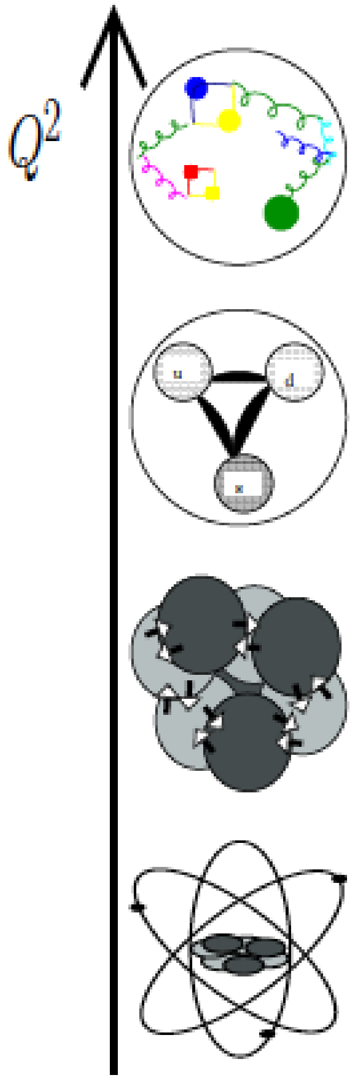
- The possibility to isolate $\gamma_v p \rightarrow n\pi^+\pi^0$ exclusive events in the accumulated CLAS data is shown.

- Due to the small angular coverage of the CLAS calorimeter CLAS12 data are needed.

- A Monte Carlo simulation of the investigated channel with CLAS12 reconstruction procedure is needed.

- The adaptation of the reaction model to the $\gamma_v p \rightarrow n\pi^+\pi^0$ channel is currently in progress.

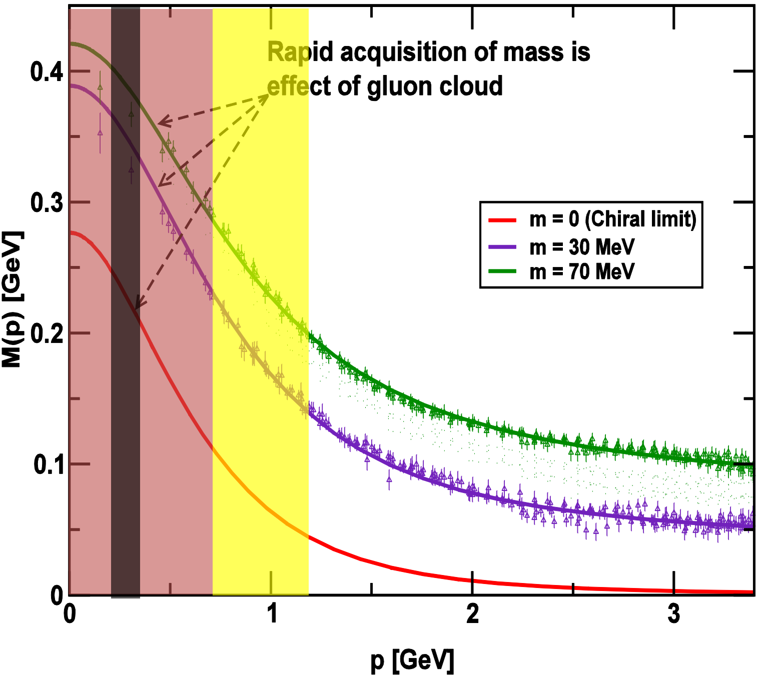
Structure of Matter



$\begin{pmatrix} q_r \\ q_g \\ q_b \end{pmatrix}$ = Current Quarks [q=(u,d,s)]	gluons	Standard Model's pQCD
$\begin{pmatrix} \mathbf{u} \\ \mathbf{d} \\ \mathbf{s} \end{pmatrix}$ = Constituent Quarks	*Effective forces*	*Models*: CQM
$\begin{pmatrix} p \\ n \end{pmatrix}$ = Nucleons	*Effective forces*	*Models*: Yukawa
Nucleus, e^-	photon	Standard Model's QED

The Study of N*

arXiv:1009.3458[nucl-th]

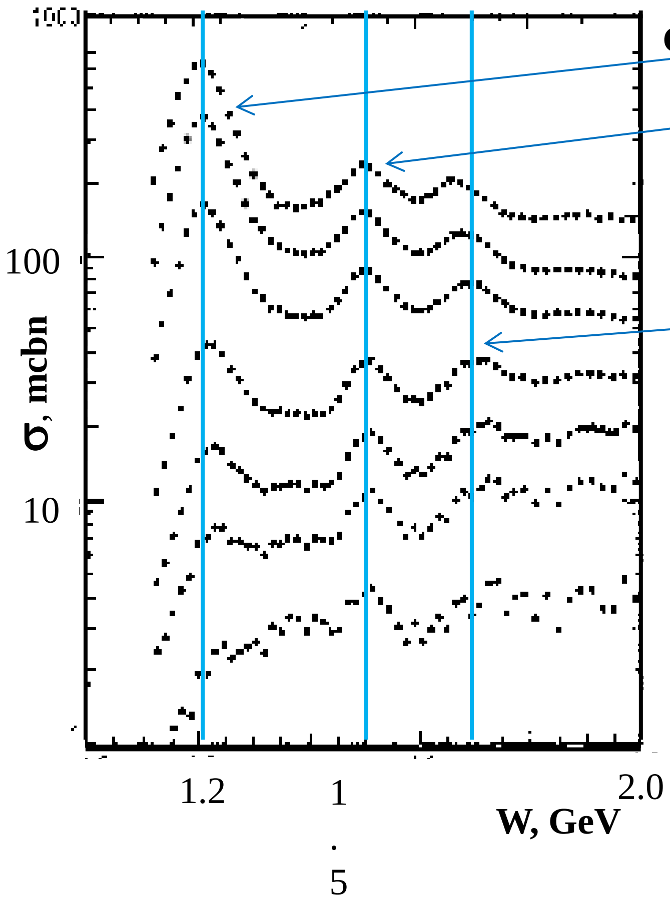


- **Pin-down active degrees of freedom in N* structure at various distances.**
- **Study the non-perturbative strong interactions which are responsible for N* formation and their emergence from QCD.**
- **Uniquely access the origin of more than 97% of dressed quark masses generated through dynamical chiral symmetry breaking, and explore the origin of confinement .**

N* studies are key to the exploration of non-perturbative strong interactions and confinement.

N* States in Inclusive Electron Scattering

Total virtual photon cross sections
F. Foster and G.Hughes, Rep. Progr. Phys. 46, 1445 (1983).



The peak content

$P_{33}(1232)$

$P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$., $S_{31}(1620)$

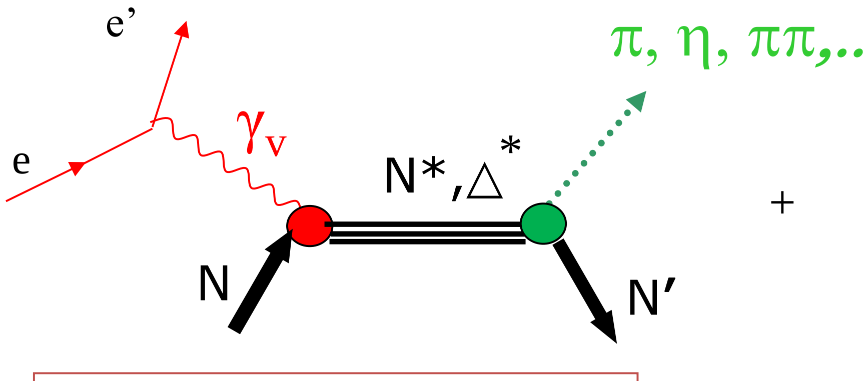
$S_{11}(1650)$, $F_{15}(1685)$, $D_{33}(1700)$, $P_{11}(1710)$, $P_{13}(1720)$. $3/2^{+}(1720)$.

What we knew 30 years ago:

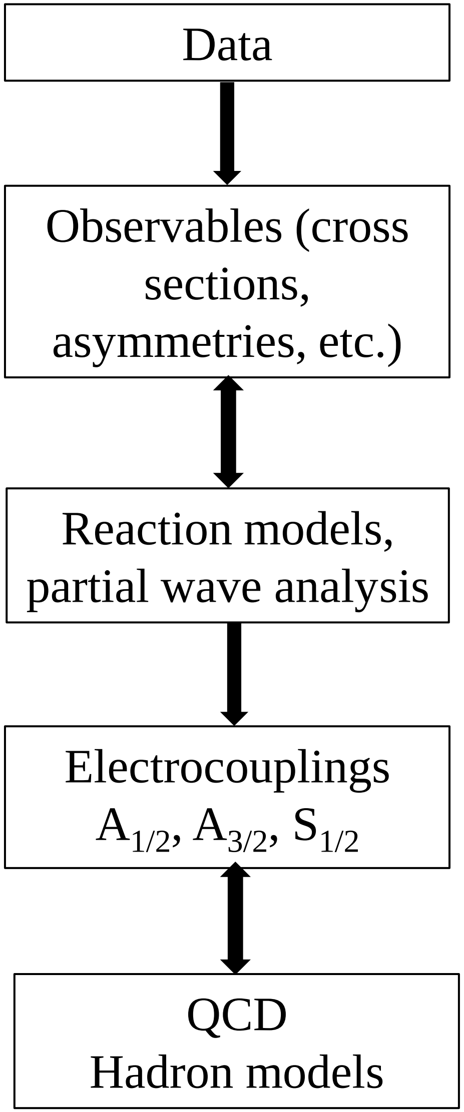
- three resonant peaks.
- different Q^{2}-evolution of these peaks
- different structure of different N*-states.

Substantial overlap between different N*- states and sizable non-resonant contributions prevent extraction of N* parameters from inclusive data only. The data on exclusive meson electroproduction are needed for exploration of the N* structure.

Extraction of $\gamma_v NN^*$ Electrocouplings from the Data on Exclusive Meson Electroproduction off Protons



- $A_{1/2}(Q^2)$, $A_{3/2}(Q^2)$, $S_{1/2}(Q^2)$



- **Separation of resonant/non-resonant contributions within the framework of reaction models; Breit Wigner ansatz for parameterization of resonant amplitudes; fit to the data.**

- **Consistent results on $\gamma_v NN^*$ electrocouplings from different meson electroproduction channels and different analysis approaches demonstrate reliable extraction of N* parameters.**

Electroproduction Data from CLAS

Hadronic final state	Covered W-range, GeV	Covered Q^2-range, GeV^2	Measured observables
$\pi^+ n$	1.1-1.40 1.1-1.55 1.1-1.7	0.15-0.40 0.3-0.6 1.7-4.2	$d\sigma/d\Omega$ $d\sigma/d\Omega$ $d\sigma/d\Omega$, A_b
$\pi^0 p$	1.1-1.40 1.1-1.7 1.1-1.7	0.15-0.40 0.4-0.7 0.75-6.0	$d\sigma/d\Omega$ $d\sigma/d\Omega$, A_b,A_t,A_{bt} $d\sigma/d\Omega$
ηp	1.5-2.0	0.2-4.0	$d\sigma/d\Omega$
$K^+\Lambda$	1.65-2.35 1.65-2.35	0.65-2.55 1.4-2.6	$d\sigma/d\Omega$ P’
$K^+\Sigma^0$	1.7-2.1 1.8-2.5 1.7-2.6	0.5-2.55 1.5-3.50 1.8-3.50	$d\sigma/d\Omega$ P’ $d\sigma/d\Omega$
$\pi^+\pi^- p$	1.3-1.6 1.4-2.1	0.2-0.6 0.5-1.5	Nine 1-fold differential cross sections

- $d\sigma/d\Omega$–CM angular distributions
- A_b,A_t,A_{bt}-longitudinal beam, target, and beam-target asymmetries
- P’ –recoil polarization of strange baryon

Almost full coverage of the final hadron phase space in πN, $\pi^+\pi^- p$, ηp, and KY electroproduction

The data are available in the CLAS Physics Database:

http://depni.sinp.msu.ru/cgi-bin/jlab/db.cgi

Kinematical Variables

$ep \to e' p' p^+ p^-$ in single photon exchange approximation $g_v p \to p' p^+ p^-$

From experiment → $$\frac{d^7\sigma_e}{dW\,dQ^2 d^5\tau} = \Gamma_v \frac{d^5\sigma_v}{d^5\tau}$$ ← From JM model (fit to the data)

Final hadrons are described by 5 independent variables

$$M_{\pi^+\pi^-},\; M_{\pi^+p},\; \theta_p,\; \varphi_p,\; \alpha_{(p,p'),(\pi^+,\pi^-)}$$

$$M_{\pi^+\pi^-},\; M_{\pi^-p},\; \theta_{\pi^+},\; \varphi_{\pi^+},\; \alpha_{(p,\pi^+),(\pi^-,p')}$$

$$M_{\pi^+\pi^-},\; M_{\pi^+p},\; \theta_{\pi^-},\; \varphi_{\pi^-},\; \alpha_{(p,\pi^-),(\pi^+,p')}$$

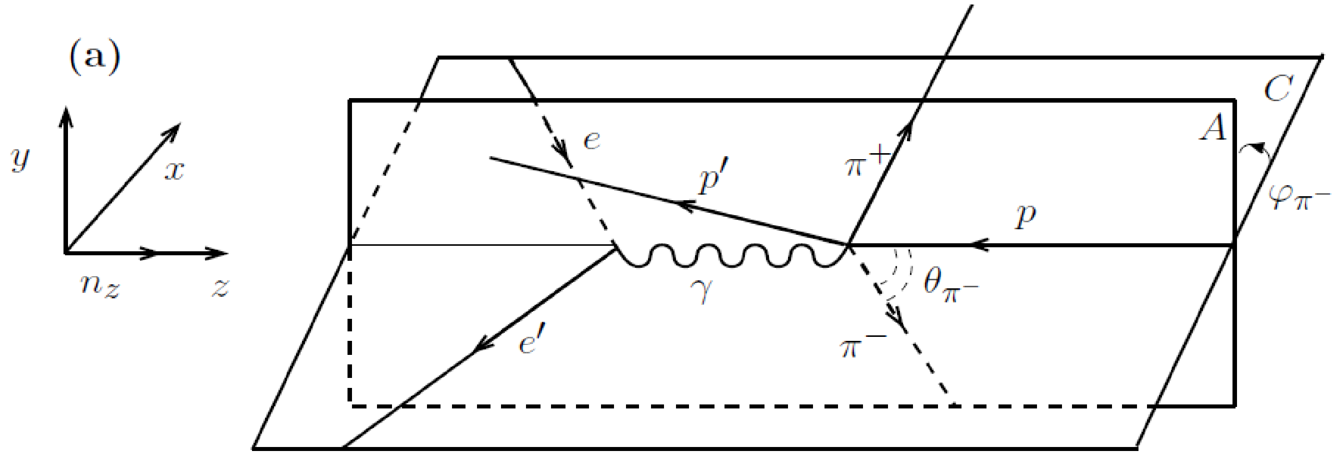
$$\frac{\mathrm{d}\sigma}{\mathrm{d}M_{\pi^+\pi^-}} = \int \frac{\mathrm{d}^5\sigma}{\mathrm{d}^5\tau}\mathrm{d}\tau_{\pi^+\pi^-},$$

$$\mathrm{d}\tau_{\pi^+\pi^-} = \mathrm{d}M_{\pi^-p}\mathrm{d}\Omega_{\pi^-}\mathrm{d}\alpha_{(p,\pi^-),(\pi^+,p')},$$

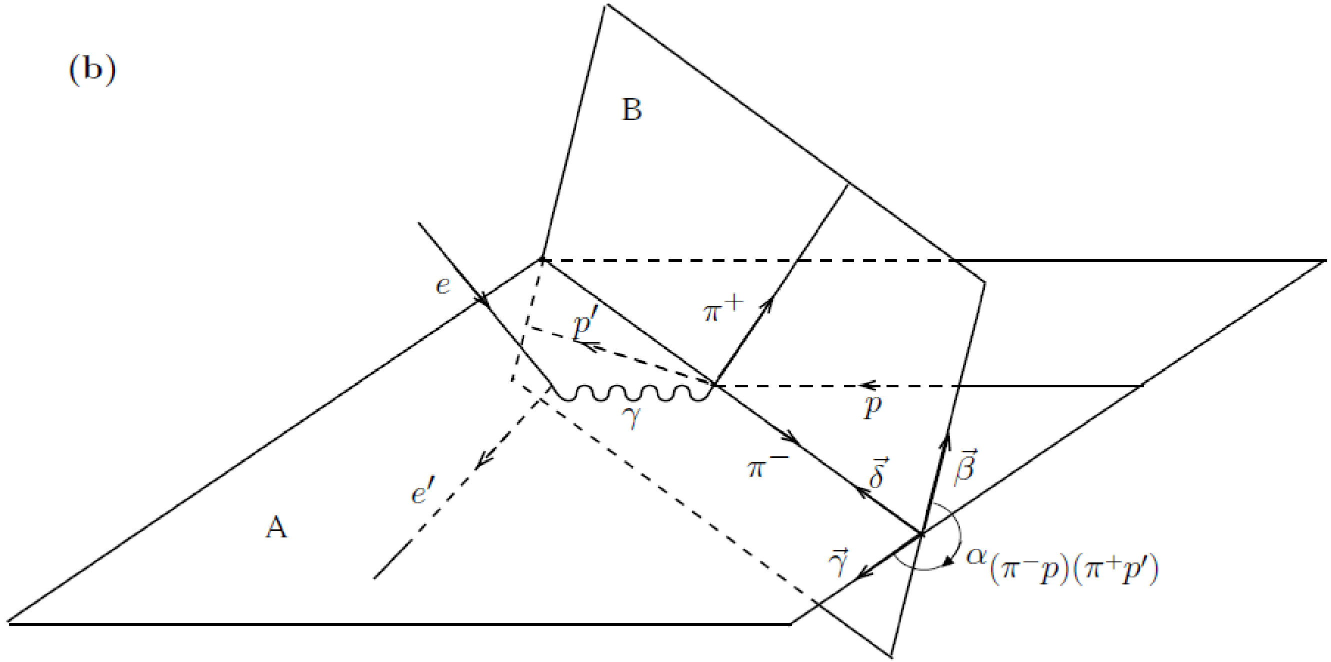
$$\mathrm{d}^5\tau_{\pi^+\pi^-} = \mathrm{d}M_{\pi^+\pi^-}\mathrm{d}M_{\pi^-p}\mathrm{d}\Omega_{\pi^-}\mathrm{d}\alpha_{(p,\pi^-),(\pi^+,p')}$$

Final Hadrons Angles Definition

(a)



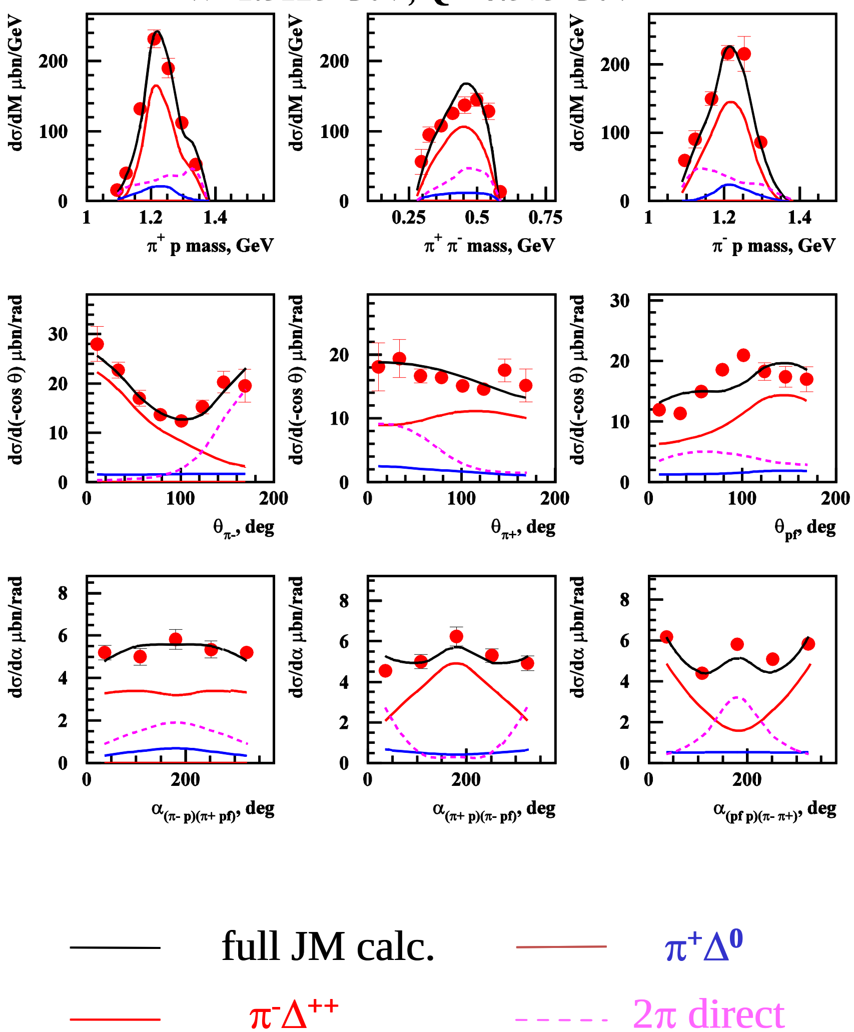
(b)

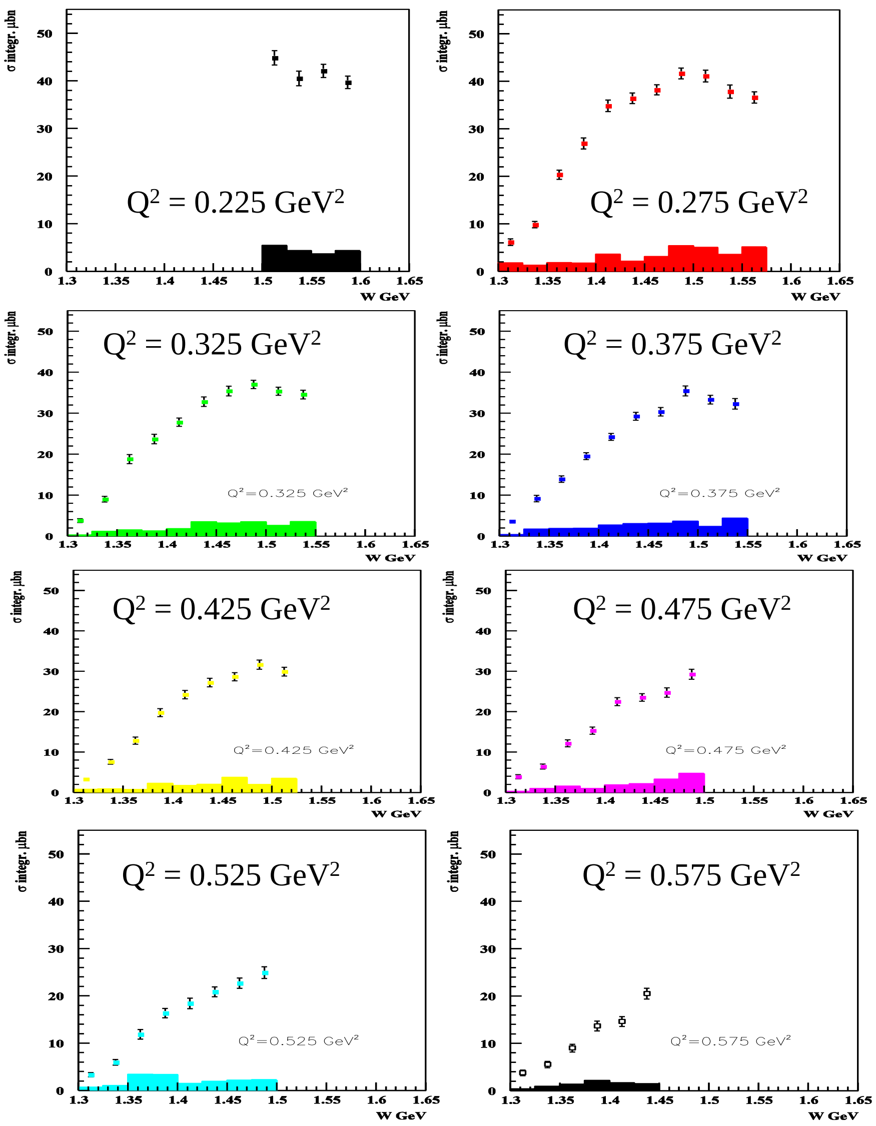


CLAS Data on $\pi^+\pi^-p$ Differential Cross Sections and their Description within the JM Model

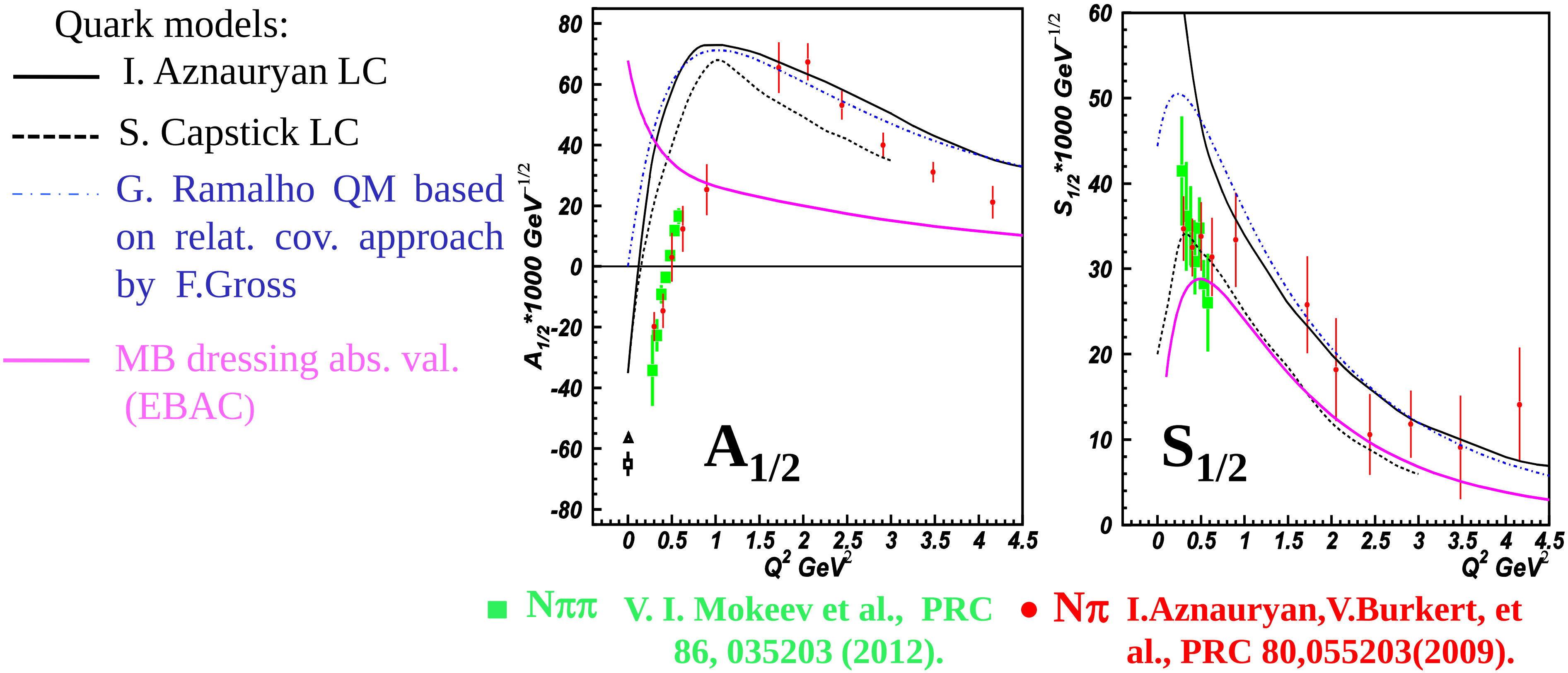
G.V.Fedotov et al., PRC 79 (2009), 015204

W=1.5125 GeV, Q^2=0.375 GeV^2



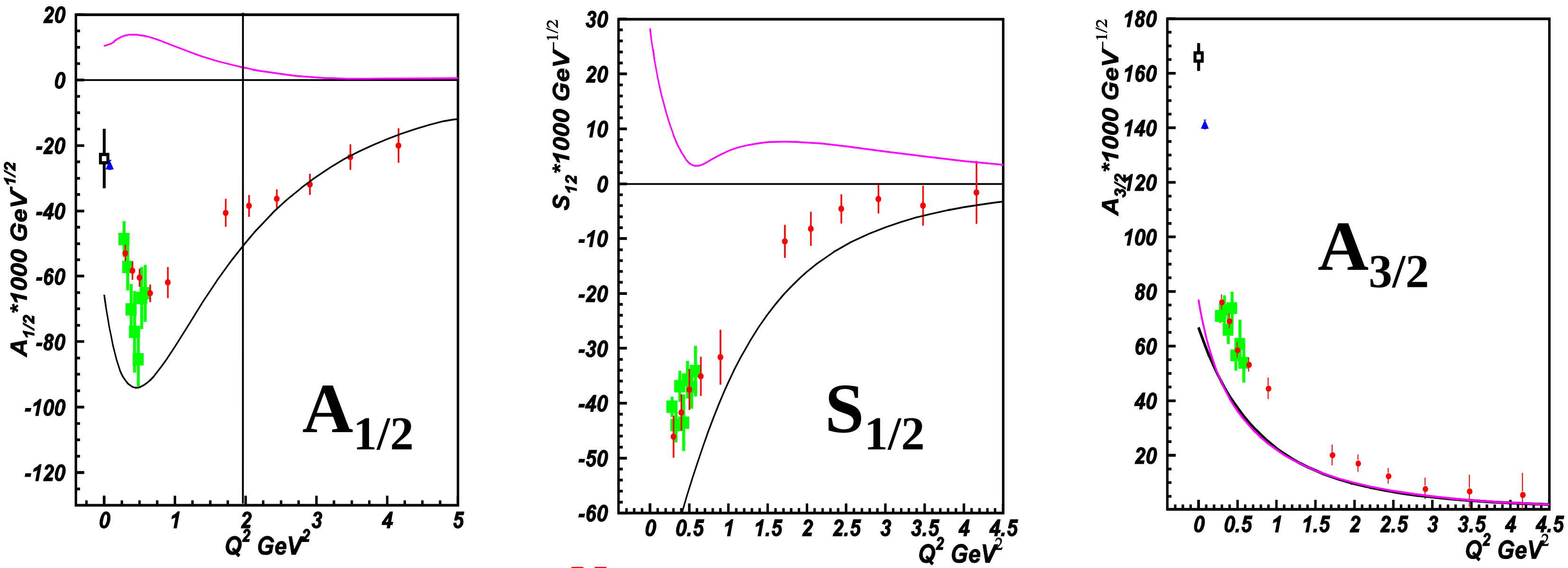


P_{11}(1440) Electrocouplings from the CLAS Data on $N\pi$/$N\pi\pi$ Electroproduction



- **The electrocouplings are consistent with *P_{11}(1440)* structure as a superposition of the contributions from: a) quark core as a first *radial excitation of the nucleon as a 3-quark ground state and b) meson-baryon dressing*.**

$D_{13}(1520)$ Electrocouplings from the CLAS Data on $N\pi/N\pi\pi$ Electroproduction



■ $N\pi\pi$ V. I. Mokeev et al., PRC 86, 035203 (2012).

● $N\pi$ I.Aznauryan,V.Burkert, et al., PRC 80,055203(2009).

—— M.Giannini/E.Santopinto hCQM

—— MB dressing abs val. (EBAC)

- good agreement between the results from $N\pi$ & $N\pi\pi$ channels
- substantial contributions from quark core at $Q^2 > 2.0$ GeV^2 and sizable MB cloud at $Q^2 < 1.0$ GeV^2

The results on $A_{1/2}$ electrocouplings at $Q^2 > 2.0$ GeV^2 for the first time offer almost direct access to the quark core!

Kinematic Variables

$ep \rightarrow e'p'p^+p^-$ **in single photon exchange approximation** $g_v p \rightarrow p'p^+p^-$

From experiment → $$\frac{d^7\sigma_e}{dW\,dQ^2 d^5\tau} = \Gamma_v \frac{d^5\sigma_v}{d^5\tau}$$ ← **From JM model (fit to the data)**

Final hadrons are described by 5 independent variables

$$M_{p^+p^-}, M_{p^+p'}, q_p, j_p, a_p$$

$$M_{p^+p^-}, M_{p^-p'}, q_{p^+}, j_{p^+}, a_{p^+}$$

$$M_{p^+p^-}, M_{p^+p'}, q_{p^-}, j_{p^-}, a_{p^-}$$

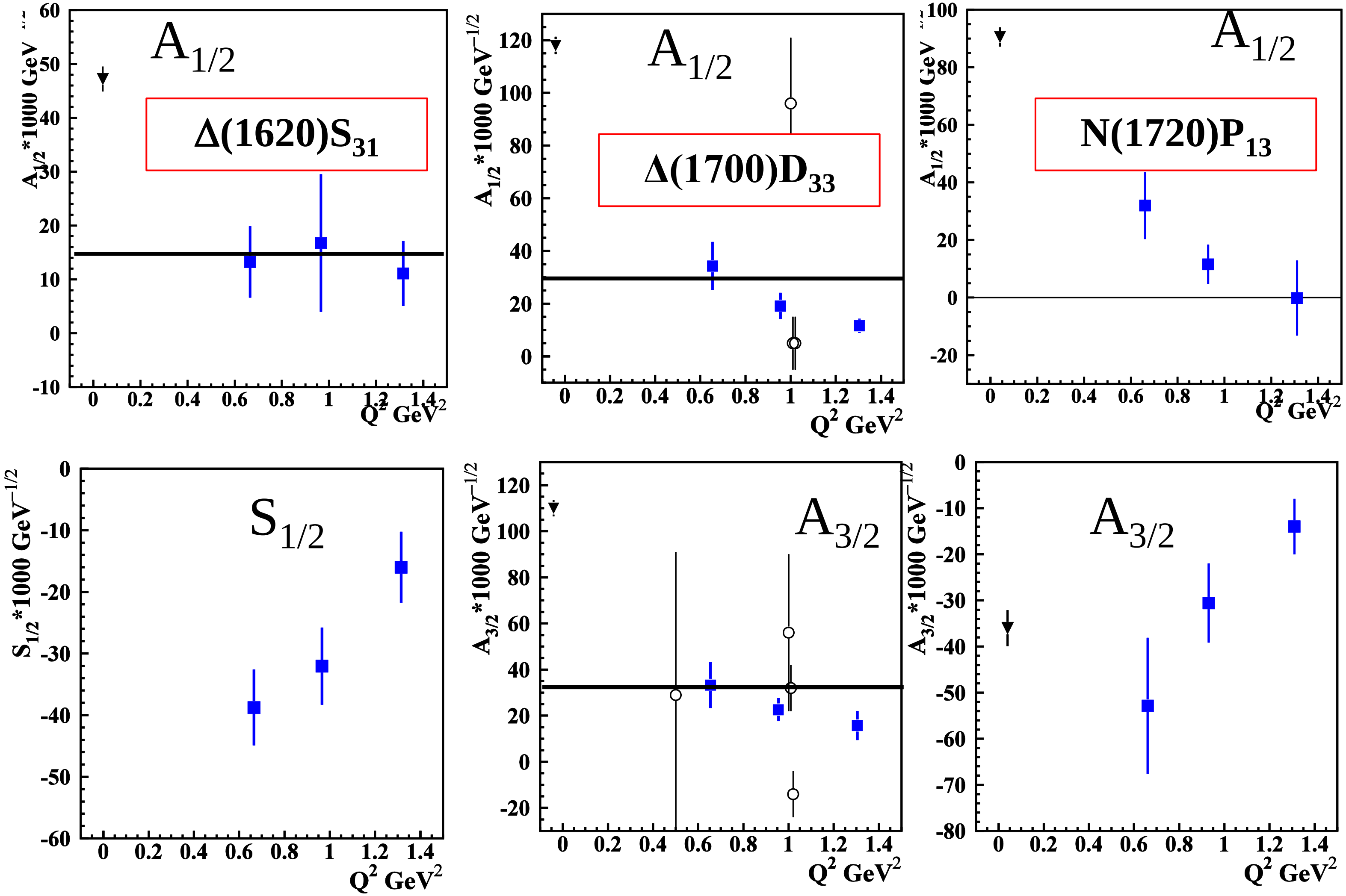
All three sets of hadronic variables are used to extract cross sections. The difference between total cross sections obtained by integration over various sets of hadronic variables is interpreted as systematic uncertainty.

High Lying Resonances Electrocouplings from the $\pi^+\pi^-p$ CLAS Data Analysis

○ Nπ world V.D.Burkert, et al., PRC 67, 035204 (2003).

■ Nππ CLAS preliminary

▼ Nπ Q^2=0, CLAS M.Dugger, et al., PRC 79,065206 (2009).



Only 3 Q^2 points are available, new analysis brings at least 8 more

Motivation for Analysis at Higher W

- Considerable extension of the information on electrocouplings of high mass resonances at W from 1.6 to 1.8 GeV. Extraction N* electrocoupling in Q^2-bins of smaller sizes than in previous data adding at least 8 new points to electrocoupling Q^2-dependences.

- Valuable cross check for available information on electrocouplings of these resonances

Advantages of New Analysis

- New fast analysis code written on C++/ROOT with using THnSparse
- Combination of all available topologies allows to minimize contributions from zones with zero acceptance
- New advanced method of filling out zones with zero acceptance based on new 2π event generator, which is currently under development

Advantages of New 2π Event Generator

- Based on JM15 + newest data
- Written on C++
- New EG will work up to W > 2 GeV and at extremely low Q^2 (available with CLAS12, when FT in use)
- New EG will provide output both in BOS- and LUND-format (EVIO also if needed) – compatible with CLAS12 reconstruction software
- Takes into account cross section change with beam energy
- Allows to obtain cross section value from EG
- EG generates flat distributions and applies the cross section as weight to each event

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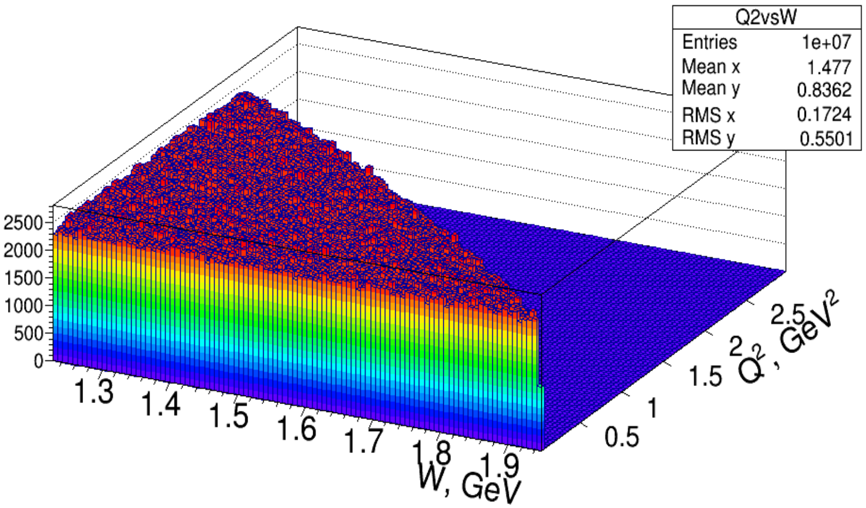
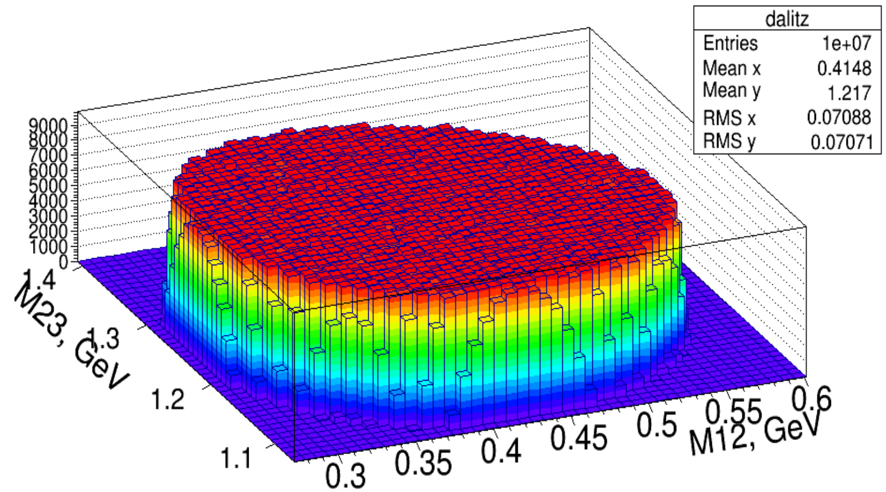
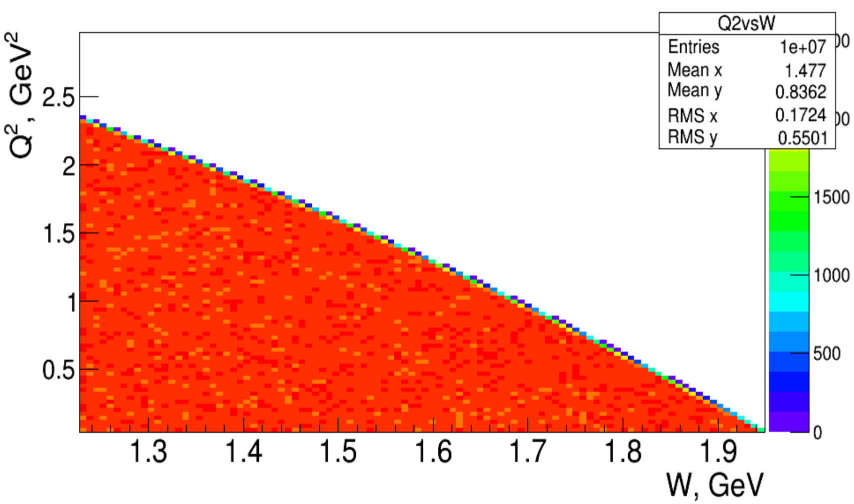
Flat Generation

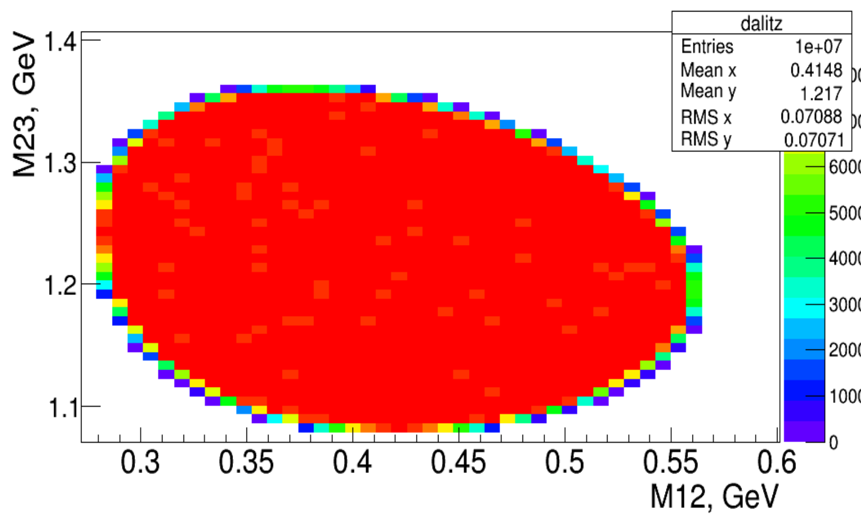
For Ebeam = 2 GeV:

For W = 1.5 GeV:









80

Data Included

Electroproduction:

•

• **1) CLAS data at E_{beam} = 2.445, E_{beam} = 4 GeV**

M. Ripani et al. [CLAS Collaboration], Phys. Rev. Lett. 91, 022002 (2003)

V. I. Mokeev et al. [CLAS Collaboration], Phys. Rev. C 86, 035203 (2012)

2) CLAS data at E_{beam} = 1.515 GeV

G. V. Fedotov et al. [CLAS Collaboration], Phys. Rev. C 79, 015204 (2009)

Photoproduction:

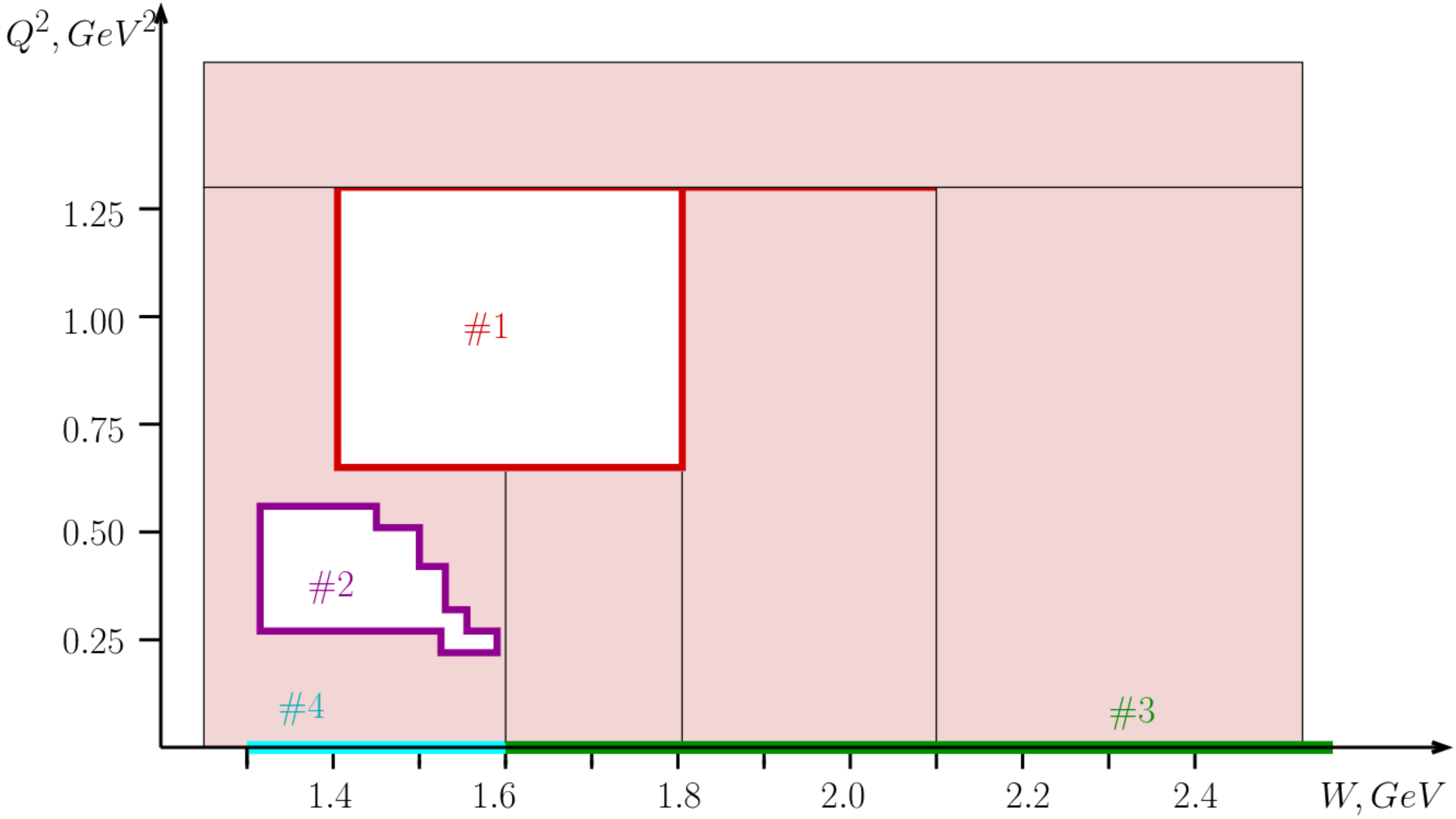
3) CLAS g11a experiment

E. Golovach et.al. CLAS ANALYSIS NOTE (under review).

4) SAPHIR Eur. Phys. J. A 23, 317 (2005).

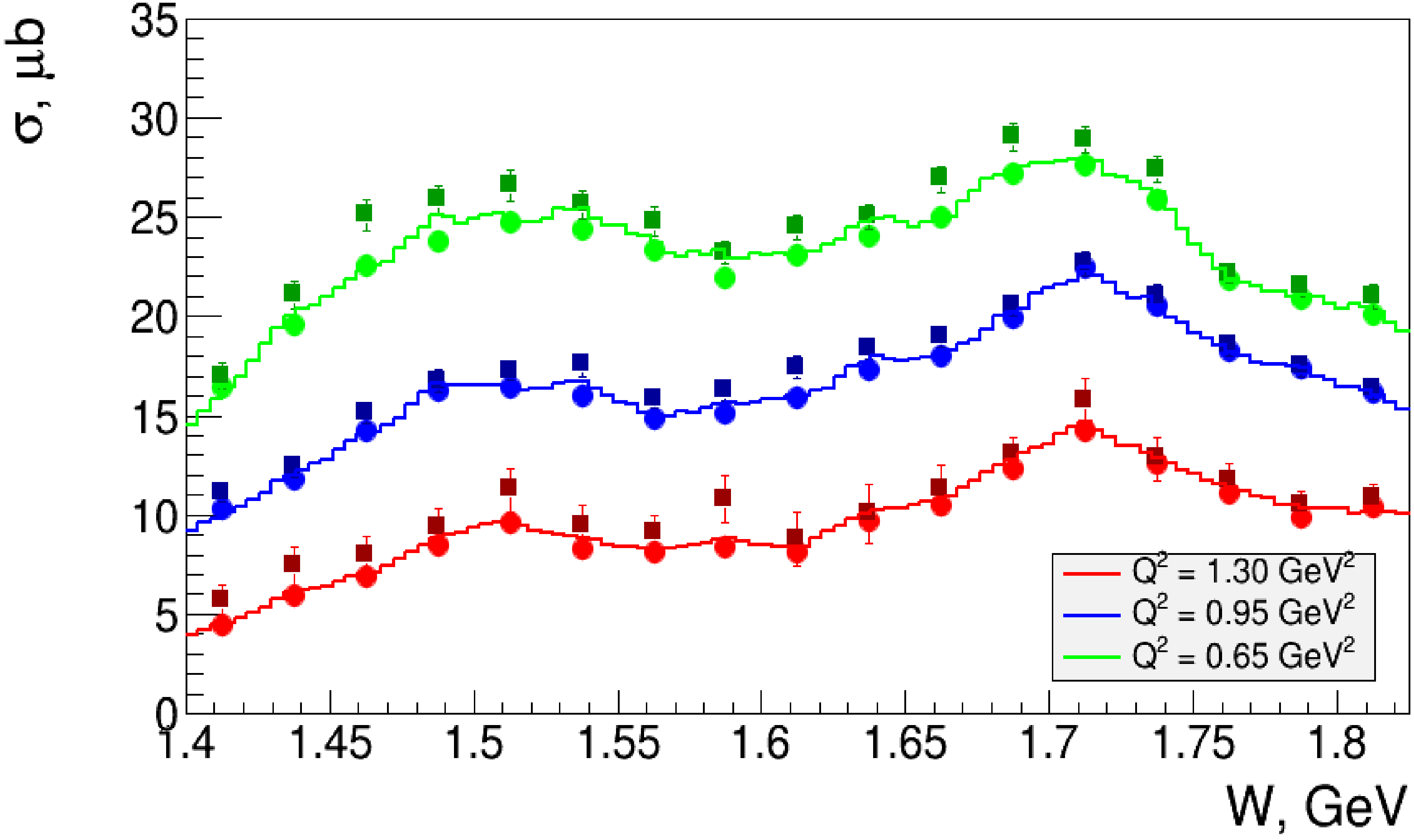
ABBHM Collab., Phys. Rev. 175, 1669 (1968)

Data Included



82

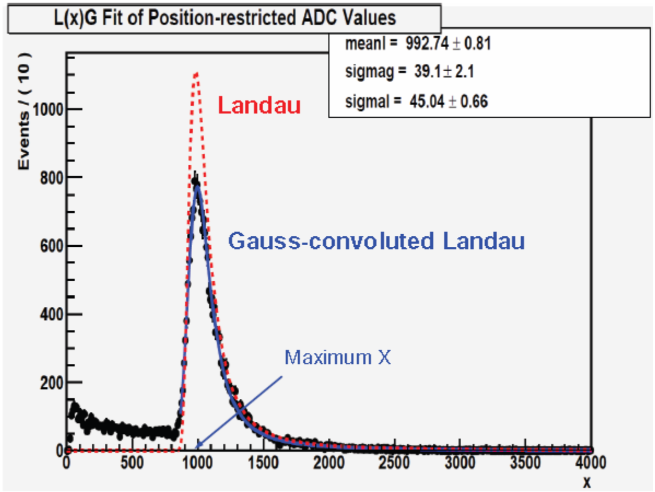
W-Dependences of Integrated Cross Sections

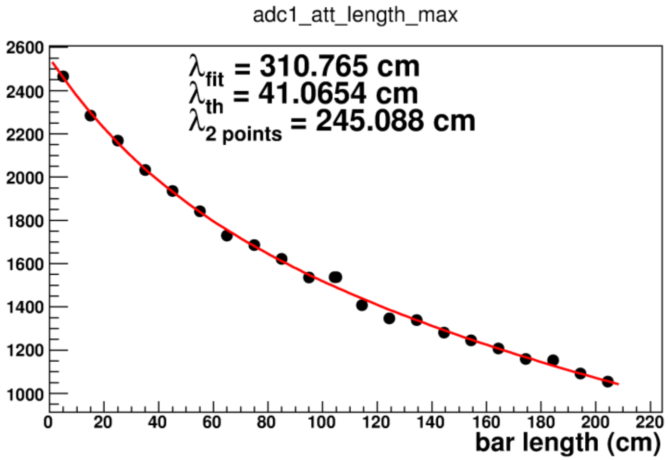


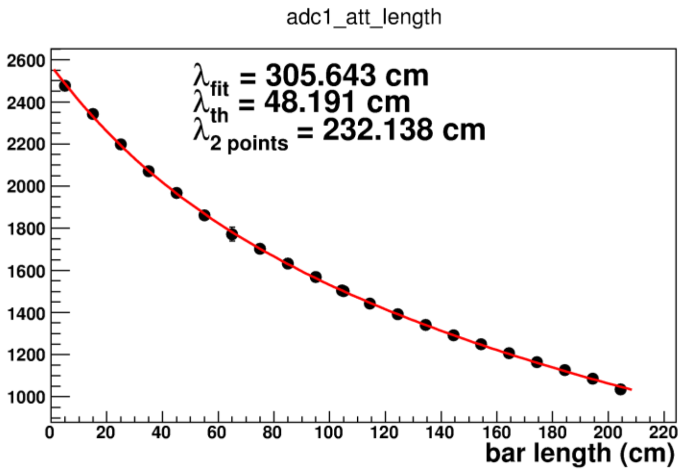
Squares – exp. data, circles – JM model, curves – EG

83

Light Attenuation



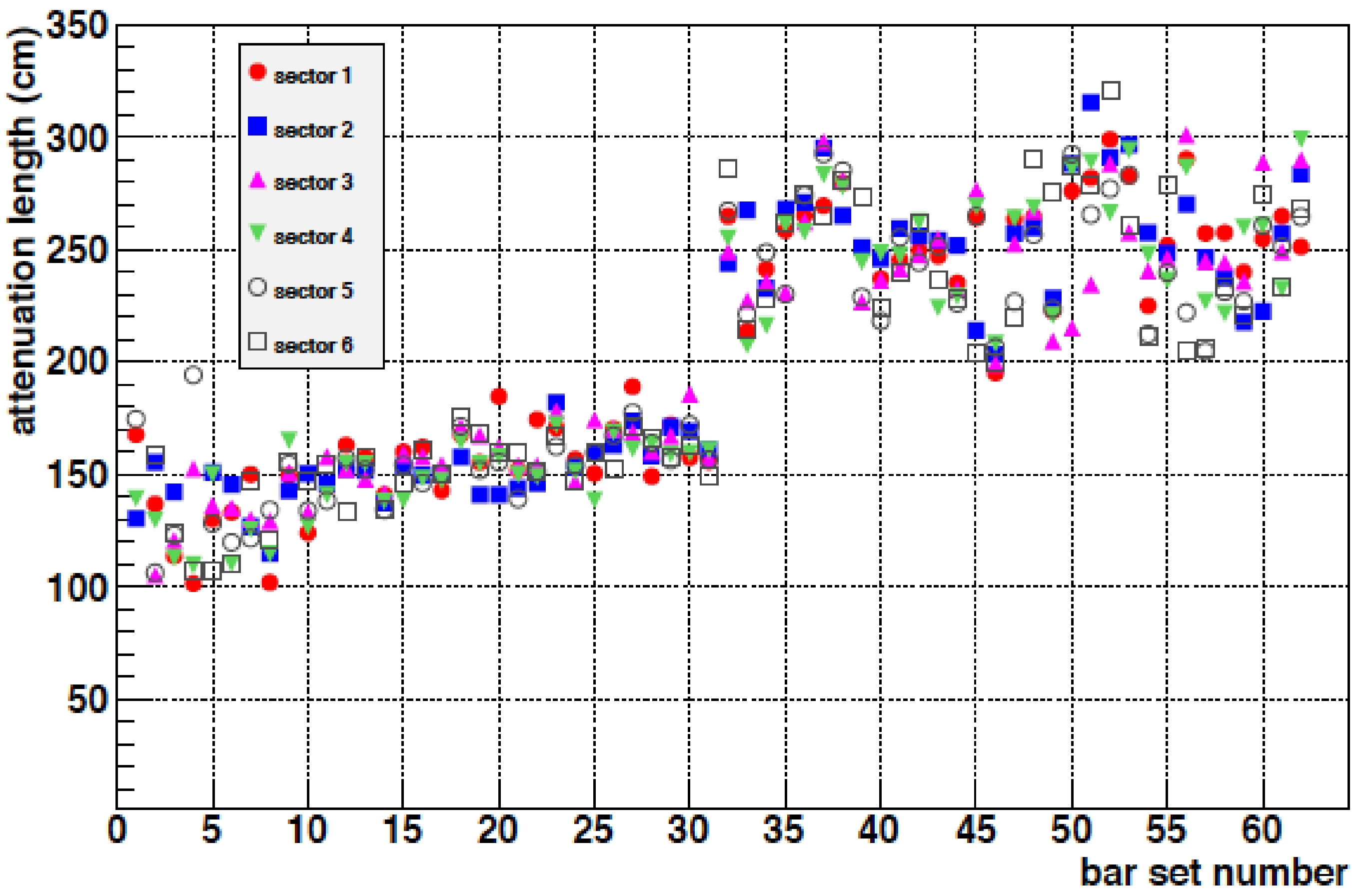




$$N = N_{0T} e^{-x/\lambda_{th}} + N_{0B} e^{-x/\lambda_B}$$

Results from maximum and Gauss-convoluted Landau fits for BC-408 are in good agreement, and the 2 point attenuation length is smaller than guaranteed by manufacturer.

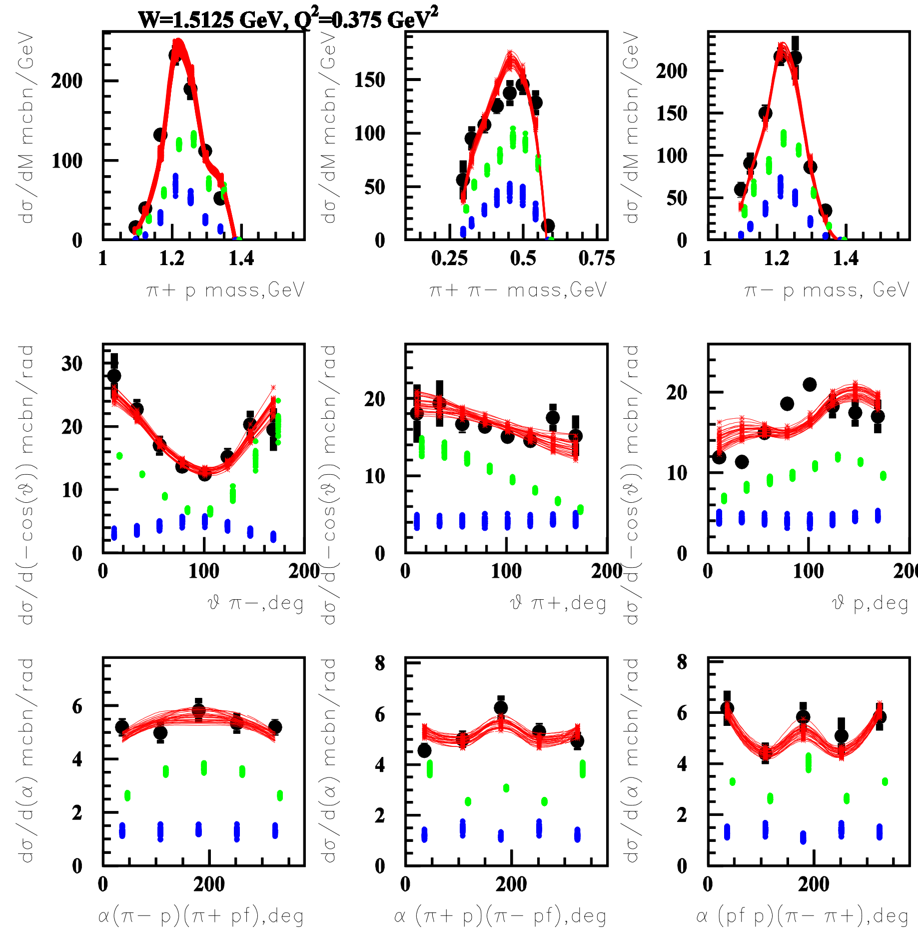
Light Attenuation



Average bulk attenuation length versus set number. Various symbols correspond to the different scintillators in the set or in other words to CLAS12 sectors.

Resonant & non-resonant parts of Nππ cross sections as determined from the CLAS data fit within the framework of JM model

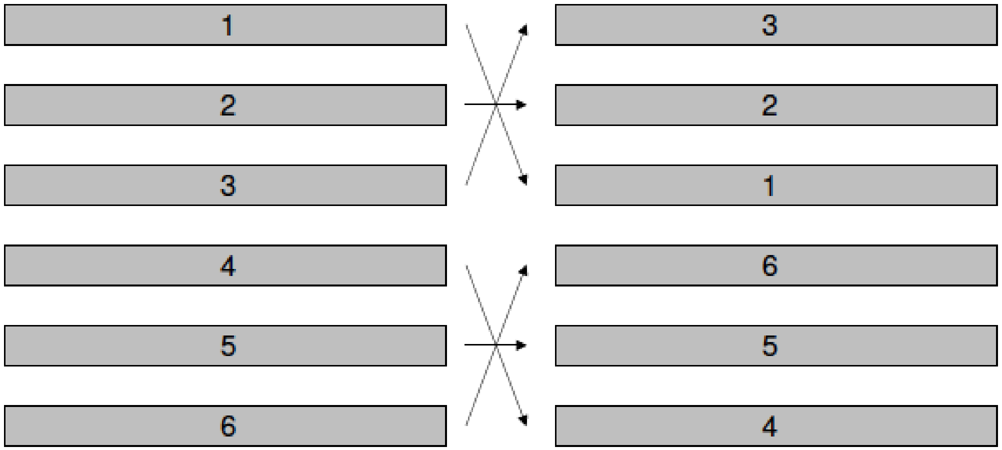
Reliable and unambiguous separation between resonant and non-resonant contribution was achieved



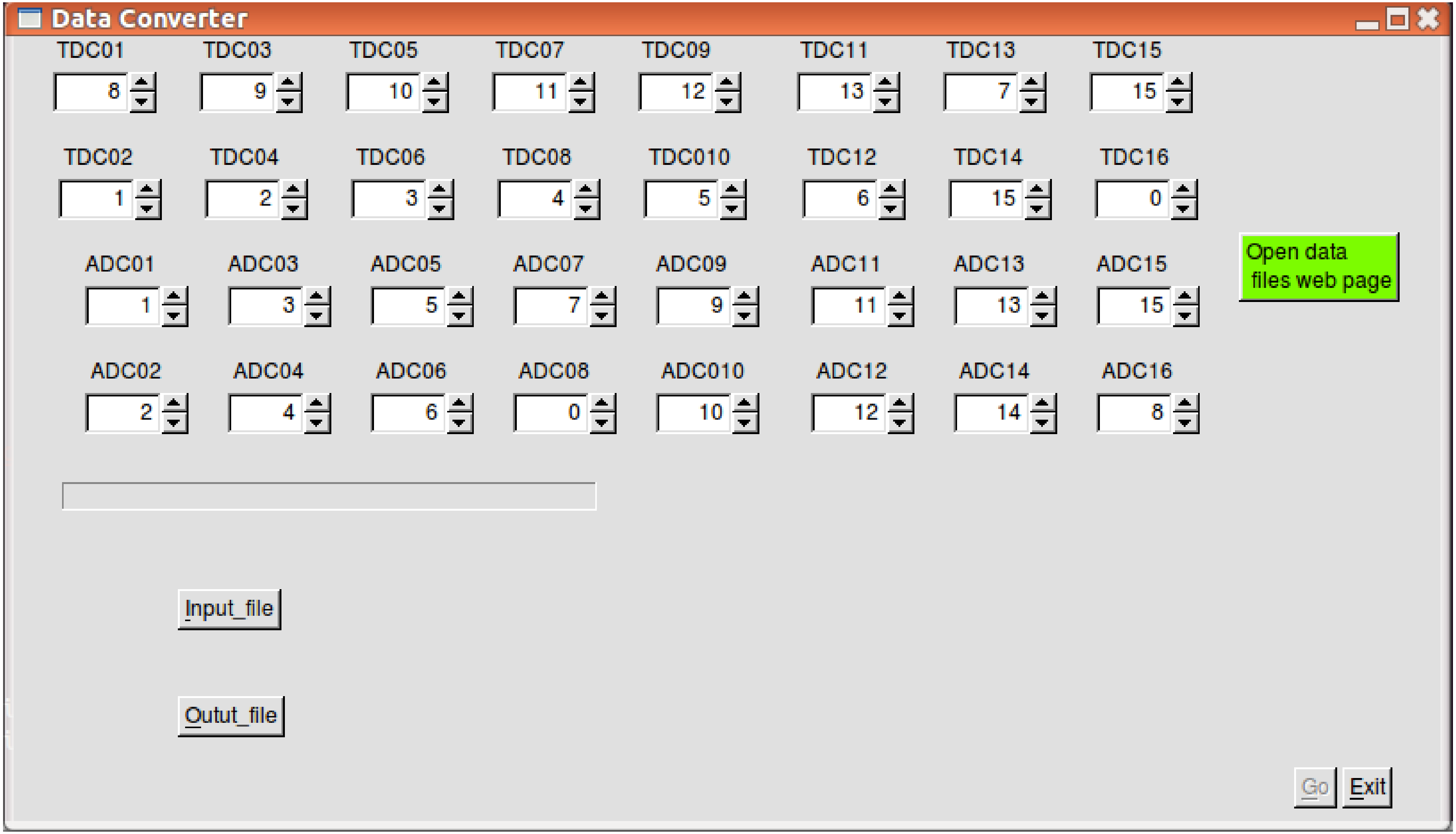
full cross sections resonant part non-resonant part

Normal and Complimentary Orders

Counter	# of Dependent T_k	Dependent Combinations
1	2	(1,2,3), (1,3,5)
2	3	(1,2,3), (2,3,4), (2,4,6)
3	4	(1,2,3), (2,3,4), (3,4,5), (1,3,5)
4	4	(2,3,4), (3,4,5), (4,5,6), (2,4,6)
5	3	(3,4,5), (4,5,6), (1,3,5)
6	2	(4,5,6), (2,4,6)

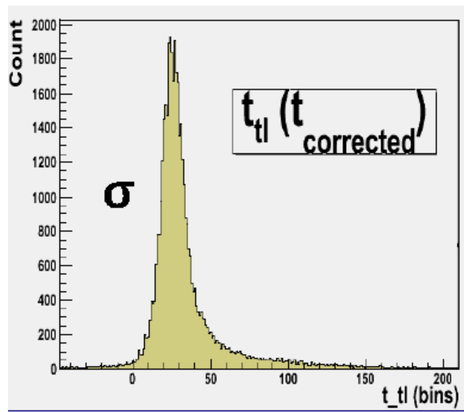


Data Converter GUI



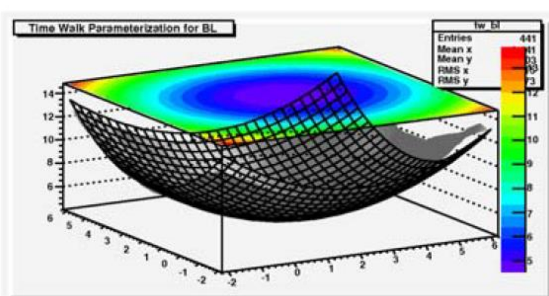
Position-Specific Time-Walk Corrections

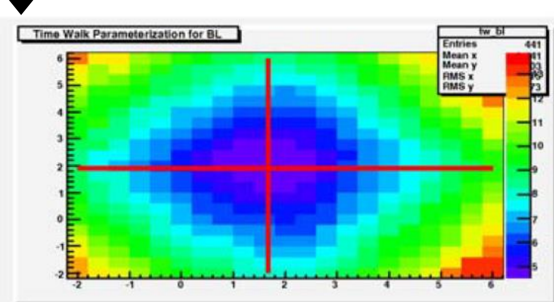
$$t_{corrected} = \left(TDC_i - \frac{\lambda_i}{\sqrt{ADC_i}}\right) - \left(TDC_{ref} - \frac{\lambda_{ref}}{\sqrt{ADC_{ref}}}\right)$$



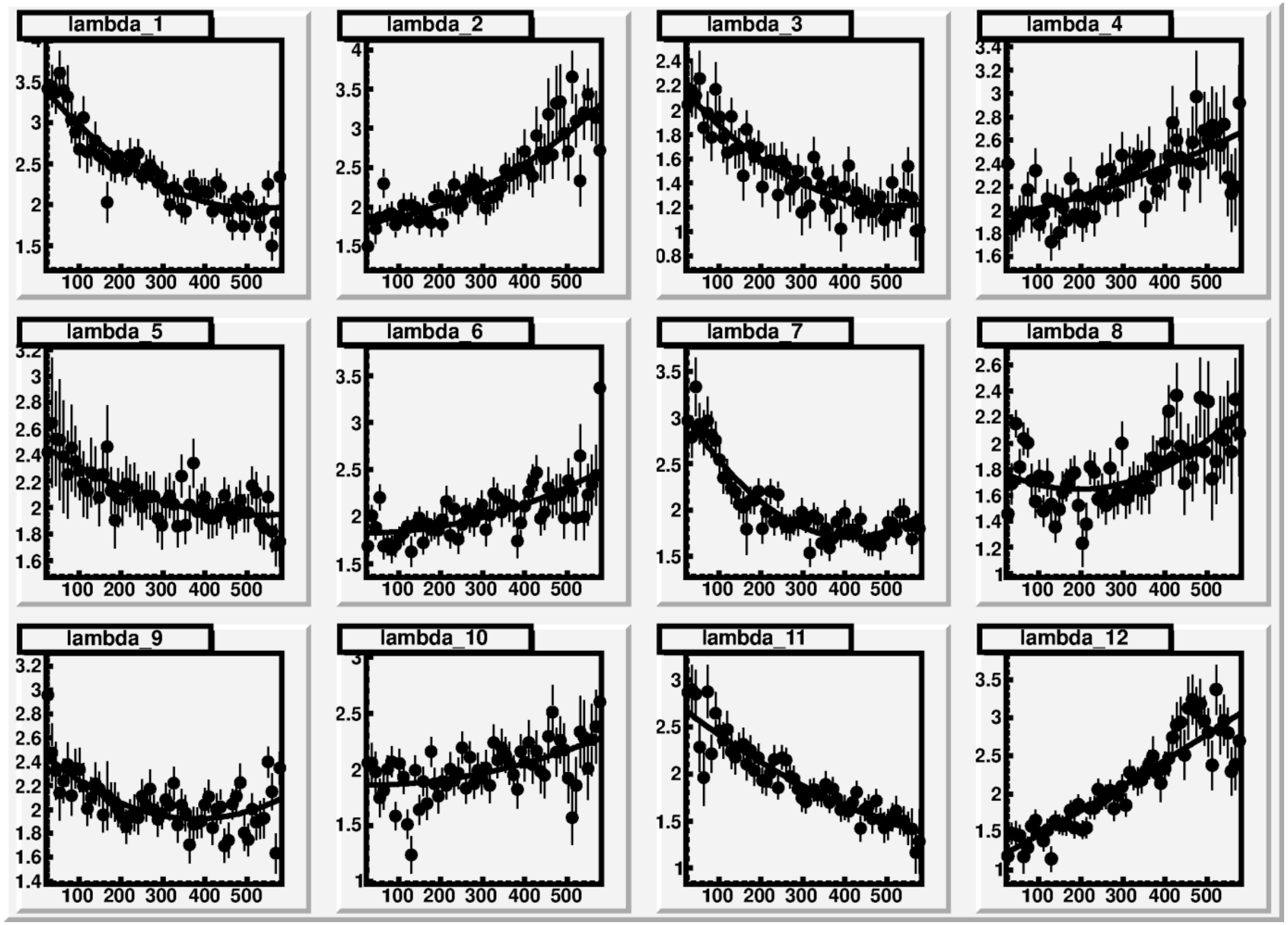
σ as color code

Paraboloid fit



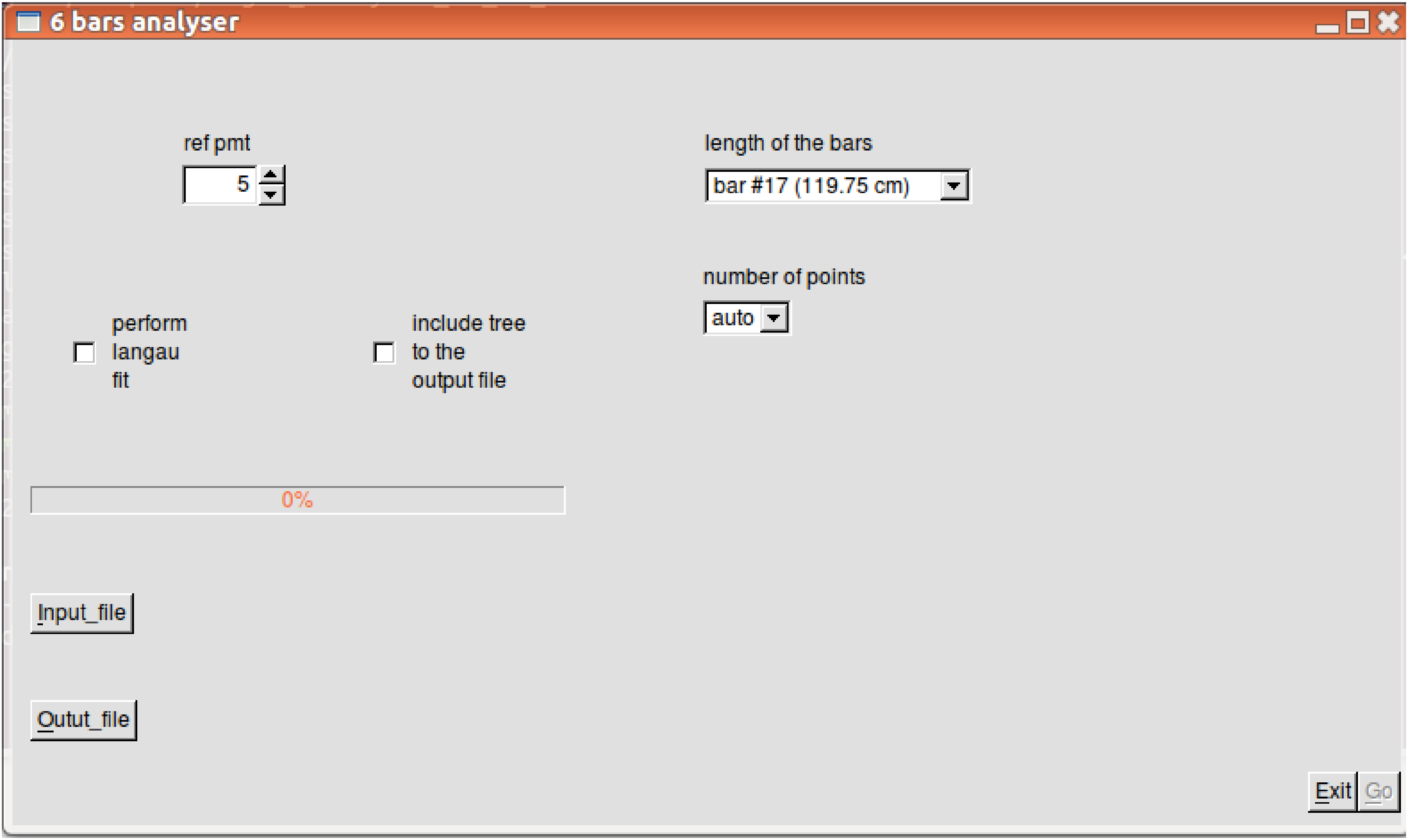


Time-Walk Parameters

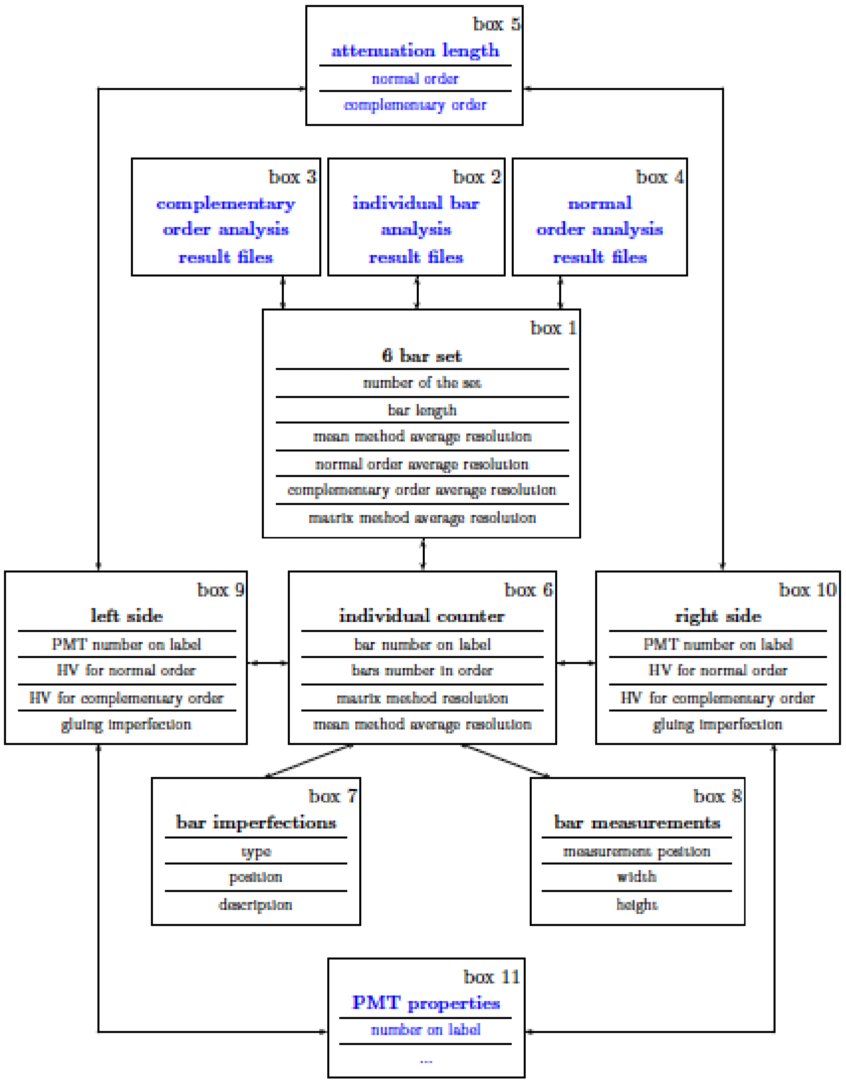


Fit of position dependence of time-walk parameters

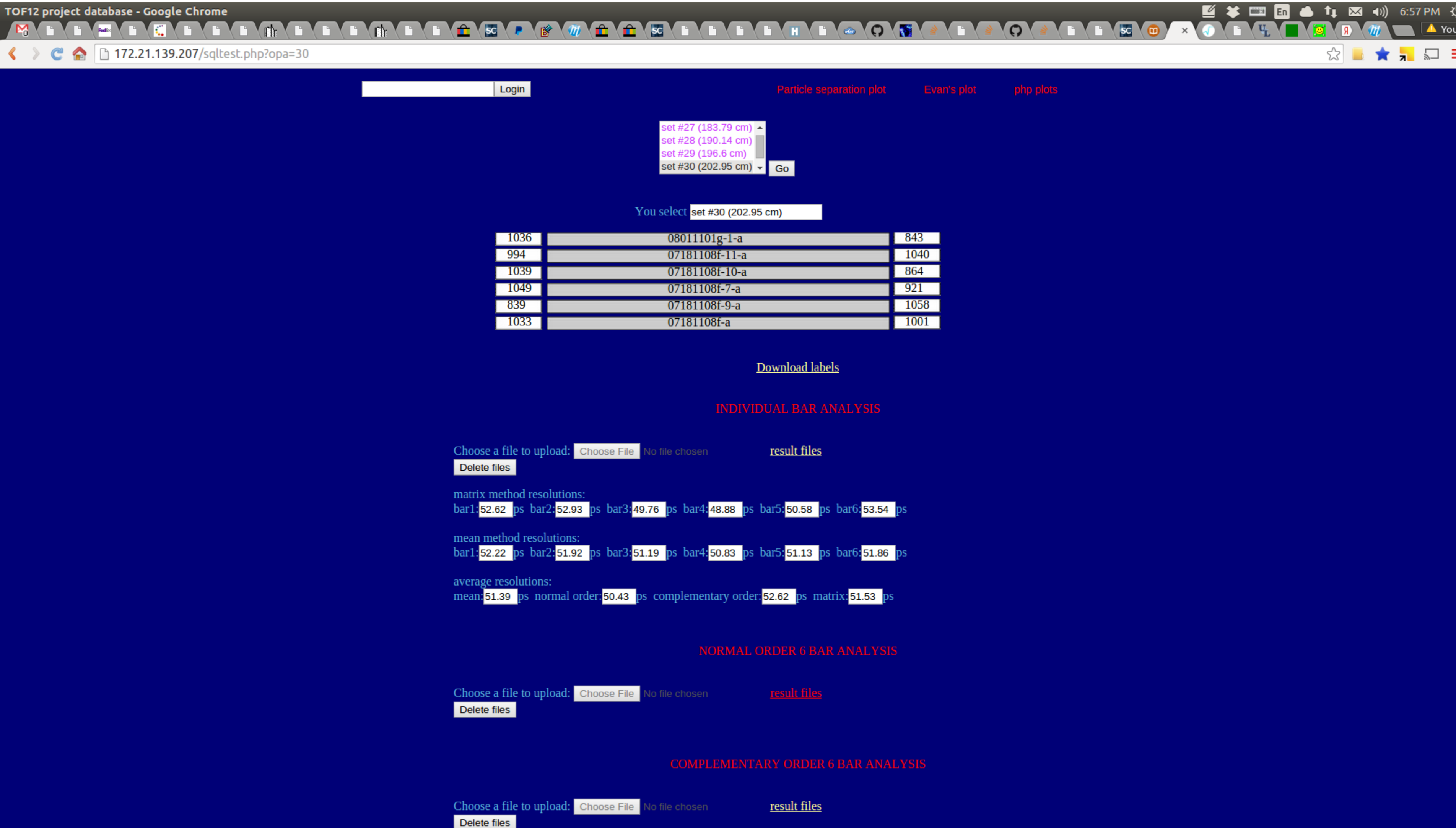
Time-Walk Corrections Automated Program Interface



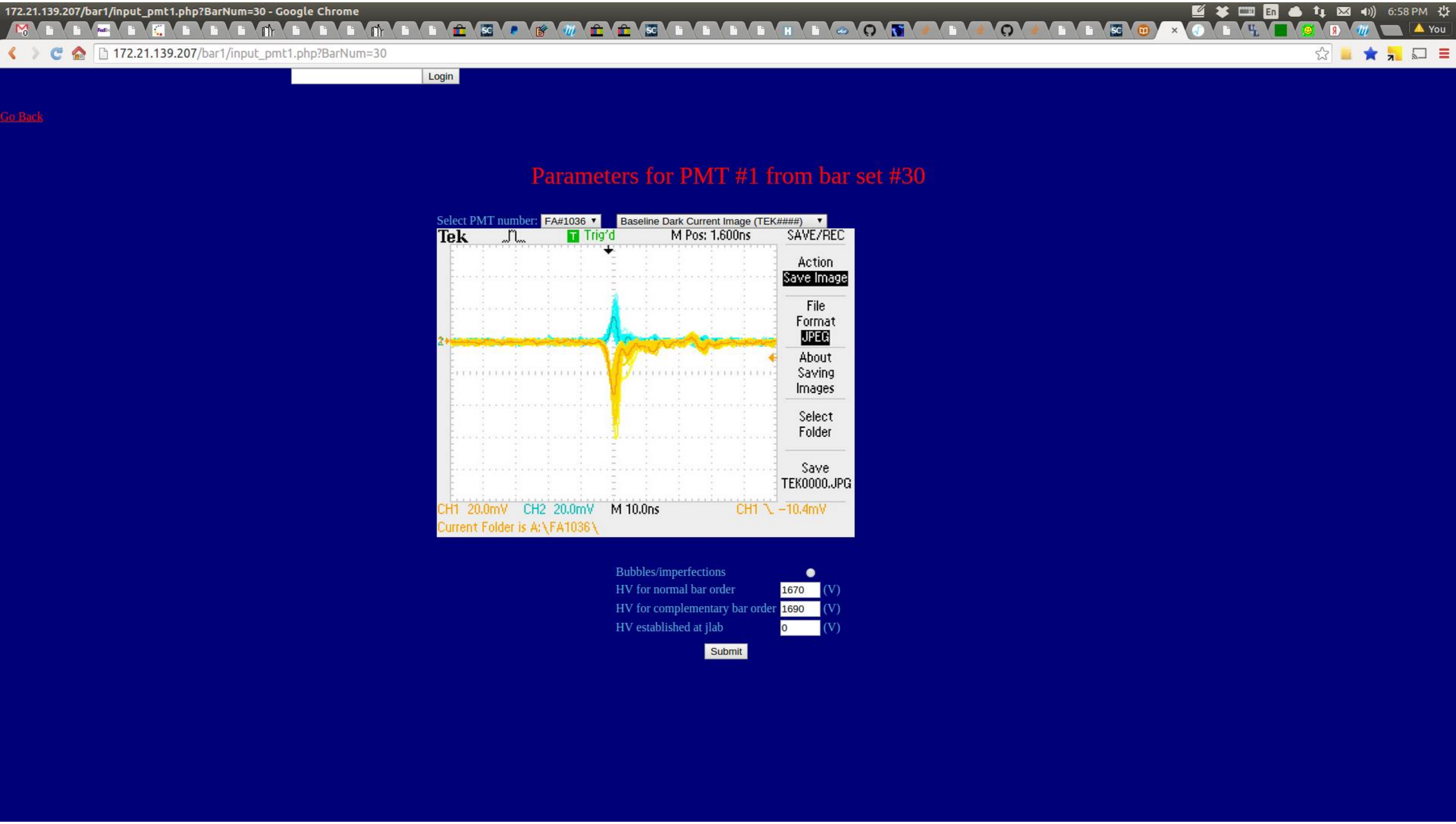
MySQL Database Scheme



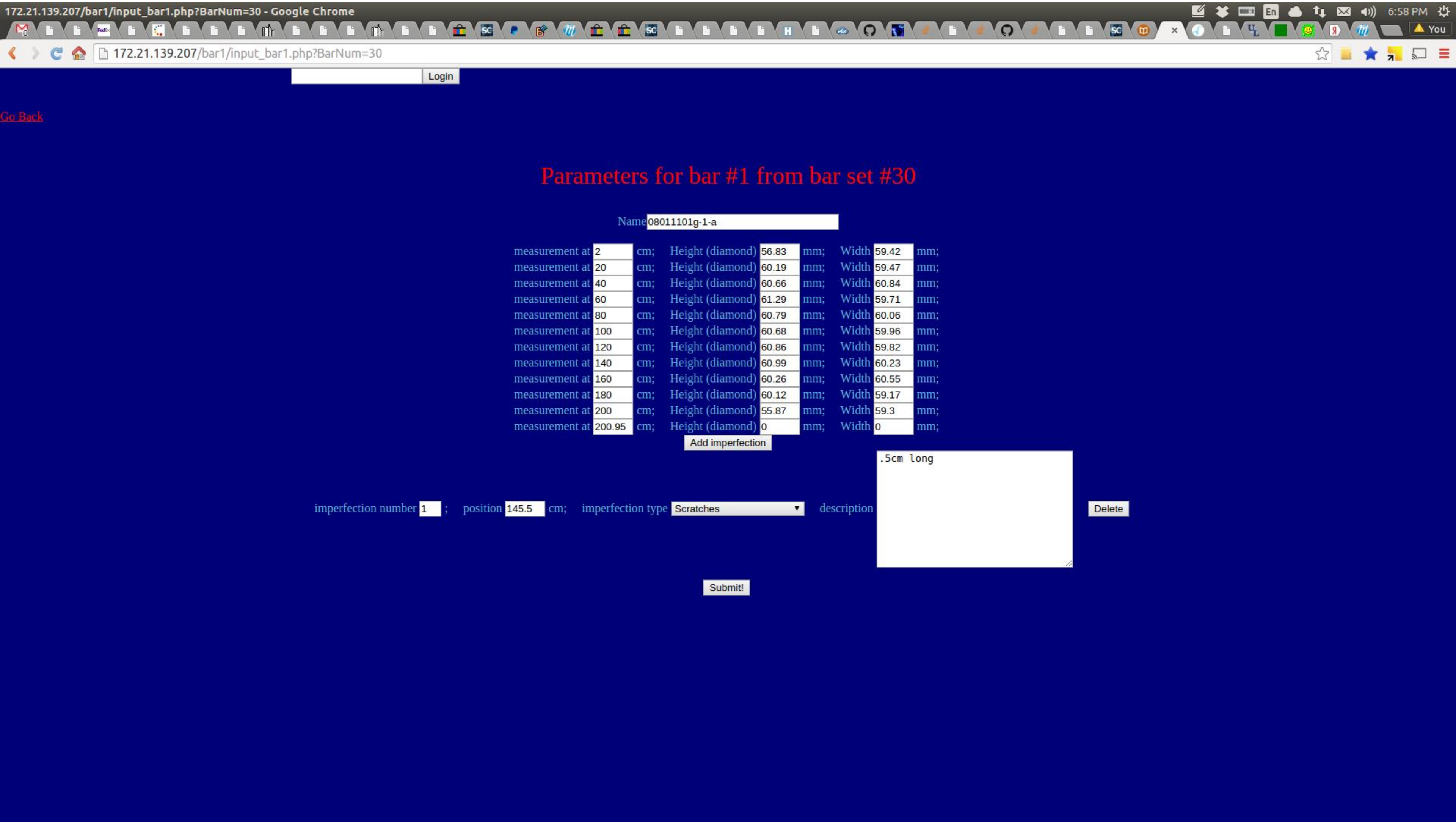
Database Main Page



Individual PMT Properties



Individual Scintillator Properties



172.21.139.207/bar1/input_bar1.php?BarNum=30 - Google Chrome

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Go Back

Parameters for bar #1 from bar set #30

Name 08011101g-1-a

measurement at		Height (diamond)		Width	
2	cm;	56.83	mm;	59.42	mm;
20	cm;	60.19	mm;	59.47	mm;
40	cm;	60.66	mm;	60.84	mm;
60	cm;	61.29	mm;	59.71	mm;
80	cm;	60.79	mm;	60.06	mm;
100	cm;	60.68	mm;	59.96	mm;
120	cm;	60.86	mm;	59.82	mm;
140	cm;	60.99	mm;	60.23	mm;
160	cm;	60.26	mm;	60.55	mm;
180	cm;	60.12	mm;	59.17	mm;
200	cm;	55.87	mm;	59.3	mm;
200.95	cm;	0	mm;	0	mm;

Add imperfection

imperfection number 1 ; position 145.5 cm; imperfection type Scratches description .5cm long Delete

Submit!

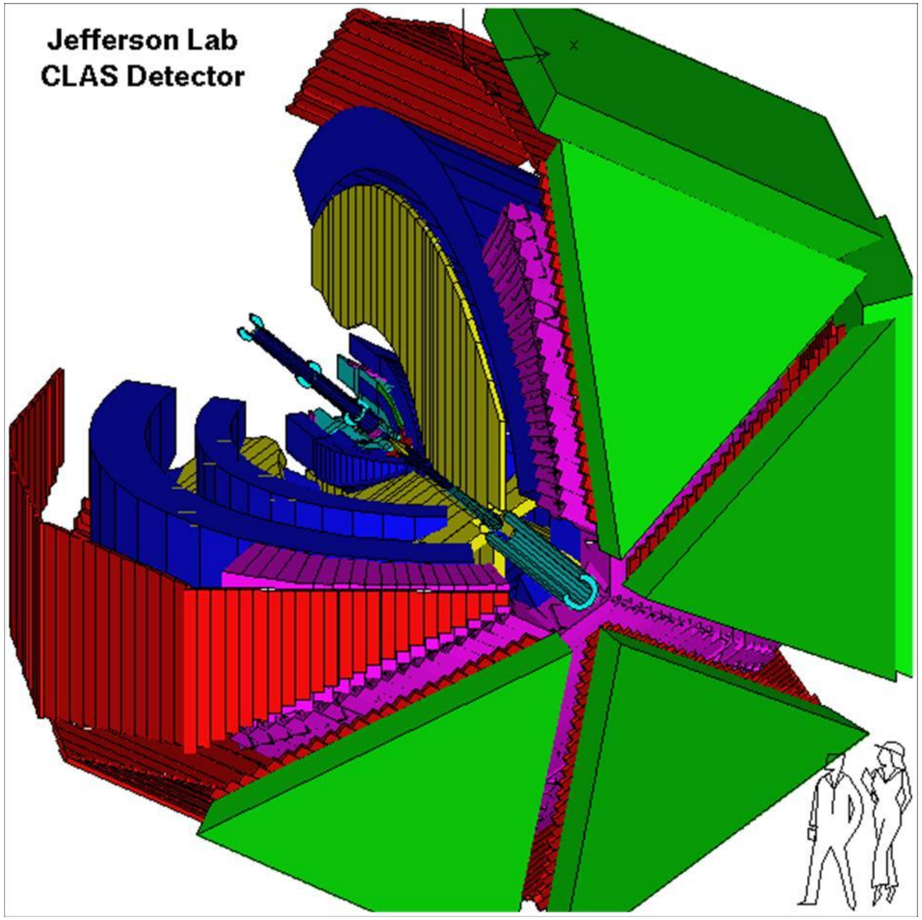
CEBAF Large Acceptance Spectrometer

Torus magnet
6 superconducting coils

Liquid D_2 (H_2) target;
e minitorus

Drift chambers
argon/CO_2 gas, 35,000 cells

Time-of-flight counters
plastic scintillators,
684 PMTs



Large angle calorimeters
Lead/scintillator, 512 PMTs

Gas Cherenkov counters
e/π separation, 216 PMTs

Electromagnetic calorimeters
Lead/scintillator, 1296 PMTs

The unique combination of the CEBAF continuous electron beam and the CLAS detector makes Hall-B@JLAB the only facility operational worldwide, that is capable of measuring unpolarized cross sections and polarization asymmetries of most exclusive meson electroproduction channels with substantial contributions at W < 3.0 GeV and $Q^2 < 5.0$ GeV^2.

Summary of the CLAS Data on Exclusive Meson Electroproduction off Protons in N* Excitation Region

Hadronic final state	Covered W-range, GeV	Covered Q^2-range, GeV^2	Measured observables
$\pi^+ n$	1.1-1.40 1.1-1.55 1.1-1.7	0.15-0.40 0.3-0.6 1.7-4.2	$d\sigma/d\Omega$ $d\sigma/d\Omega$ $d\sigma/d\Omega$, A_b
$\pi^0 p$	1.1-1.40 1.1-1.7 1.1-1.7	0.15-0.40 0.4-0.7 0.75-6.0	$d\sigma/d\Omega$ $d\sigma/d\Omega$, A_b,A_t,A_{bt} $d\sigma/d\Omega$
ηp	1.5-2.0	0.2-4.0	$d\sigma/d\Omega$
$K^+\Lambda$	1.65-2.35 1.65-2.35	0.65-2.55 1.4-2.6	$d\sigma/d\Omega$ P'
$K^+\Sigma^0$	1.7-2.1 1.8-2.5 1.7-2.6	0.5-2.55 1.5-3.50 1.8-3.50	$d\sigma/d\Omega$ P' $d\sigma/d\Omega$
$\pi^+\pi^- p$	1.3-1.6 1.4-2.1	0.2-0.6 0.5-1.5	Nine 1-fold differential cross sections

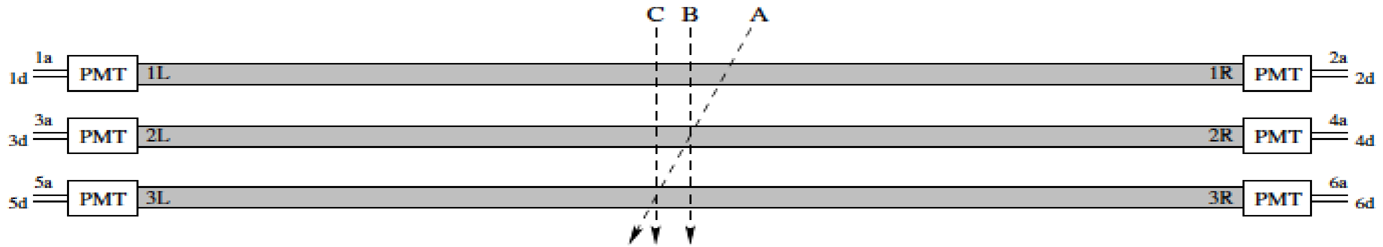
- $d\sigma/d\Omega$–CM angular distributions
- A_b,A_t,A_{bt}-longitudina beam, target, and beam-target asym-metries
- P' –recoil polarizatio of strange baryon

Almost full coverage of the final hadron phase space in πN, $\pi^+\pi^- p$, ηp, and KY electroproduction

The data are available in the CLAS Physics Database:

http://depni.sinp.msu.ru/cgi-bin/jlab/db.cgi

Three-Bar Method



Reference time is subtracted from the raw time reported by TDC $t = t_{raw} - t_{ref}$

The time at which cosmic-ray ray particle interacts with i[th] scintillator $t_i' = \frac{t_{iL} + t_{iR}}{2} - \frac{L}{2v_{eff}}$

Counter interaction time for each of the three counters are finally given by

$$\begin{aligned} t_t' &= \tau + \varepsilon_t, \\ t_m' &= \tau + \varepsilon_m + \delta, \\ t_b' &= \tau + \varepsilon_b + 2\delta, \end{aligned}$$

τ is the actual time at which the cosmic-ray particle interacts with top counter; ε_i are all errors contributions to the measured time; δ is the time it takes the cosmic ray particle to travel between adjacent counters.

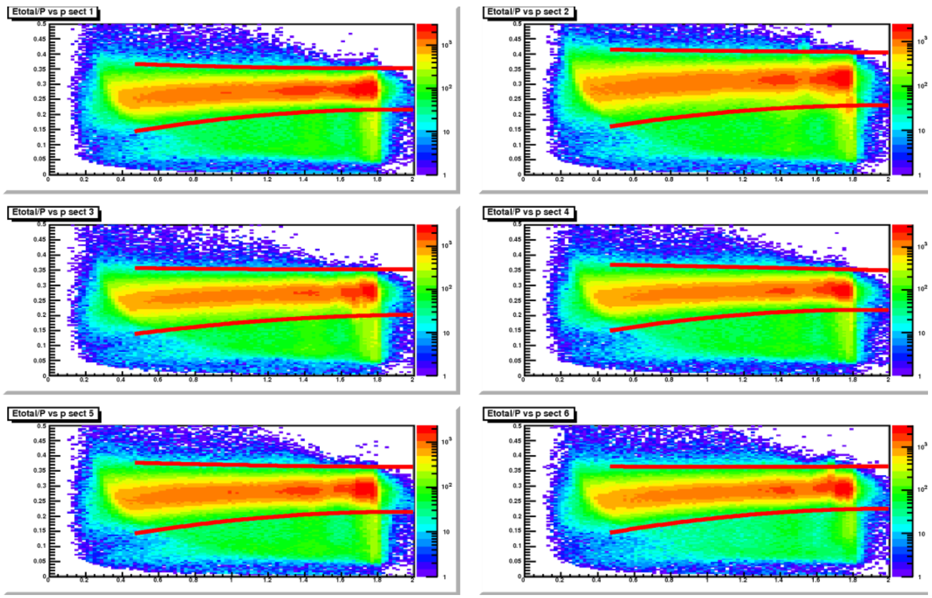
Quantity $T = \frac{t_t + t_b}{2} - t_m = \frac{\varepsilon_t + \varepsilon_b}{2} - \varepsilon_m$ is composed to be constant, only smeared by uncertainties ε_i

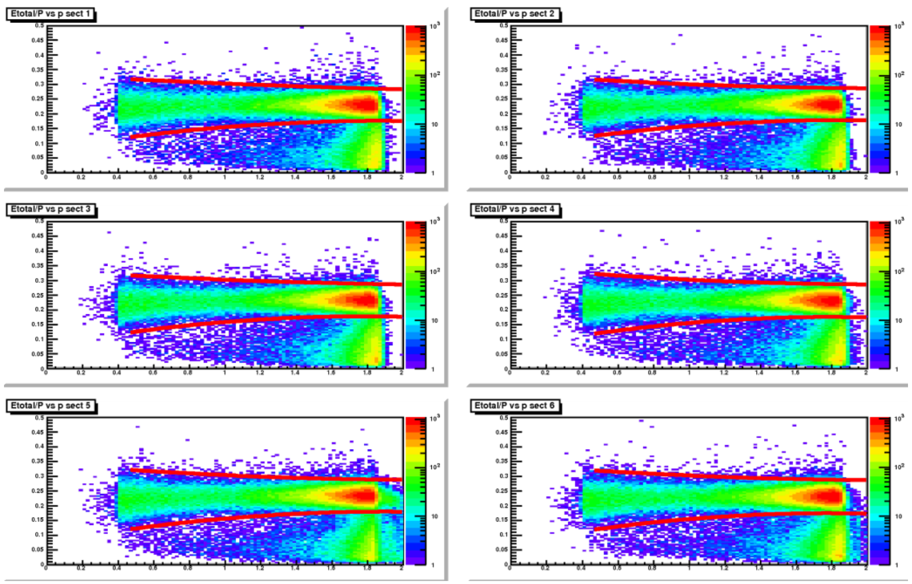
Statistical uncertainty in T is defined by $\sigma_T^2 = (\sigma_{\varepsilon_t}^2 + \sigma_{\varepsilon_b}^2)/4 + \sigma_{\varepsilon_m}^2$

Assuming all counters are identical $\sigma_\varepsilon = \sqrt{\frac{2}{3}}\sigma_T$

Electron identification

Electrons were identified as first in time particles, which produce signals in SC, DC, EC and CC in coincidence

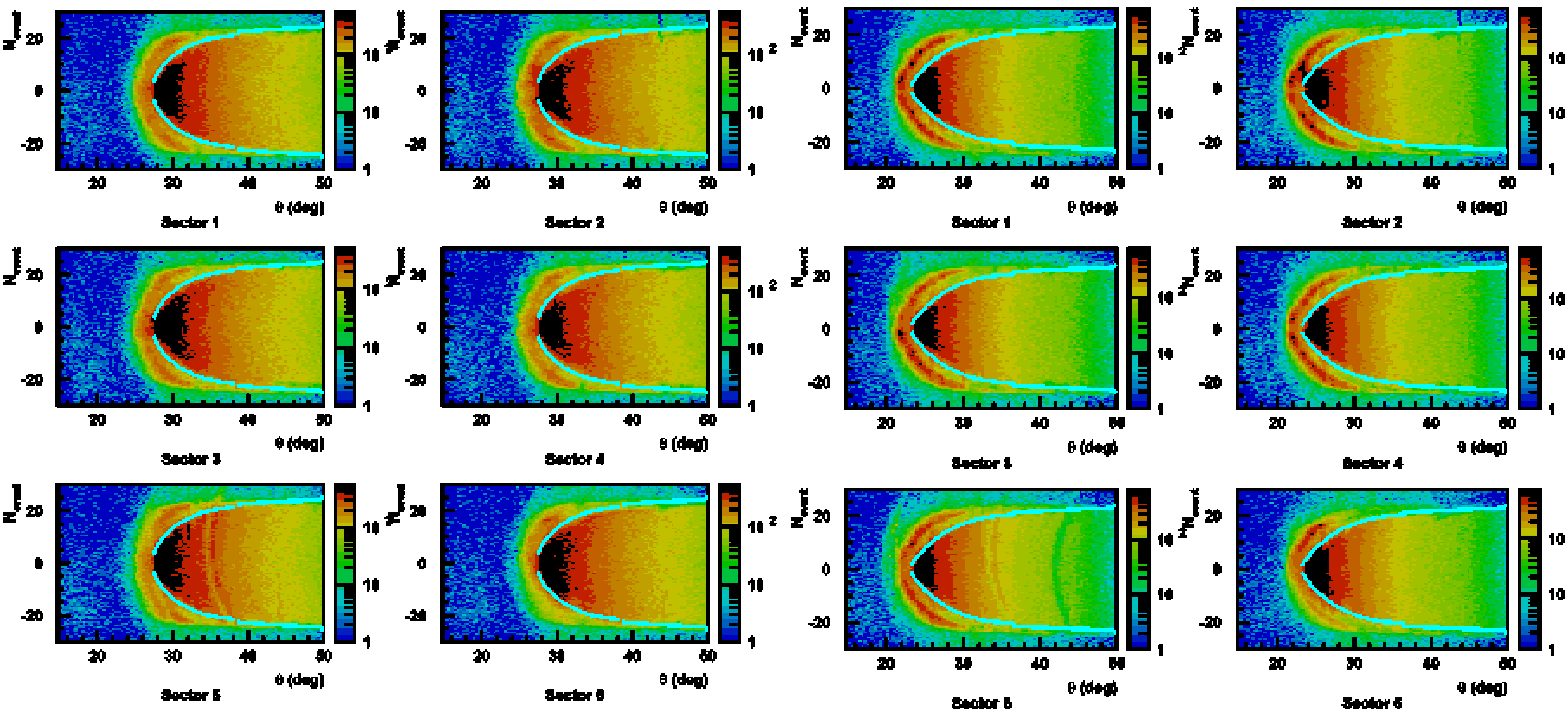




- Total energy deposited in calorimeter divided by electron momentum versus electron momentum.
- Left 6 plots represents real data distributions for 6 CLAS sectors, while 6 plots on the right side represent Monte-Carlo.
- We identify events between two red curves as electrons.

Electron fiducial cuts

$0.7<P_e<0.8$ GeV $1.0<P_e<1.1$ GeV



- Example of 2-dimensional distributions for electron.
- Electron momentum from 0.7 to 0.8 GeV (left plots) and from 1.0 to 1.1 GeV (right plots).
- Each plot represent one CLAS sector. Events in inside blue curves were accepted to analyze.

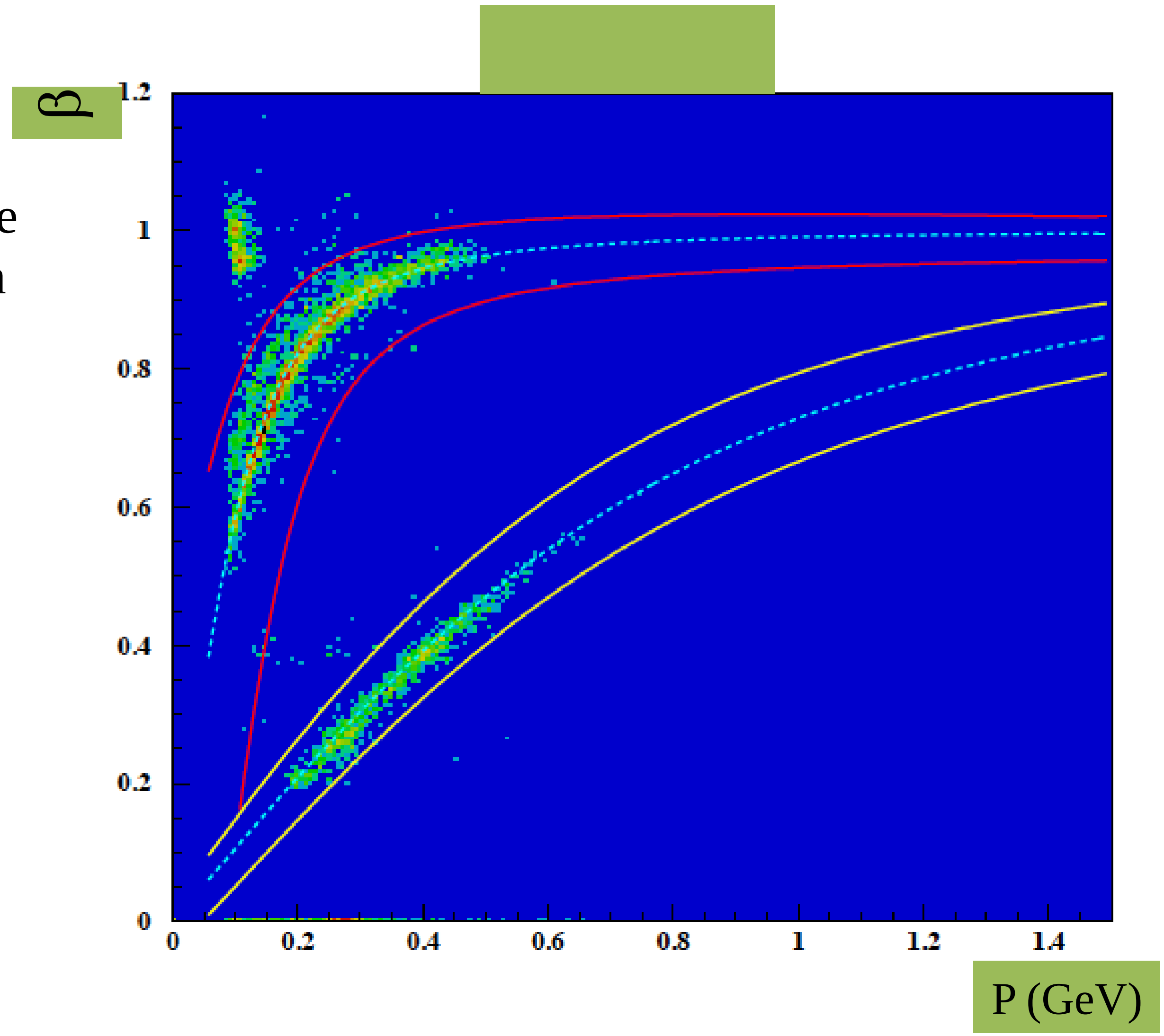
Hadrons identification

β vs momentum plotted for one particular TOF scintillator for positively charged particles

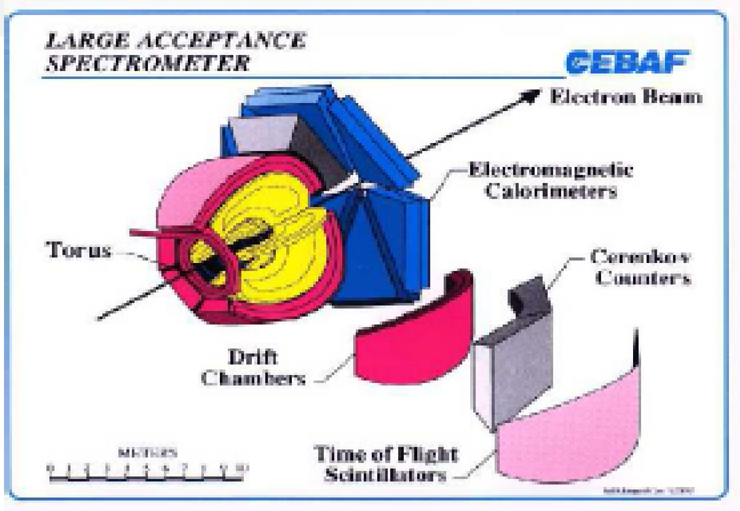
DC determines particle charge (track curvature), track length (L) and momentum

SC gives timing (t)

$\beta = L/(ct)$

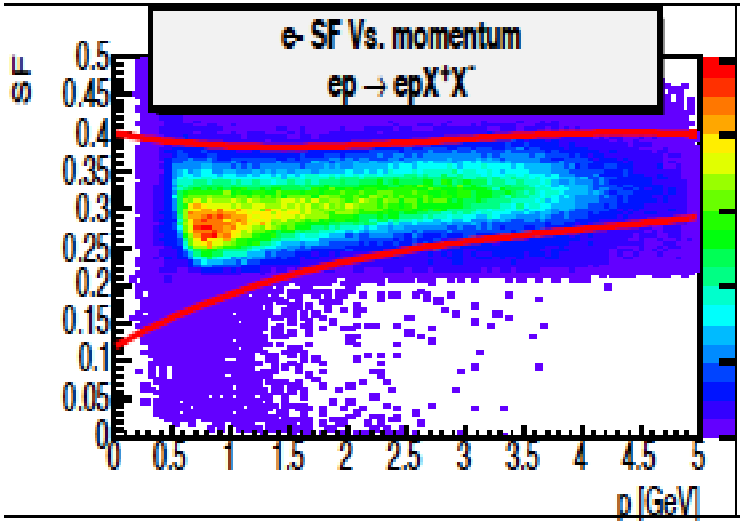


Particle identification

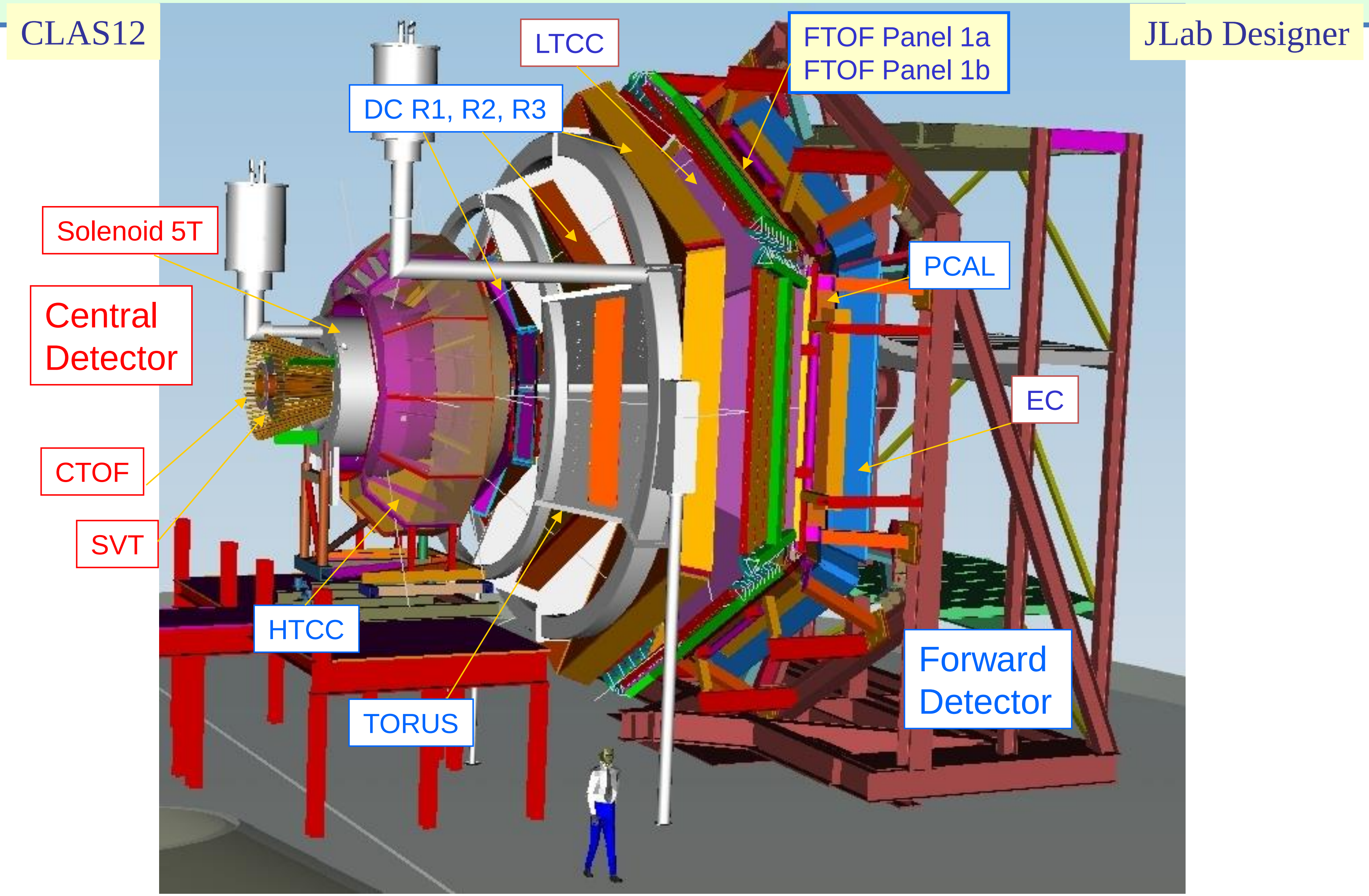


Electrons

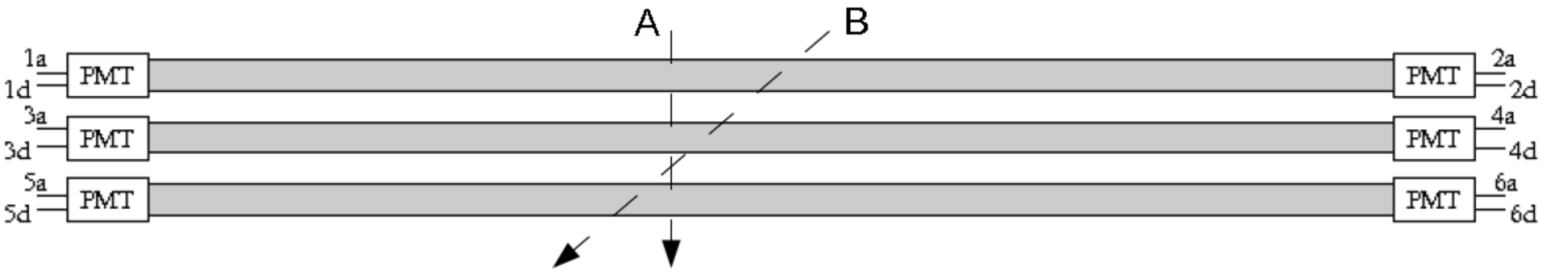
- Coincident "hit" in DC,CC,SC,EC
  - Only electrons trigger CC
- Additionally, use the EC
  - $SF = \frac{E_{EC}}{p} = \text{constant}$



Forward Time-of-Flight upgrade



Three-bar method



$$T = \frac{t_t + t_b}{2} - t_m = \frac{t_1 + t_2 + t_5 + t_6 - 4t_r}{4} - \frac{t_3 + t_4 - 2t_r}{2} = \frac{t_1 + t_2 + t_5 + t_6}{4} - \frac{t_4 + t_3}{2}$$

$$t_r = t_3$$

$$\sigma_{counter} = \frac{\sigma_{pmt}}{\sqrt{2}}$$

$$t_t = \frac{t_{tl} + t_{tr}}{2} \qquad t_{tl} = t_1 - t_r \qquad t_{tr} = t_2 - t_r$$

$$t_m = \frac{t_{ml} + t_{mr}}{2} \qquad t_{ml} = t_3 - t_r \qquad t_{mr} = t_4 - t_r$$

$$t_b = \frac{t_{bl} + t_{br}}{2} \qquad t_{bl} = t_5 - t_r \qquad t_{br} = t_6 - t_r$$

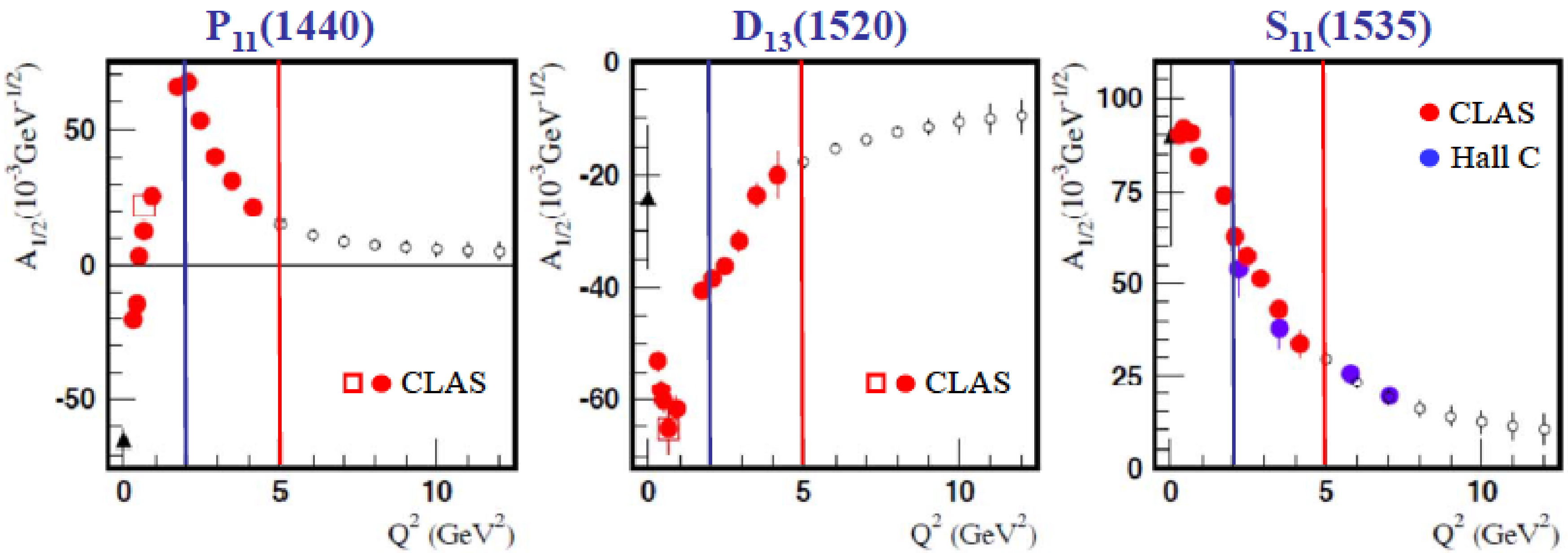
$$\sigma_{tb} = \sigma_1 = \sigma_2 = \sigma_5 = \sigma_6$$

$$\sigma_m = \sigma_3 = \sigma_4$$

$$\sigma_T^2 = \frac{1}{4}\sigma_{tb}^2 + \frac{1}{2}\sigma_m^2$$

$$\sigma_{counter} = \sqrt{\sigma_T^2 - \frac{1}{2}\sigma_{ref}^2}$$

Anticipated N* electrocouplings from Combined Analysis of Nπ/Nππ

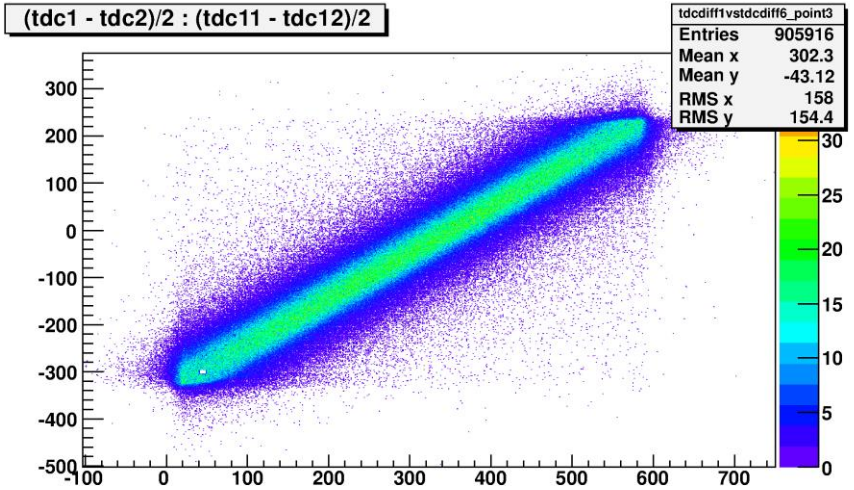


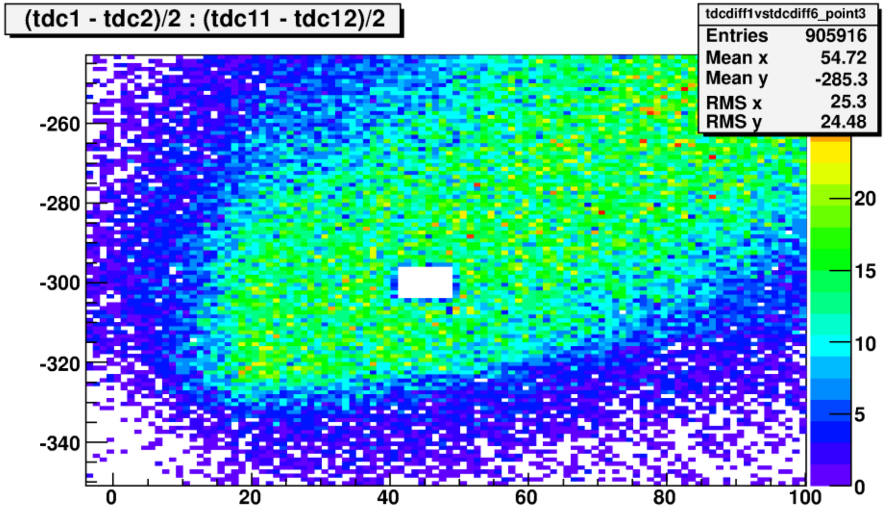
Open circles represent projections and all other markers the available results with the 6-GeV electron beam

- Examples of published and projected results obtained within 60d for three prominent excited proton states from analyses of Nπ and Nππ electroproduction channels. Similar results are expected for many other resonances at higher masses, e.g. $S_{11}(1650)$, $F_{15}(1685)$, $D_{33}(1700)$, $P_{13}(1720)$, …

- The approved CLAS12 experiments E12-09-003 (NM, Nππ) and E12-06-108A (KY) are currently the only experiments that can provide data on $\gamma_v NN^*$ electrocouplings for almost all well established excited proton states at the highest photon virtualities ever achieved in N* studies up to Q^2 of 12 GeV^2.

Position Cut

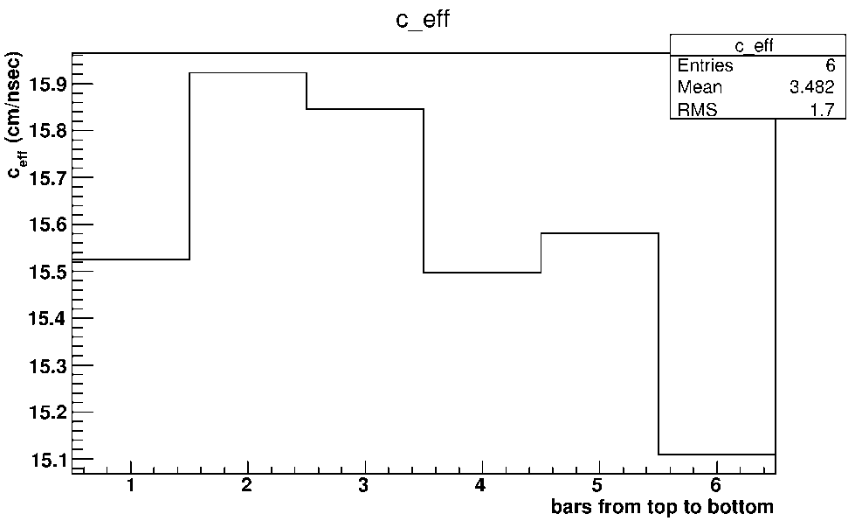




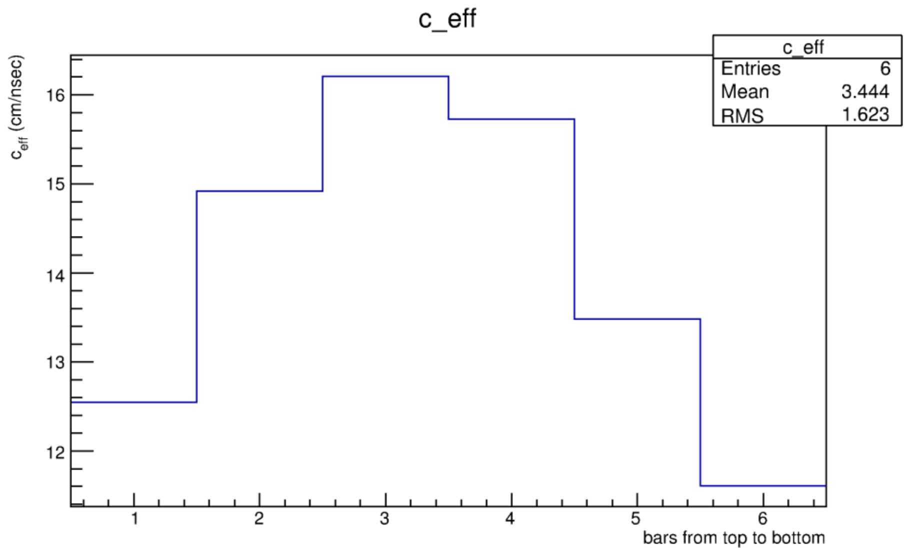
To select vertical tracks we bin over the top and bottom bars length.

Optimized bin width ~ 3cm

Effective speed of light



210 cm bars



30.08 cm bars

Single-Differential Cross Sections

$$\frac{d\sigma}{dM_{\pi^+\pi^-}} = \int \frac{d^5\sigma}{d^5\tau} d\tau^4_{M_{\pi^+\pi^-}}; \qquad d\tau^4_{M_{\pi^+\pi^-}} = dM_{\pi^+p} d\Omega_{\pi^-} d\alpha_{\pi^-}$$

$$\frac{d\sigma}{dM_{\pi^+p}} = \int \frac{d^5\sigma}{d^5\tau} d\tau^4_{M_{\pi^+p}}; \qquad d\tau^4_{M_{\pi^+p}} = dM_{\pi^+\pi^-} d\Omega_{\pi^-} d\alpha_{\pi^-}$$

$$\frac{d\sigma}{d(-cos\theta_{\pi^-})} = \int \frac{d^5\sigma}{d^5\tau} d\tau^4_{\theta_{\pi^-}}; \qquad d\tau^4_{\theta_{\pi^-}} = dM_{\pi^+\pi^-} dM_{\pi^+p} d\varphi_{\pi^-} d\alpha_{\pi^-}$$

$$\frac{d\sigma}{dM_{\alpha_{\pi^-}}} = \int \frac{d^5\sigma}{d^5\tau} d\tau^4_{\alpha_{\pi^-}}; \qquad d\tau^4_{\alpha_{\pi^-}} = dM_{\pi^+\pi^-} dM_{\pi^+p} d\Omega_{\pi^-}$$

$$d^5\tau = dM_{\pi^+\pi^-} dM_{\pi^+p} d\Omega_{\pi^-} d\alpha_{\pi^-}$$